Using a Shortest Path Algorithm for Identifying Areas of Interest in An Area Of Operations

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Modeling and Simulation Workgroup
of the
8th International Command and Control Research and Technology Symposium

Abstract

Sensor coverage of a unit's area of operations will be critical to maintaining situational awareness for interim and objective force ground units. As such, sensor deployment plans that offer a high probability of covering lines of communication (LOC's) and avenues of approach (AA) while limiting the number of sensors employed will become an important part of the Intelligence Preparation of the Battlefield. Manual methods for determining these LOC's and AA's can be time consuming when applied over large areas of terrain. We propose a method that combines mathematical morphology and a greedy heuristic (single source shortest path algorithm) in order to identify channeling terrain along likely routes of enemy movement. The results of the analysis can then be graphically reviewed for quality and used as necessary in the Intelligence Preparation of the Battlefield (IPB) process.

I. Introduction

Interim and objective force commanders at the brigade level will be responsible for security and intelligence gathering in areas of operations (AO) as large as 2500 square kilometers [IBCT 2000] As a comparison, this area roughly corresponds to the AO assigned to today's legacy divisions [DIVOPS 1996]. In addition to the increased size of the AO, the commander's staff will be significantly smaller than the current division's staff, and, based on the expected operational tempo, have considerably less time to conduct the intelligence preparation of the battlefield (IPB). In order to facilitate these two mission requirements, brigades will be equipped with ground sensors designed to be employed in a network that enhances situational awareness and targeting capabilities. Consequently, a smaller staff will be required to produce in a shorter period of time, the standard intelligence products as well as a sensor emplacement plan that supports the commander's critical intelligence requirements and provides maximum AO coverage.

A component of the IPB includes extensive analysis of the terrain and its suitability in supporting enemy operations. Much of the information an intelligence analyst uses to conduct this analysis is readily available in digital format and stored in Geographic Information Systems (GIS). Thus, with the right set of tools, an analyst could rapidly identify mobility corridors and chokepoints throughout the AO. Knowledge of this restrictive terrain coupled with likely enemy routes through the area of operations could single out tactically significant terrain for nomination as Named Areas of Interest (NAI) for review by the analyst. Additionally, the terrain identified by this analysis could be used to generate a sensor employment plan that would provide coverage for most activities in the AO. Finally, these automated processes can be executed in minutes rather than hours. Thus, the analyst is left with more time to explore the information and develop the intelligence products.

The research described in this paper differs from the bulk of the literature in this field in that it focuses on an often overlooked component of the sensor emplacement process: selecting locations for which to provide sensor coverage. The preponderance of research in the field of AO sensor coverage assumes either total area coverage or total coverage of pre-selected areas of interest within an AO. The goals of this research are to develop a strategy for automating components of the NAI selection process as well as developing a first pass solution for a sensor employment plan. With this research, we attempt to replicate and enhance components of the current IPB process in an automated fashion as opposed to replacing the IPB process with technology. The underlying methods supporting the automated analysis in this paper are standard processes with accepted results. What is new is the combining of these processes in such a way as to generate tactically significant locations within the AO.

The balance of this paper is organized as follows: In section 2 we summarize the related work pertaining to methods of providing sensor coverage in an area of operations. In section 3 we present several key processes that are critical to understanding the proposed strategy. Sections 4 and 5 cover the methodology and some simulation results. Finally, in section 6 we present areas for further research.

II. Related Work

Previous research in sensor coverage has mostly been concerned with efficient total area coverage designed to minimize the number of sensors required to meet minimum tolerances for detection[Chakrabarty et al 2002], [Meguerdichian et al 2001], [Howard et al 2002]. Underlying assumptions that support this approach are that sensors will be cheap, readily available, and relatively small. Even if we assume that the three previously mentioned assumptions bear out, the logistical costs with respect to employment time, delivery assets, storage, and transportation associated with such a policy could be prohibitive.

