

**Emergent Behaviour –  
Theory and Experimentation Using the MANA Model**

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## Abstract

In this paper we explore the relation between the local self-organisation of force elements, and the resultant emergent behaviour and effectiveness of the force. This is a fundamental question relating to Network Centric Warfare. We do this by developing a mathematical ‘metamodel’ of a complex and self-organising model of warfare named MANA. The mathematical metamodel makes a number of predictions about the emergent behaviour we should see. A large number of experimental runs and analysis of the simulation are then carried out, which show that the metamodel indicates the correct behaviour – our theory is not falsified. The paper then points the way forward to further work to complete this analysis, by considering the size distribution of clusters of self-organising units on the battlespace.

## Introduction

MANA (‘Map Aware Non-uniform Automata’) is a cellular automata based model of conflict which has been developed by the New Zealand Defence Technology Agency. Over several years we have been working with its main author, Dr Michael Lauren, through The Technical Co-operation Program (TTCP-JSA-TP3) on a number of theoretical developments related to MANA. These developments allow an analytical approach to the linkage between local clustering assumptions concerning the forces, and the resultant emergent behaviour seen in the battlespace. This emergent behaviour is measured in terms of both expected attrition across the whole simulated conflict and the dynamics of the time-series of casualties over simulated time.

Information Age Forces. The approach is particularly relevant to dispersed, dynamically evolving ‘Information Age’ scenarios, involving local force clustering and reclustered (a form of local self-organisation). This is in contrast to the force-on-force attrition approach to ‘Industrial Age’ warfare typified by Lanchester’s equations. In [1], Darilek, Perry *et al* discuss the implications of this thinking for the ‘Information Age’ US Army, in which autonomous units of force move and cluster across a dispersed battlespace, producing the effect of battlespace control.

Network Centric Warfare. The linkage between such local self-organisation and global emergent behaviour lies at the core of ideas such as Network Centric Warfare. This relationship is discussed particularly in the most recent book by Alberts and Hayes [2]. Initial mathematical ideas are also discussed in the book by Moffat [3]. In [3], Chapter 4, we also show how the relationship between local autonomous unit clustering and overall emergent battlespace control can be quantified using the same theoretical approach developed here for casualties. This relates to the Network Centric Warfare form of Command and Control [2] in which Command sets the initial conditions (the properties of the autonomous units etc.) and Control is an emergent phenomenon related to the way in which these units interact together in a self-organising way.

This paper pulls together a number of research developments which have occurred recently, and which were thus not folded into Reference [3]. The first of these is a coherent description of an analytical approach to emergent behaviour in the MANA model, based on work by Lauren and Moffat. We refer to this analytical approach as our ‘theory’. Subsequently, the experimental evidence we have established so far is described. This evidence consists of a number of runs of the MANA model across a number of generic

‘scenarios’ in order to test the theory. Finally we discuss the implications of these results and point the way forward.

### Theoretical Analysis

The first part of our analysis is based on the approach of Lauren [4] made more rigorous by applying the dimensional analysis of Barenblatt [5]. Consider a Blue agent in a cellular automata model of conflict, travelling at a velocity  $v$  for a time  $\Delta t$ . Let  $d = v\Delta t$ . If the Red force is fractally clustered, then by definition of the fractal dimension  $D$  of these forces, the probability of encountering a Red cluster in this box of width  $d$  is  $\propto (v\Delta t)^{-D}$  since this is directly related to the probability of this box being occupied by one or more Red agents. If now  $n_d =$  mean number of Red agents in an occupied box of side  $d$ , then the number of encounters with Red agents is  $\propto (v\Delta t)^{-D} n_d$ . We assume that we can approximate  $n_d$  by the expected size of a cluster of Red agents at time  $t$  times the number of such clusters.

At a given time  $t$  we assume that the distribution of Red clusters of size  $x$  is given by  $\varphi(x, t)$ . Let the mean value of the cluster size distribution be  $\bar{\varphi}(x, t)$  and assume it is normalised such that if  $N(t)$  is the number of clusters at time  $t$  then  $N(t)\bar{\varphi}(x, t) = R(t)$  where  $R(t)$  is the number of Red agents at time  $t$ . Returning to our reasoning above, we have that the number of encounters by a Blue agent is given by the expression  $\propto (v\Delta t)^{-D} N(t)\bar{\varphi}(x, t)$ .

The expected attrition rate of Blue agents,  $\Delta B$  over the time period  $\Delta t$ , is then given by;

$E(|\frac{\Delta B}{\Delta t}|) \propto$  (unit kill probability of Red)x (Probability of meeting a cluster in time period  $\Delta t$ )x (mean cluster size)x (number of clusters) where  $E$  denotes the expectation (i.e. the mean). Thus;

$$E(|\frac{\Delta B}{\Delta t}|) = f(k, (\Delta t)^{-D}, N(t)\bar{\varphi}(x, t))$$

(We have dropped the variable  $v$  since this is assumed constant).

Applying the dimensional balancing approach of Barenblatt, we then set up the Fundamental Gauge Class  $F =$  Force,  $T =$  Time and  $L =$  Length. In terms of these dimensions, we have;

$$[\frac{\Delta B}{\Delta t}] = FT^{-1}$$

$$[k] = FT^{-1}F^{-1} = T^{-1}$$

$$[(\Delta t)^{-D}] = T^{-D}$$

$$[N(t)\bar{\varphi}(x, t)] = F$$

Thus, equating dimensions on both sides of the relationship, we have;

$$\begin{aligned} FT^{-1} &= (k)^\alpha (\Delta t^{-D})^\beta (N(t)\bar{\varphi}(x, t))^\gamma \\ &= T^{-\alpha} T^{-\beta D} F^\gamma \end{aligned}$$

Hence;

$$\begin{aligned} \gamma &= 1 \\ \alpha + \beta D &= 1 \end{aligned}$$

and we can write  $\alpha = q(D)$ ,  $\beta D = r(D)$ , with  $r(D)$  and  $q(D)$  linear functions of  $D$ .

$$\text{Hence, } E\left(\left|\frac{\Delta B}{\Delta t}\right|\right) \propto k^{q(D)} \Delta t^{-r(D)} N(t) \bar{\varphi}(x, t) \dots \dots \dots (1)$$

with  $q(D)+r(D)=1$ .

