C3X: Correlation, Causation and Controlled eXperimentation for C2

Dr. Paul J. Hiniker
Sr. Ops Research
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The Problem

How does one test causal hypotheses on C2 effectiveness against empirical evidence?

Causation

- All we observe are covariations.
  (David Hume, 1740)

- The causal interpretation of a simple(or partial) correlation depends upon

  - the presence of a compatible causal hypothesis

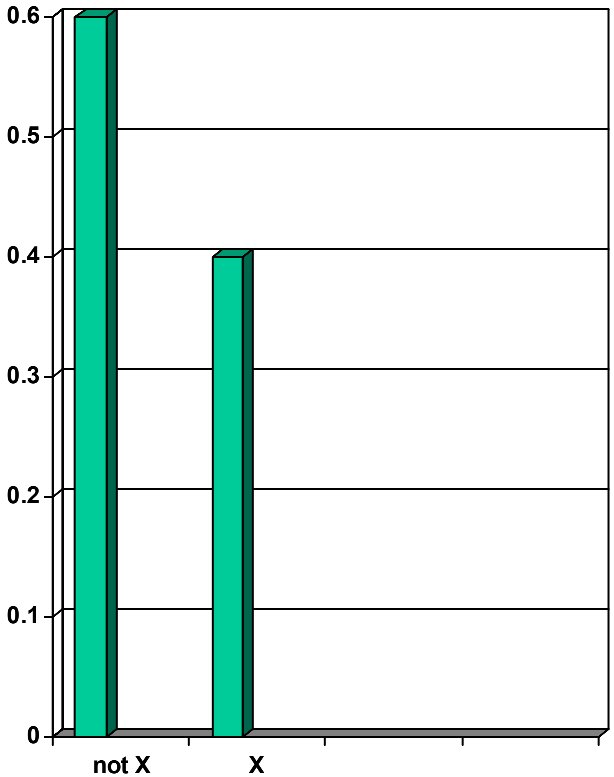
  - and the absence of a plausible rival hypothesis to explain the correlation on other grounds.

  (Herb Simon, 1957)

Causal Hypotheses & Correlation

Hypothesis: Fire engines prevent fire damage.

X---→Y $Y_X < Y_{not\ X}$ $\Phi_{XY} < 0$



	not X	X	
Y	.30	.20	.50
not Y	.20	.30	.50
	.50	.50	N=100

X = Fire engines = Fleet of 4+

Y =Percent of fires w/ damage > $500k(Expected)

40% < 60%

$$\Phi_{XY} = P_{XY} - P_X P_Y / \sqrt{(P_X Q_X P_Y Q_Y)} = -.20$$

Decomposing Correlations with Controls: Incendiary Fire Engines

X = Fire Engines Sent

Y = Fire Damage at Site

C = Size of Fire >1,000 ft

X ▸ Y

C → X, C → Y

	not X	X	
Y	.2	.3	.50
notY	.3	.2	.50
	.50	.50	100

$\Phi xy = .20$

notC

	notX	X	
Y	.2	.0	.20
notY	.6	.2	.80
	.50	.50	50

$\Phi xy\text{-}c = -.25$

C

	notX	X	
Y	.2	.6	.80
notY	.0	.2	.20
	.50	.50	50

$\Phi_{XYC} = -.25$

	notC	C	
Y	.1	.4	.50
notY	.4	.1	.50
	.50	.50	100

$\Phi_{YC} = .60$

	notC	C	
X	.1	.4	.50
notX	.4	.1	.50
	.50	.50	100

$\Phi_{XC} = .60$

Yule's (Covariance) Theorem for Dichotomous Attributes

$$\Phi_{XY} =$$

$$\Phi_{XY\text{-}C} P_{\text{-}C} \sqrt{(P_{X\text{-}C} Q_{X\text{-}C} P_{Y\text{-}C} Q_{Y\text{-}C} / P_X Q_X P_Y Q_Y)} + \Phi_{XYC} P_C \sqrt{(P_{XC} Q_{XC} P_{YC} Q_{YC} / P_X Q_X P_Y Q_Y)}$$

$$+ \Phi_{YC} \Phi_{XC}$$

- **For any two attributes, X and Y, and a third control attribute, C , the universal covariance can be decomposed into**

  a weighted average of the covariances within control subgroups , and, in addition

  a term involving the product of the covariances between Y and C and C and X.

**** N.B. In treating causality we assume, of course, that X and C are antecedent to Y.***

Controlled Experiment: Fire Engines Prevent Fire Damage

H: $X \overset{C}{\longrightarrow} Y$

	not X	X = TREATMENT	
EFFECT= Y	.32	.18	.50
not Y	.18	.32	.50
	.50	.50	N = 100 TRIALS

Random assignment of fleets(x) vs. few engines (not x)to fires:

- ***So*** $\Phi_{XC} = 0$.
- Since $\Phi_{XC} = 0$,

Experiment $\Phi_{XY} = (w' \Phi_{XY\text{-}C} + w \Phi_{XYC}) / 2 = \Phi_{XY.C}$, for $\forall$c, thus ruling out rival explanations.