In studies where total area coverage is not considered, authors assume that areas of interest have previously been identified [Cheng et al 2002], [Konstantinos et al 2003], [Haney and Blatt 2001] and the task at hand is to strategically place the sensors to cover these areas of interest. In research where GIS are coupled with sensor networks [Heidemann and Bulusu 2001], the focus has been in using the data for identifying sensor location, known as localization, and estimating target location, known as targeting, in a network. No methodology is described however, to determine how these areas of interest are identified.

In current efforts to develop sensor emplacement decision aids for the military [Mattice 2002], researchers have developed processes that estimate the probability of detection for a moving target within an area of interest, given a sensor location. The analyst however, must select the sensor locations and must have already determined the NAI's. Another set of researchers [Braswell 2003] have developed a process for predicting static enemy positions given terrain, equipment and mission. While both of these technologies are similar to the research presented in this paper, [Mattice 2002] requires prior knowledge of NAI's and [Braswell 2003] does not provide candidate NAI's for mobile targets. We present in this paper a method that nominates NAI's for mobile targets.

III. Key Processes

There are three principle components for this process. Each is based on standard practices. A better understanding of each of these components will facilitate the comprehension of the overall process presented here. The first process is the generation of a movement cost matrix developed by the NATO Reference Mobility Model (NRMM). The second process identifies all passages through the terrain of a specified size. This is accomplished using a standard mathematical morphological process called *closing*. Finally we use a shortest path algorithm to generate likely routes through the AO. A more detailed explanation of each of these processes follows.

NRMM is a vehicle performance prediction model [Haley et al 1979]. The model predicts maximum vehicle speed for a given unit of terrain. The calculations are made using vehicle specifications, terrain properties, and weather. An analyst provides the data for the terrain to be analyzed, usually available in Vector Product Interim Terrain Data (VITD) format, weather data, a vehicle type, and the dimensions for a terrain unit. The output of this model is a raster file for the entire area under analysis that determines the maximum speed the selected vehicle could safely travel within each terrain unit. This is an accredited model that has been in use since 1979. NRMM includes both wheeled and tracked vehicles for friendly and enemy forces. For the purposes of this research, we assume a unit terrain size of 50 meters by 50 meters since this is the standard unit block of information associated with VITD [Ryder 1996]. For testing our process, we do not use the actual outputs from the NRMM. Rather, in order to maintain an unclassified label for this document, we use false movement cost rasters that simulate the output of the NRMM.

NRMM creates a digital image of the terrain with respect to target mobility. The next step is to digitally inspect the image to identify restrictive terrain. For this process, we use elements of mathematical morphology that have been used in digital image processing since the early 80's [Pratt 2001]. The two basic operators of morphology are dilation and erosion. The effect of a dilation on an image is to enlarge the boundaries of the elements in the image. The effect of an erosion on an image is to erode the boundaries of the elements in an image. In order to execute either a dilation or an erosion, one must have the digital representation of the image and a component referred to as the structuring element. The dilation and erosion of an image are the results of the operation on the digital image by the structuring element.

When dilation and erosion are executed in this order, it is known as a *closing* since it has the effect of closing small openings and gaps in a digital image. In this research we use a closing to identify chokepoints and mobility corridors in the terrain. We define the AO as a binary digital image to be processed and use a basic 3X3 grid for the structuring element. The restrictive terrain is defined as the foreground image and assigned a value of one (1). All other terrain is defined as the background image and assigned a value of zero (0).

Given a digital image I of size X pixels by Y pixels, a structuring element S of size X' pixels by Y' pixels, where X' goes from $-u$ to $+u$ *and* Y' goes from $-v$ to $+v$, a dilation D of the image is defined as:

$$D(x,y) = MAX\left(I(x+x',y+y');\forall x',y'\right)$$

or, each pixel in the dilation assumes the maximum value of the image intersected with the structuring element. Symbolically, this is represented as:

$$D(x,y) = I(x,y) \oplus H(x',y')$$

An erosion E of the image is defined as:

$$E(x,y) = MIN\left(I(x+x',y+y'); \forall x',y'\right)$$

or, each pixel in the dilation assumes the minimum value of the image intersected with the structuring element. Symbolically, this is represented as:

$$D(x,y) = I(x,y) \ominus H(x',y')$$

Now that we have identified restrictive terrain in the AO, we must identify where target vehicles are likely to travel. We do this using a shortest path algorithm. A single source shortest path problem is defined as finding the least cost path between a source node and all other nodes in a graph [Cormen et al 1990]. This is a well studied problem with many efficient heuristic based algorithms leading to a provably optimal solution. If we consider the individual cells of the movement cost raster generated by the NRMM to be nodes in a grid, and we allow an arc to connect each node to its adjacent nodes (each node has at most eight arcs since we are considering movement through a 3X3 grid), then we could transform the raster representation of the AO to a network representation of the AO. By using this network, we can predict likely enemy routes throughout the AO by finding the least cost path from some set of start nodes to some set of terminal nodes. [Cherkassky et al 1996] determined that a network with such a structure as the one described above can be solved rapidly (approximately 1 second for a network with over one million nodes) using a specialized algorithm known as a double bucket implementation of Dijkstra's Algorithm. For the sake of ease in implementation and explanation, we do not implement his algorithm in this research. Instead we implement the standard Dijkstra's algorithm at the expense of longer run times.

IV. Methodology

We now present the methodology for identifying tactically significant terrain within an area of operations. The goal of our research is to quickly generate a list of potential NAI's that cover the routes a target vehicle could be expected to use in order to accomplish its mission. The objective of this research is to automate this terrain analysis, thereby providing the same results as manual methods in a small fraction of the time. We define tactically significant terrain as restrictive terrain that limits the target vehicles options for movement and serves as an indicator for the target vehicles possible objective. Several significant assumptions are made in this research:

- In order to reach its objective, a target vehicle's path will approximate the least cost path with respect to time.

- The set of paths generated from initial nodes to terminal nodes, while not exhaustive, is representative of the paths that could most likely be taken.

To facilitate the explanation of the process, we use a small piece of artificial terrain 1.3 km by .8 km shown in figure 1 and its movement raster representation, after having been notionally processed through NRMM using 50 m by 50 m terrain units, figure 2. The dark gray line is a paved road, the light gray line is a dirt road, the black line is a river, and the circled feature in figure 1 is a fordable location in the river.

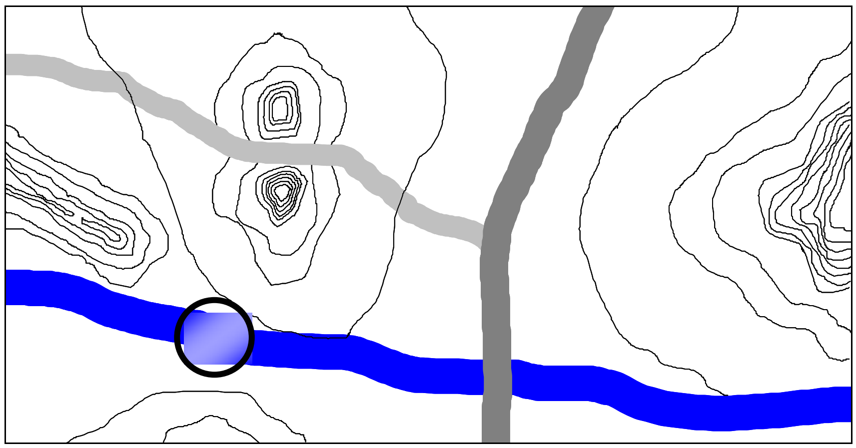


Figure 1: Graphical representation of sample terrain used to describe process

We begin with the movement raster and modify it to prevent inappropriate movement across restrictive terrain. Specifically, if the movement cost from cell i to cell j is represented by the following function:

$$c_{it} = \frac{1}{2}\left(c_i + c_t\right)\sqrt{d}$$

where: c_i = cost in seconds to traverse the initial node
c_j = cost in seconds to traverse the terminal node

$$d = \begin{cases} 2 \text{ if } j \text{ is diagonally adjacent to } i, \\ 1 \text{ if } j \text{ is strictly adjacent to } i, \\ 0 \text{ otherwise,} \end{cases}$$