**Correspondence Requirement**

When there is no dispersed fractal clustering, and Red agents form a single unit of force (i.e. one single cluster) we require that the relation at (1) reverts to a form of Lanchester equation. In this case, the mean cluster size,  $\bar{\varphi}(x, t) = R(t)$  for all time  $t$ , and the number of clusters  $N(t)=1$ . The fractal dimension  $D=0$ , corresponding to a single unit of force at a point. It follows from above that  $\alpha = q(D) = 1$  and  $\beta D = r(D) = 0$ . Thus in equation (1) we have  $E\left(\left|\frac{\Delta B}{\Delta t}\right|\right) \propto kR(t)$  which is of Lanchester square law form. In addition, if there is no clustering, and the Red agents are uniformly spread across the battlespace, corresponding to  $D=2$ , then  $\alpha = 1$  and  $\beta = 0$  is the only integer (non-fractal) solution, corresponding to a Lanchester square law again.

**Correlation in Time**

We assume that the system is similar to a turbulent flow system where turbulence is multiscaling (in the sense that the effect is recursive at different levels of system resolution). This leads to the assumption of *multifractal statistics*. This means that the form of expression above applies in a analogous way to all of the moments of the random variable  $\left|\frac{B(t + \Delta t) - B(t)}{\Delta t}\right|$ . Thus we have that;

$$E\left(\left|\frac{B(t + \Delta t) - B(t)}{\Delta t}\right|^p\right) \propto \Delta t^{f(D,p)}$$

where  $f$  is a function only of the battlespace casualties fractal dimension  $D$  and the order of the moment  $p \geq 1$ .

In particular, (noting that  $\Delta t$  is a constant relative to the expectation  $E$  over time  $t$ ), the second moment ( $p=2$ ) should have the relationship;

$$E\left(\left|\frac{B(t + \Delta t) - B(t)}{\Delta t}\right|^2\right) = \frac{1}{\Delta t^2} E(|B(t + \Delta t) - B(t)|^2) \propto \Delta t^{f(D,2)}$$

and hence;

$$E(|B(t + \Delta t) - B(t)|^2) \propto \Delta t^{f(D,2)+2}$$

Initial work by Lauren [6], [7], indicates that this relationship is in fact of the form;

$$E(|B(t+\Delta t) - B(t)|^2) \propto \Delta t^D \dots\dots\dots(2)$$

In other words,  $f(D, 2) + 2 = D$ . We recall that in equation (2),  $B(t)$  is the number of Blue casualties at time  $t$ ,  $E$  denotes expectation over time  $t$ , and  $D$  is the fractal dimension associated with Red casualty locations on the battlefield, corresponding to local clustering of Red agents. In this way, clustering in time of Blue casualties is related to clustering in space of Red (and hence Red casualties) on the battlefield.

**Swarming**

In the Network Centric Warfare context, we can hypothesise that agile clustering and re-clustering of forces by Red (such as *swarming* effects), measured by the Red fractal dimension  $D$ , creates clustering in time of Blue casualty effects and hence a particular fractal based form for the plot of Blue casualties over time.

If we expand the square term on the left hand side of Equation 2, we have;

$$\begin{aligned} E(|B(t+\Delta t) - B(t)|^2) &= E[B(t+\Delta t)^2 - 2B(t)B(t+\Delta t) + B(t)^2] \\ &= 2s^2 - 2\text{corr}(B(\Delta t)) \end{aligned}$$

where  $s^2$  is the second moment of the random variable  $B(t)$ , and  $\text{corr}(B(\Delta t))$  is the auto-correlation for the time-series of Blue casualties  $B(t)$  with lag  $\Delta t$ . We assume (as is usual) that  $\Delta t$  is small relative to the complete timespan  $T$  over which we are taking expectations  $E$ , with  $0 \leq t \leq T$ .

Since  $s^2$  is a constant, we can thus see from Equation 2 that;

$$\text{corr}(B(\Delta t)) \propto \Delta t^D$$

Considering now a range of values of the lag  $\Delta t$ , then, as shown by Lauren, if a variable such as  $\text{corr}(B(\Delta t))$  varies as  $\Delta t^D$  with  $0 \leq D \leq 2$ , it follows from the Weiner-Khinchine relation [8] that;

$$F\{\text{corr}(B(\Delta t))\}(f) \propto |f|^{-(D+1)} \dots\dots\dots(3)$$

where  $F$  is the Fourier Transform.

Let  $B*B$  denote (in functional terms), the auto-correlation of the time series  $B$ , where we think of  $B$  as a function of time  $t$ . From standard Fourier Transform theory, we have that

$$F(B*B)(f) = F(B)(f)\overline{F(B)(f)} = |F(B)(f)|^2$$

where again  $F$  is the Fourier Transform function, and  $f$  a specific frequency value. Thus the Fourier transform  $F$  of the auto-correlation function is just the square of the Fourier Transform of the time series  $B(t)$ . This is, by definition, the power spectrum of the Blue casualty time series  $B(t)$ .

It thus follows that;

$$|F(B)(f)|^2 = F\{\text{corr}(B(\Delta t))\}(f) \propto |f|^{-(D+1)} \dots\dots\dots(4)$$

Putting this together then, the left hand side of equation 4 is the power spectrum of the time series of Blue casualties  $B(t)$ . According to Equation 4, this should vary with frequency  $f$  in accordance with a power law, where the exponent (the exponent  $(D+1)$  in equation 4) is proportional to the fractal dimension of the locations of Red casualties on the battlefield, corresponding to the agile clustering and reclustered (such as swarming behaviour) of the Red agents. Since the Red fractal dimension  $D$  lies in the range 0 to 2, our theory predicts that this exponent should lie in the range -1 to -3.

**Initial Experimental Evidence**

In order to derive experimental evidence of these various relationships, an international defence research exchange was set up by the study team under TTCP-JSA-TP3. Through this process, access was obtained to the Maui High Performance Computer Centre (MHPCC) owned by the US Air Force. This is a supercomputer cluster which allows, through the US Marine Corps Project Albert process of ‘data farming’, the ability to run models such as MANA many thousands of times across large parameter sets. This was ideal for our purpose of attempting to obtain experimental evidence for our postulated relationships.

***The First Term of the Metamodel***

One clear implication of the metamodel discussed earlier is that the expected rate of Blue attrition  $E(|\frac{\Delta B}{\Delta t}|)$  is related to Red unit firepower effectiveness  $k$  through the relationship

$$E(|\frac{\Delta B}{\Delta t}|) \propto k^\alpha$$

where  $\alpha$  is a power law exponent. A similar relation should also hold between expected Red attrition and Blue unit effectiveness. In the full metamodel at equation (1), this exponent is related to the local clustering between the agents, as measured by the fractal dimension  $D$ , implying that a non-integer exponent should be anticipated. We analyse this using regression analysis, as part of testing the hypothesis that there is such a power law relationship between expected attrition and unit effectiveness, across a number of sample scenarios.

***Sample Scenarios in the MANA Simulation Model***

The MANA model is based on the foundation ideas of complexity and emergence. It has been developed using as a start point the ISAAC model developed by the US Marine Corp. ISAAC was developed in order to explicitly explore in an experimental fashion the relation between local unit interactions and the resultant emergent properties and behaviour of the overall force, and was inspired by approaches such as the simulation of artificial life in model ecosystems.