- Experiment $\Phi_{XY} = -.28$
- So less fire damage is due to more fire engines on site.

Causal Modeling with Non-Experimental Data

- So to prevent spurious correlation, conduct of a controlled experiment guarantees $ø_{cx} = 0$ and ensures a valid test of a causal hypothesis.

- However, for non-experimental causal modeling,with one or more independent variables, one must verify that the residual error terms of all the variables are uncorrelated:

$$r_{Uy\,Uxi} = r_{Uxi\,Uxj} = 0, \text{ for all } X_i.$$

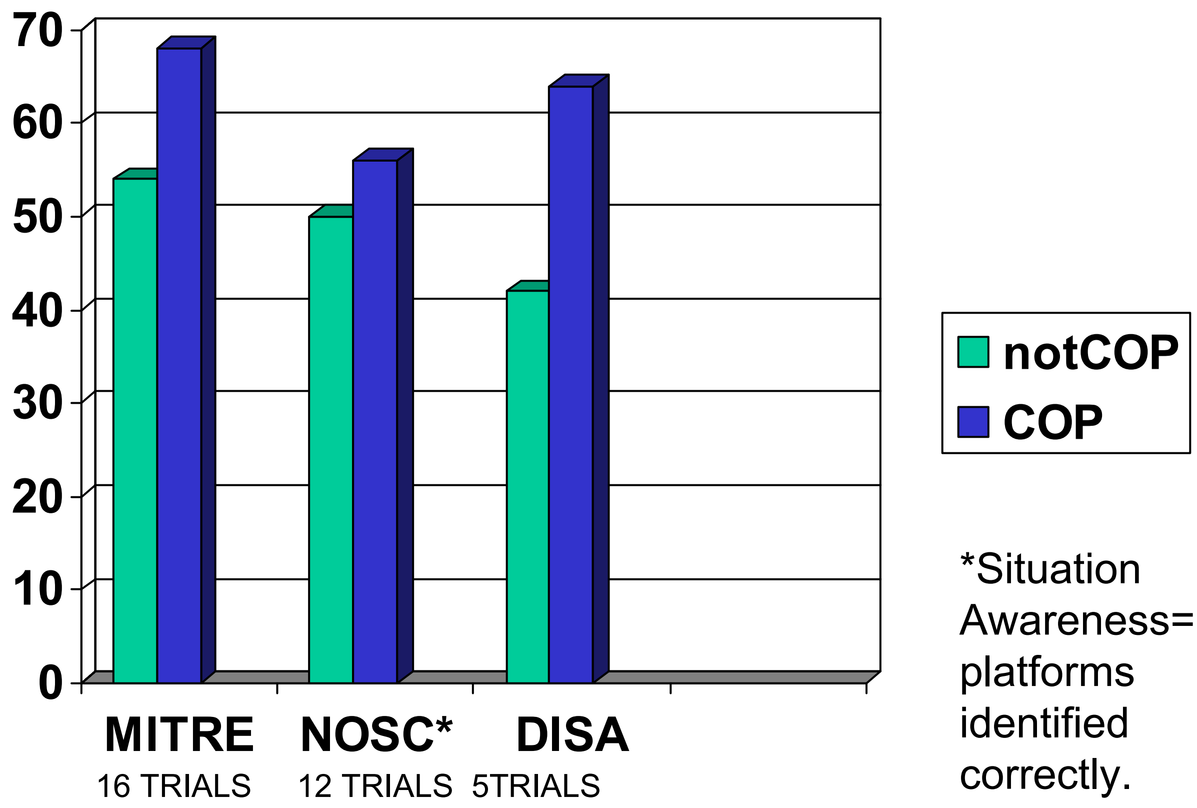
Otherwise, there could exist some extraneous variable(s), C_i, affecting both Y and X_i, hence forming part of u_y and u_{xi}, which would then be correlated; this would spuriously contribute to the correlations implied by the model.

- Thus simple correlation can neither prove nor disprove a causal hypothesis.

Controlled Experiments in C2

H: Use of a shared Common Operational Picture by a combat team(X) causes improved combat effectiveness(Y, in % platforms lost that are Red).

H: X ⟶ Y



*Situation Awareness= % platforms identified correctly.

Some Causal Hypotheses on NCW

- A basic assumption underlying most technological acquisitions for defense is the belief that the acquired capability will cause improved military effectiveness; therefore, controlled experimentation should be an integral part of the acquisition process.

- Net Centric Warfare (NCW) doctrine clearly includes such assumptions and several specific causal hypotheses such as the following:

H: Increased Shared Situation Awareness and Collaborative Planning by a distributed combat team causes increased decision loop speed and increased combat effectiveness.

- Such causal hypotheses warrant experimental testing.

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