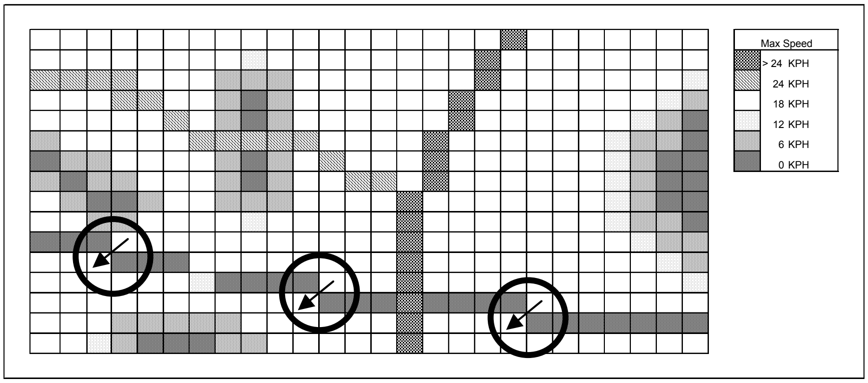


Figure 2: Movement Raster after processing through NRMM

Then in each of the circled areas in figure 2, diagonal movement across the river, represented by the arcs, can be accomplished with no cost associated for crossing the river since the initial node and terminal node are non-restrictive terrain. To remedy this, we iterate through the raster and identify locations where restrictive terrain is only diagonally adjacent, and replace the counter diagonal raster cells with a restricted terrain identifier. Figure 3 shows the modified raster.

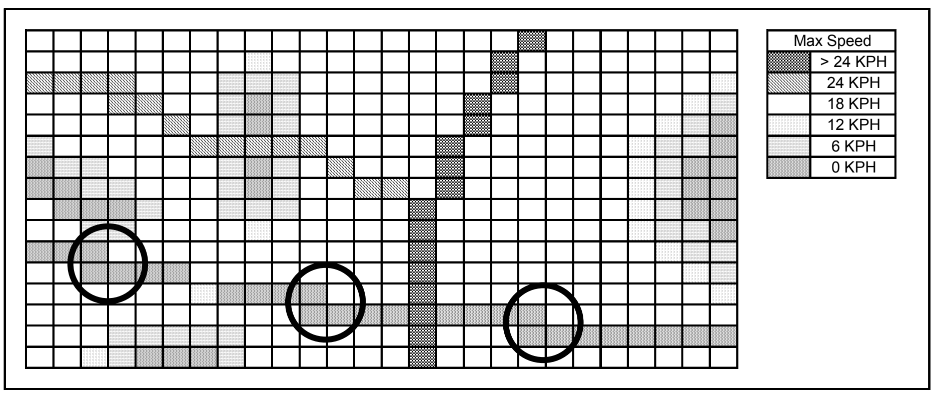
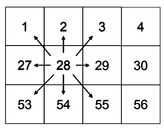


Figure 3: Modified movement raster with impeded diagonal movement across restricted terrain

The value in each raster cell is then converted from maximum safe speed to seconds required to traverse the cell, and finally, each arc cost is generated and loaded into a two dimensional movement cost matrix, *C*, to be used in the shortest path algorithm. An extract of *C* is presented in Table 1.

In preparation for the closing operation, we create a binary representation of the movement cost raster by substituting a one (1) for restrictive terrain and a zero (0) for non-restrictive terrain in each cell address and load the data into an array. In this paper we refer to this array as *B*.



Initial Node			Terminal Node			Arc Cost
Cell Number	Max Safe Speed	Cell Cost (c_i)	Cell Number	Max Safe Speed	Cell Cost (c_j)	c_{ij}
28	17	10.6	1	17	10.6	14.9
28	17	10.6	2	17	10.6	10.6
28	17	10.6	3	18	10	14.5
28	17	10.6	27	17	10.6	10.6
28	17	10.6	29	18	10	10.3
28	17	10.6	53	22	8.2	13.2
28	17	10.6	54	21	8.6	9.6
28	17	10.6	55	22	8.2	13.2

Table 1: Extract from Cost matrix and conversion from speed to seconds

For this research we consider the two slowest movement categories ($\leq$ 6 kph and 0 kph) to be classified as restrictive terrain. Figure 4 shows the binary representation of the sample terrain.