In the MANA simulation model, a scenario is defined by the battlespace and the characteristics of the two sides (Blue and Red). Each of the two forces is composed of a

number of units. These units have the ability to locally sense their environment, and then decide, on the basis of a number of simple algorithms, how to move around the battlespace. Waypoints can also be defined which help to steer the flow of forces in broad general directions. For more information on MANA see Reference [9]. For more information on the ISAAC simulation model see [10].

In MANA, the developer, Dr Michael Lauren has noticed four types of emergent behaviour [11]. They are:

- A Lanchester style battle with two single blocks of force fighting;
- A line of force on force battle which is stable and moves in a wave;
- A line of force on force battle which is unstable;
- Swarming behaviour or manoeuvre warfare.

We have used these as a basis for developing a number of scenarios for our experimental investigation. Specifically, we investigated a line of force that is stable and moves in a wave (the ‘Meet’ scenario), swarming behaviour (the ‘Swarm’ scenario) and a Lanchester style battle (the ‘Lanchester’ scenario). The measure of effectiveness that we used for these scenarios was the casualties suffered by the forces, in order to compare with the theoretical metamodel prediction of a power law relationship.

In the next part of the paper, we summarise the results of the first part of the analysis, carried out in the early part of 2003 and documented fully in [12]. We do this in order to link to the theory laid out earlier, and also to link to further experimental results that have been produced since then.

### ***Experimental Results for the ‘Meet’ scenario***

Figure 1 shows how expected Blue casualties (‘attrition’) as a proportion of the initial force, varies as Red unit firepower is varied along the x-axis. In MANA, unit firepower varies between 0 and 100, and the x-axis shows the normalised values. A number of these plots are superimposed, each representing a different value of Blue unit firepower effectiveness. Figure 2 shows these same results plotted on a Log-Log scale in order to investigate the power law behaviour predicted by theory. (A power law relationship becomes a straight line on a Log-Log scale).

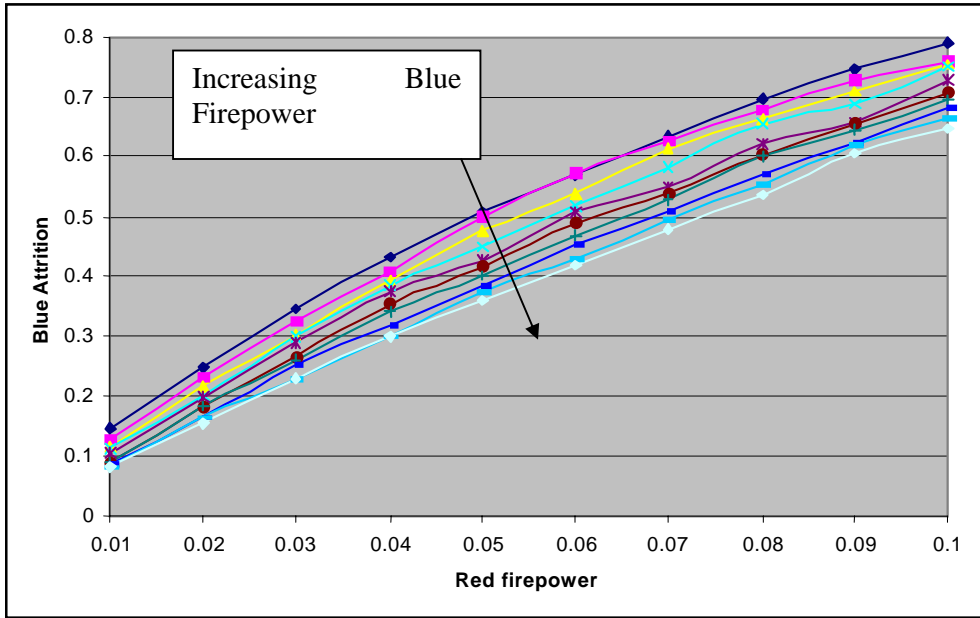


Figure 1; Blue attrition against Red unit firepower for different values of Blue unit firepower.

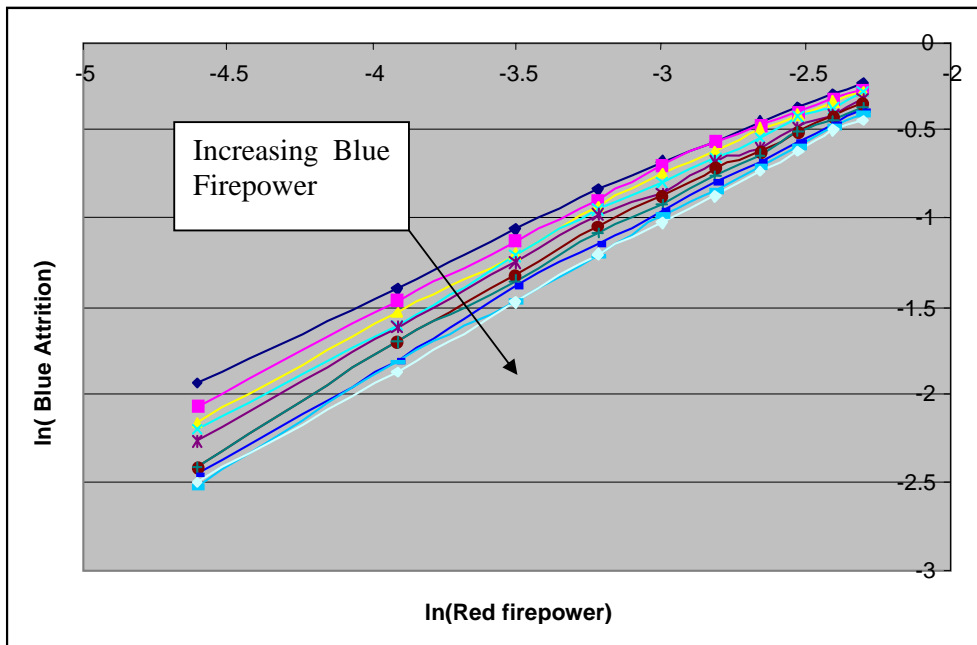


Figure 2; Figure 1 data shown on a Log-Log plot.



Looking at Figure 2, we see that the Log-Log plots form very obvious straight lines, indicating that the data has a power law relationship between individual agent firepower and enemy attrition. The gradients of these lines (close to integer values in this case) were found using linear regression and are shown in Table 1. They remain stable across the various cases considered. These gradients are the exponent values of the first term of the metamodel.

Blue fire power	Points	Gradient	Intercept
0.01	8	0.95	0.73
0.02	8	0.92	0.62
0.03	8	0.9	0.54
0.04	8	0.87	0.46
0.05	8	0.94	0.67
0.06	8	0.87	0.5
0.07	8	0.94	0.68
0.08	8	0.93	0.61
0.09	8	0.89	0.5
0.1	8	0.85	0.37

Table 1; Regression line fits for Blue force attrition; 'Meet' scenario.

Figure 3 and Figure 4 show the equivalent data for Red casualties.

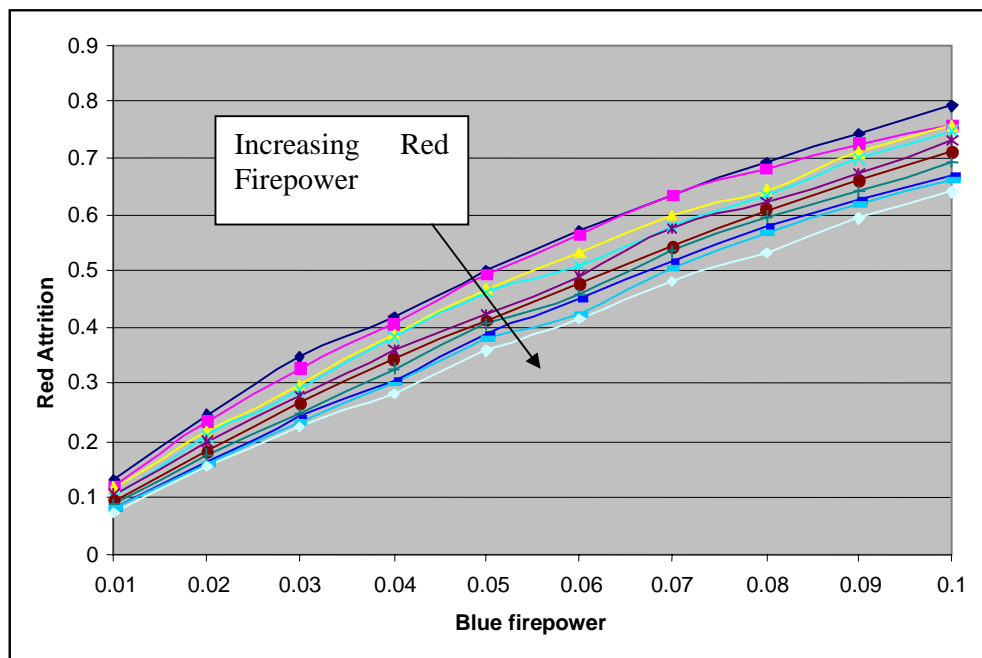


Figure 3; Red attrition against Blue unit firepower for different values of Red unit firepower.

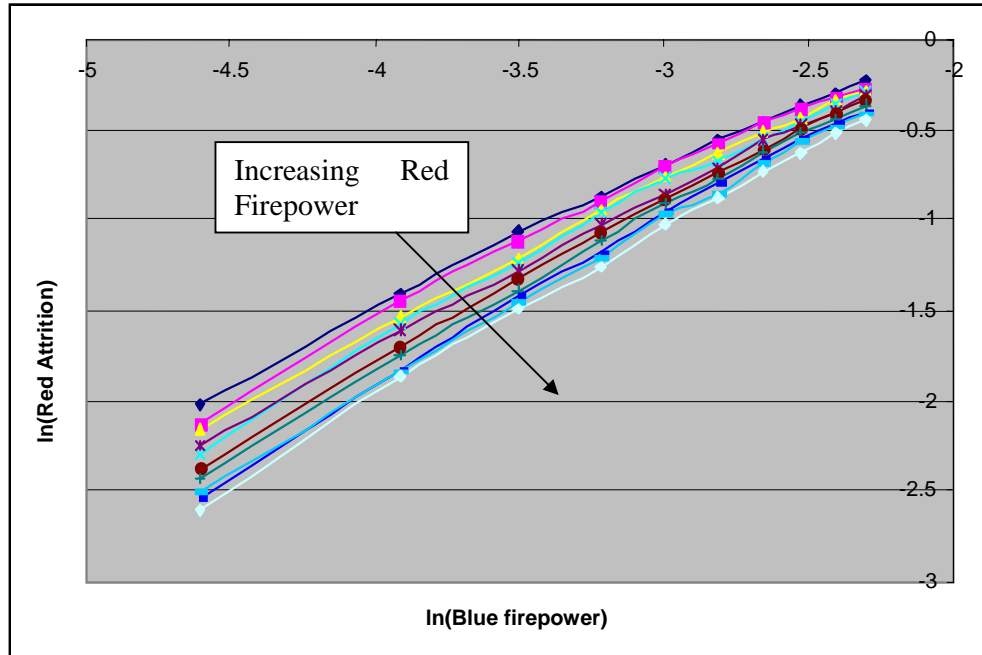


Figure 4; Figure 3 data shown on a Log-Log plot.

Table 2 shows the linear regression analysis results for the plots in Figure 4. Again, the gradient remains reasonably stable across the cases considered. There is evidence here of non-integer gradients, corresponding to fractal effects, as discussed in the earlier theory. The intercepts here are higher than for the other tables – we do not yet understand why this should be.

Red fire power	Points	Gradient	Intercept
0.01	8	0.74	1.5
0.02	8	0.76	1.55
0.03	8	0.8	1.59
0.04	8	0.8	1.6
0.05	8	0.82	1.59
0.06	8	0.85	1.66
0.07	8	0.88	1.69
0.08	8	0.91	1.74
0.09	8	0.9	1.7
0.1	8	0.9	1.64

Table 2; Regression line fits for Red force Attrition; ‘Meet’ scenario

***‘Meet’ scenario with increased force attraction***

The MANA model input data includes parameters that determine the attraction or repulsion of agents to other friendly and enemy agents, in each of the possible states (alive, injured etc.). This determines the behaviour of the forces. The MANA runs for the ‘Meet’ scenario were carried out for a range of values of the attraction to alive friendly entities. This parameter determines how closely the agents cluster together, and hence how much the effects are concentrated. The analysis discussed so far for this scenario is for the case where both Red and Blue had an attraction of (-20) to agents of their own type (so they were

repelled by agents of their own type). This was the lowest value. The analysis for the highest value of 0 is shown below. The full range of attraction values is not shown as the two extreme values indicate the trend in the data.

The following two figures show the data for Red firepower against Blue attrition.

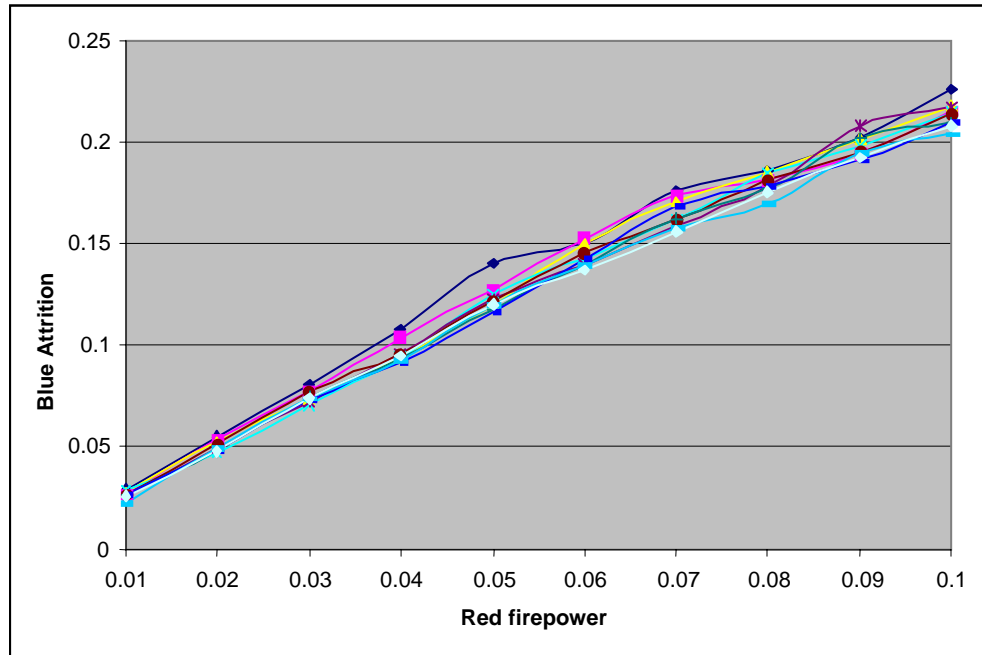


Figure 5; Blue attrition against Red unit firepower for different values of Blue unit firepower; for the highest attraction to own force agents (both Blue and Red forces)

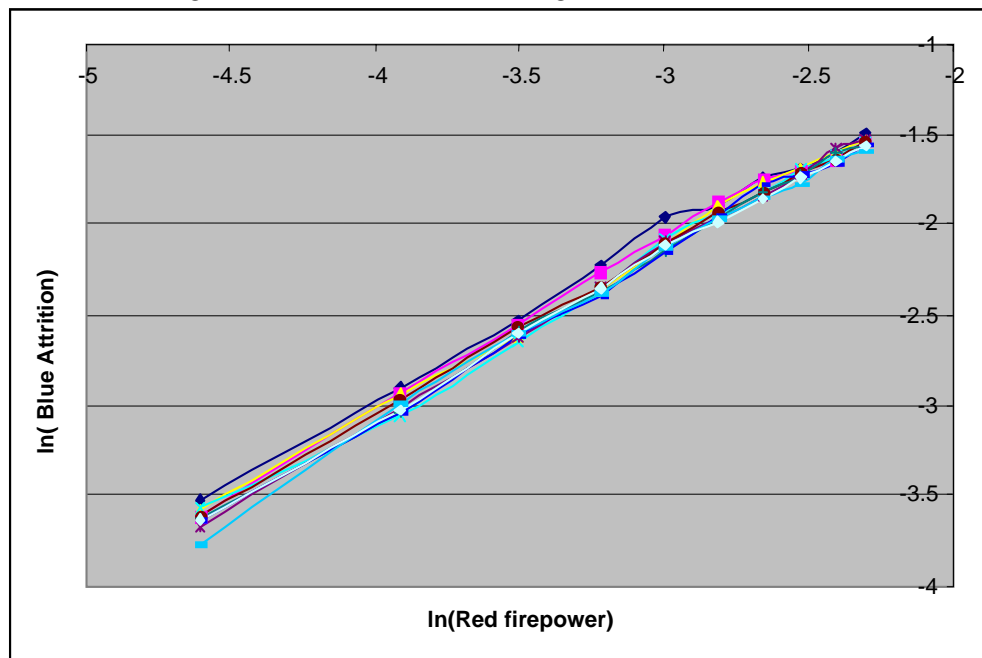


Figure 6; Figure 5 data shown on a Log-Log plot

Looking at the Log-Log plots in Figure 6, we see that there is little order in the plots as the Blue firepower increases. Comparing Figure 2 and Figure 6 shows that the plots for Figure 6 are much more closely clustered together. Table 3 shows the gradients of the plots in

Figure 6. Although the gradients are tightly clustered, there is no order to the intercepts, implying that these are all essentially the same line.

Blue fire power	Points	Gradient	Intercept
0.01	8	0.88	0.56
0.02	8	0.88	0.55
0.03	8	0.92	0.65
0.04	8	0.97	0.76
0.05	8	0.94	0.68
0.06	8	0.89	0.55
0.07	8	0.94	0.68
0.08	8	0.94	0.67
0.09	8	0.9	0.53
0.1	8	0.92	0.58

Table 3; Regression line fit; Blue force attrition; ‘Meet’ scenario; high attraction to own force

### Metamodel Implications

These experimental results for the ‘Meet’ scenario confirm the power law relationship between casualties and unit firepower effectiveness. Similar results were obtained for the ‘Lanchester’ and ‘Swarm’ scenarios, and the details are at reference [12]. This confirms the power law relationship of attrition versus unit firepower for these scenarios. Thus we have established that the first term of the metamodel at equation 1 is of the correct form (i.e. a power law relationship) for the ‘Lanchester’, ‘Meet’ and ‘Swarm’ scenarios.

### Discussion

The results across all the scenarios appear to confirm (or at least do not contradict) the power law relationship between attrition rate and unit firepower which comes from Equation 1. Equation 1 provides a linkage between casualties and the agile clustering of forces in a dispersed scenario where Command and Control is fully decentralised and self-organising effects such as ‘swarming’ can occur. This is of clear relevance to the idea of fully decentralised Command and Control, envisaged in the full implementation of Network Centric Warfare as discussed in [1], [2]. There is evidence in some of the results of fractal clustering effects corresponding to non-integer power law exponents.

### The Second Term of the Metamodel

By writing Equation 1 as:

$$E\left(\left|\frac{\Delta B}{\Delta t}\right|\right) \propto \Delta t^{-r(D)},$$

we can focus on the second term of the metamodel. Equation 4 showed that the resulting power spectrum of Blue casualties  $P(f)$  is in the form of a power law,  $P(f) = a|f|^{-(D+1)}$  where  $a$  is some constant. This implies that  $\log P(f) = \log a - (D+1)\log|f|$ . In other words, when the power spectrum is plotted on a Log-Log plot, we expect a straight line with gradient  $-(D+1)$ . As explained above, the metamodel predicts that this gradient will be between -1 and -3.

To test this, a variety of MANA scenarios were run and the time series representing Blue casualty data were recorded. We then looked for evidence of a power law relationship between the power spectrum of this data  $P(f)$  and the frequency  $f$ .

### ***Data***

The data required for this part of the work was time series of casualty data, i.e. the number of agents killed at each time step. MANA can record such step-by-step data, however in our work we have used "contact data".

Contact data measures the number of enemy agents that each friendly agent can see. Therefore, enemy agents can be counted more than once. The maximum possible number of contacts at each step is the total number of alive enemy agents, multiplied by the total number of alive friendly agents.

We have used contact data because there is not enough information from casualty data to calculate a good power spectrum. For example, quite often, there are no casualties for several time steps. This is because it is (for example) the companies that are modelled, not the individual men, and therefore the probability of killing an agent (a company) is small. However, we assume that every time there is contact between agents (i.e. they can detect one another), there will be casualties (perhaps only on a small scale). Therefore, contact data can be used to represent casualties. Note that when contact increases, the number of casualties is likely to increase.

### ***Scenarios in MANA***

Several MANA scenarios were used in order to analyse different types of situations. These scenarios were:

- Western Front in World War II (developed by Lauren, and used as comparison) [11]
- The 'Lanchester', 'Meet', and 'Swarm' scenarios mentioned earlier.

### ***Power Spectrum Analysis***

A power spectrum is defined to be the squared modulus of the Fourier transform. When using discrete data, the discrete Fourier transform is used, and the squared modulus is found for each of the elements of the Fourier transform.

Using sampled (discrete) data rather than the 'actual' continuous data means that we can only estimate the power spectrum. There is a limit to the number of data points that can be produced, as well as the length of the time step. When using the discrete Fourier transform, there are standard methods for overcoming the problems of 'aliasing' and 'frequency leakage' that are associated with this situation. [13]

In our work we have sought to improve the estimate by:

- Averaging over a partition of the data, in which each segment has had its power spectrum taken;
- Using a window function to reduce leakage; experimental evidence suggests that this may distort the gradient of the power spectrum, so this has not been applied to our data;
- Replicating MANA runs and aggregating the resulting power spectra.

As no tool existed to do all of this analysis automatically, and so that we had maximum control over the algorithms used, we are developing our own algorithms to carry this out. The details of the algorithms are given below.

### ***Power Spectrum Analysis Tool***

The mathematical definition of the power spectrum ( $P$  as a function of frequency  $f$ ) is the absolute value of the Fourier transform (FT) of the time series data ( $c_i$ ), squared. We are dealing with a sample of discrete data. Therefore, we need to use the discrete Fourier transform,

$$FT_k = \sum_{j=0}^{N-1} c_j e^{2\pi i j k / N} \quad \text{for } k = 0, 1, 2, \dots, N$$

So  $P(f_k) = |FT_k|^2 \quad \text{for } k = 0, 1, 2, \dots, N$

The periodogram estimate (PE) is used to normalise the power spectrum such that Parseval's theorem holds (i.e. the sum of all the power terms equals the mean squared amplitude of the transform).

$$PE(f_0) = \frac{1}{N^2} P(f_0)$$

$$PE(f_k) = \frac{2}{N^2} P(f_k) \quad \text{for } k = 1, 2, \dots, \left(\frac{N}{2} - 1\right)$$

$$PE(f_{N/2}) = \frac{1}{N^2} P(f_{N/2})$$

Note that the values of  $P$  from points  $(N/2 + 1)$  to  $(N - 1)$  are equal to the values at the points  $(N/2 - 1)$  to 1, respectively, because the input data are always real. Therefore, it is not necessary to calculate these.

To prevent 'leakage', a window function can be used. This weights the input data so that the beginning and end of the time series are given less emphasis. The weighting function we used is the Parzen window:

$$w_j = 1 - \frac{\left| j - \frac{1}{2}(N-1) \right|}{\frac{1}{2}(N+1)}$$

Therefore, the FT is calculated as;

$$FT_k = \sum_{j=0}^{N-1} w_j c_j e^{2\pi i j k / N} \quad \text{for } k = 0, 1, 2, \dots, N/2$$

To reduce the variance of the estimate of the power spectrum then calculated, data is partitioned into segments (each must have a multiple of 2 data points). The power spectrum estimate calculated from each set of data is then averaged at each frequency to increase confidence in the result.  $K$  segments reduce the variance of the estimate by a factor of  $K$ . In the results presented below, we did not use the window function (i.e. all weights are set to 1). This was because it appeared to distort the calculation of the best fit approximation to the power spectrum for our data sets.

### *Initial Results*

The method was first tried on Lauren's MANA model of a generic battle on the European Western Front in World War II [11]. In this scenario, each agent is a company, and so the probability of killing an agent is very low. Therefore it makes sense to use contact data rather than casualty data.

Our results from runs in MANA were compared to Lauren's, to help validate our method. From this we were able to alter our calculations as described above, to improve our confidence in the results. However, there is still a discrepancy between our results. Even for exactly the same data, Lauren records a gradient of -1.7 while we find -2.3 in Figure 7.

Four graphs follow in Figures 7 to 10. These depict the Log-Log plot of the power spectrum of the contact data from a single MANA run in each of our scenarios. The graphs illustrate (using regression analysis) that a power law (indicated by a straight line) explains 79% to 94% of the data in our experiments. In addition, the gradients of the straight lines are between -1 and -3, as expected from our theory. Therefore there is no evidence falsifying our theory, so far.

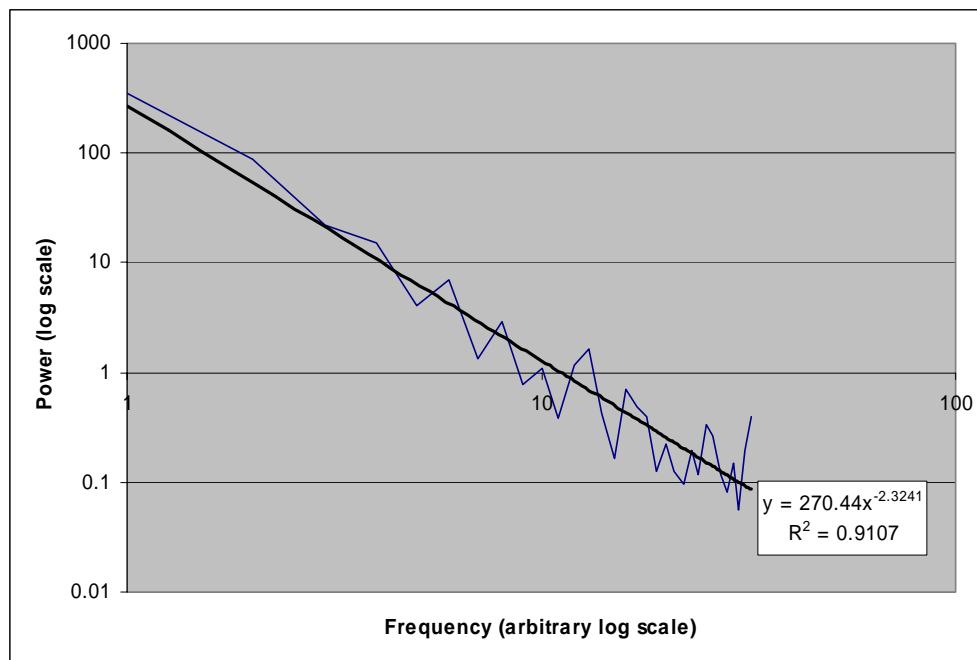


Figure 7: Power spectrum of contact data from Lauren's European Western Front Scenario with line of best fit

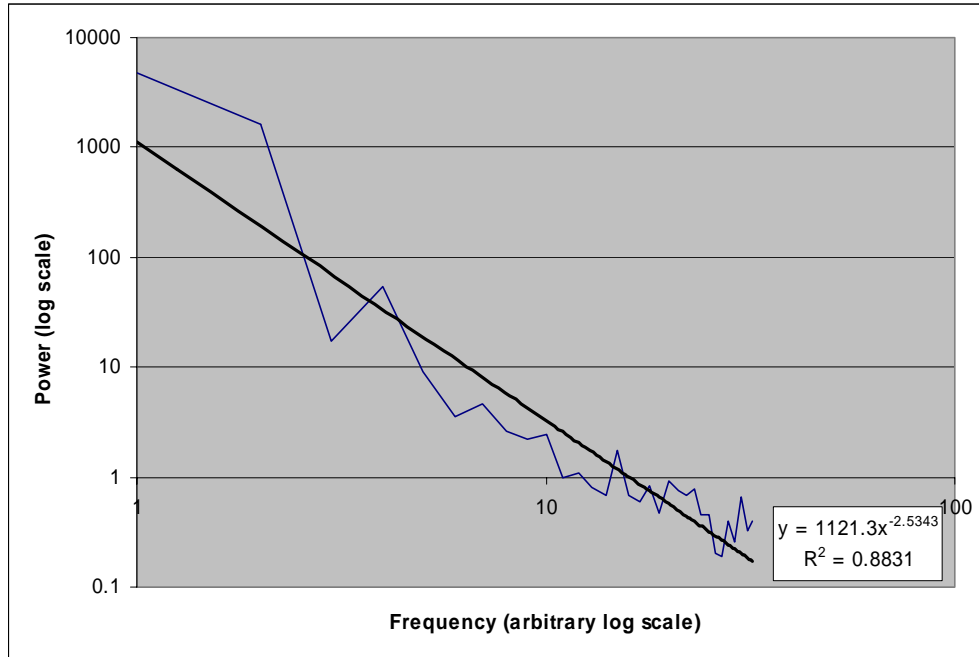


Figure 8: Power spectrum of casualty data from 'Lanchester' scenario with line of best fit

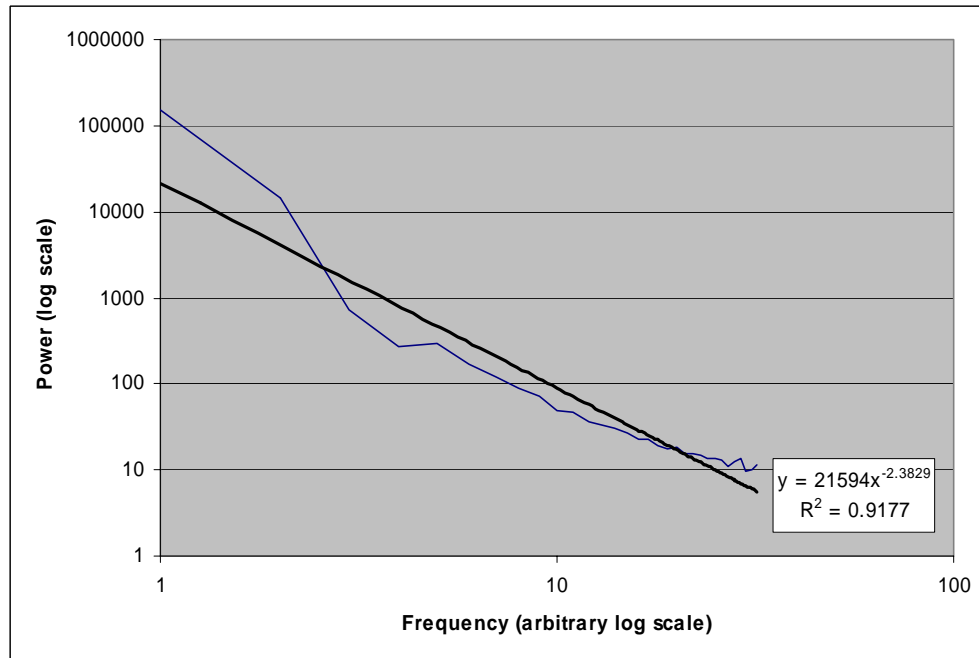


Figure 9: Power spectrum of contact data from 'Meet' Scenario with line of best fit



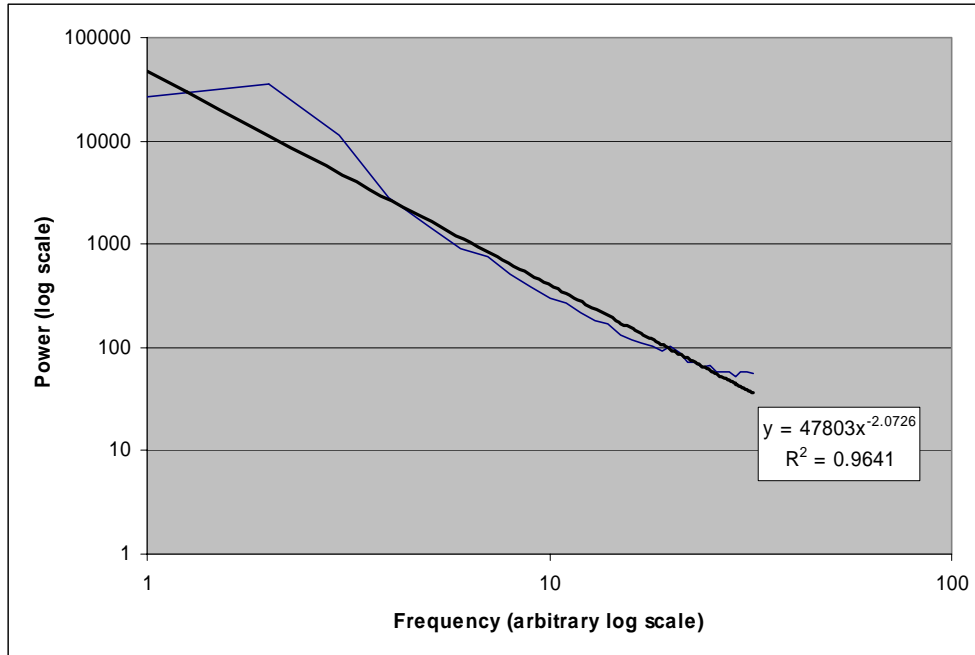


Figure 10: Power spectrum of contact data from 'Swarm' scenario with line of best fit

In order to reduce the variance of the estimate further, we have replicated the MANA scenarios hundreds of times so that the power spectra of these runs can be averaged, to produce a better estimate. The results of this analysis are displayed in Figures 11 and 12.

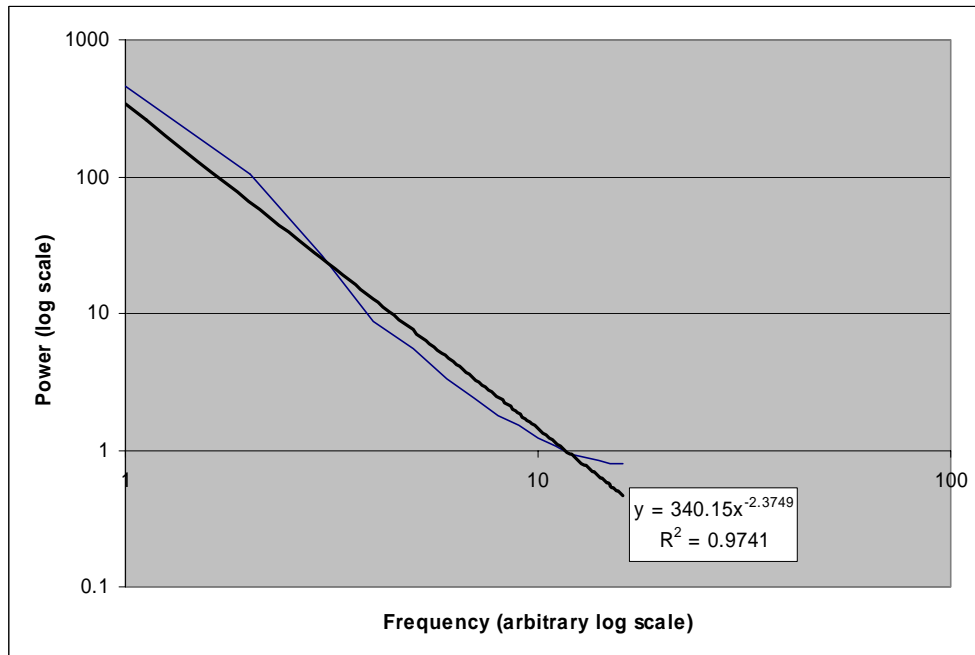


Figure 11: Aggregated results from 255 replications of European Western Front scenario power spectra, with line of best fit.

It can be seen that a straight line explains 97% of the data in the aggregated case in Figure 11. This is a better fit than the individual replication shown in Figure 7, in which a straight line explained 91% of the data. This result confirms the gradient of about -2.3 that was seen in the single replication.

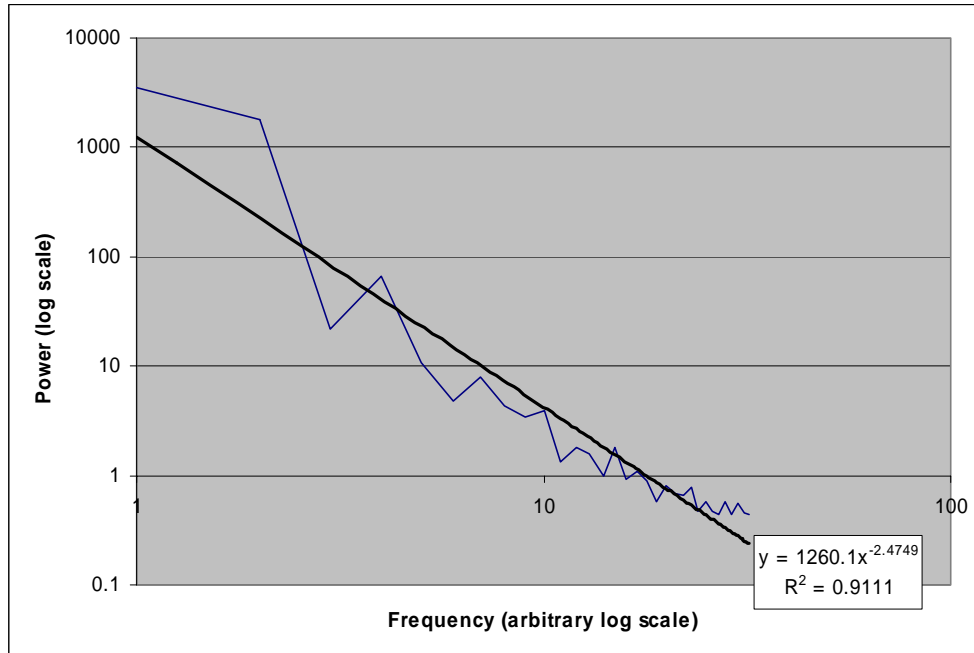


Figure 12: Aggregated results from 250 replications of ‘Lanchester’ scenario power spectra, with line of best fit.

In Figure 12, 91% of the data is now explained by a straight line, compared to 88% in the single replication. A gradient of about  $-2.5$  is seen, which confirms the result from Figure 8.

Figures 11 and 12 show that over many replications of these scenarios, a power law relationship is not disputed and the gradient is still between  $-1$  and  $-3$ , as expected.

## Conclusions

We have developed a theory that predicts the emergent behaviour of the MANA model when the emergent behaviour is characterised by force casualties, either aggregated or considered as a time series over the time of the scenario. For the first and second terms of this metamodel, experimental evidence indicates that the expected power laws are a reasonable assumption. So far, the experimental evidence does not falsify our theory.

The third term of the metamodel is concerned with the cluster size distribution. From our theory, we expect to see a coherent relationship between Blue casualties, average Red cluster size, and the number of Red clusters.

Initial experimentation with ISAAC, the predecessor of MANA, in our earlier work suggests that there is coherence in these cluster sizes. See Chapter 5 of reference [3] for more information on this. However, much further work is required to establish the correct relationship.

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