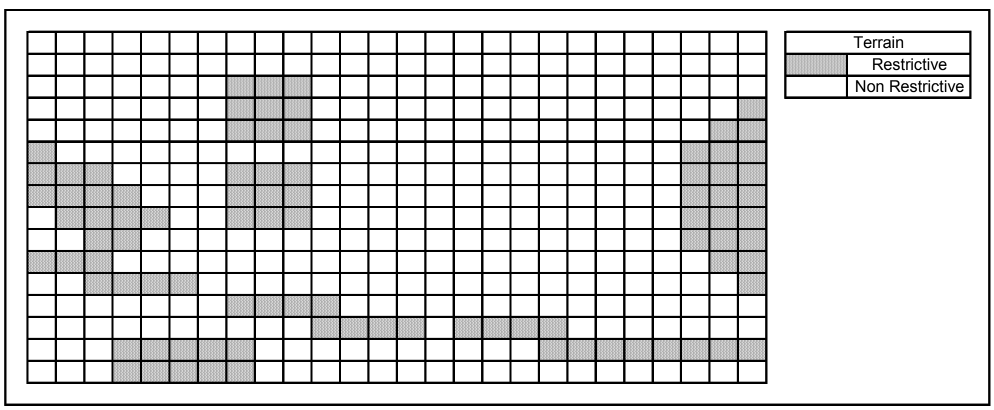


Figure 4: Binary image of restrictive terrain

After running the closing operation, we identify gaps by taking the converse of the intersection of the binary image with the closing. Symbolically, this is represented as $G = \overline{\left(B_i \cap C\right)}$ (where G is the gap matrix and each element of G is a 0 or 1). This matrix represents all gaps within the area of interest up to a specified size. For this example, we assume a gap size of 50 meters. It is this matrix that will be used to determine the tactically significant gaps.

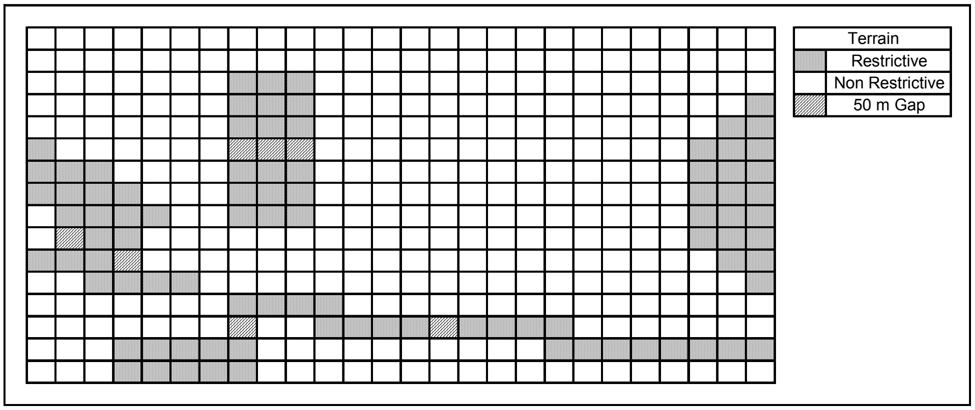


Figure 5: Binary image after closing operation with 50 meter gaps identified

After all gaps have been identified, the process seeks likely paths through the AO by executing the shortest path algorithm for all combinations of initial nodes and terminal nodes. This component of the algorithm uses the cost matrix derived from the movement raster. While it is possible to record the paths for future analysis, there is no requirement in this process to store each individual path. Instead, we seek how many paths have passed through each node. These values are stored in a matrix *P*, with the same dimensions as the binary image array mentioned above. See Table 2 for a representation of *P*. In this example, the first row of cells served as the initial nodes and the non-zero cells in the last row served as the terminal nodes (see figure 3). Thus each initial node has 23 paths originating from it, and each terminal node has 26 paths ending in it.

P_{ij}	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26
1	23	23	23	23	23	23	23	23	44	23	23	23	23	23	23	23	23	23	184	161	138	115	92	69	46	23
2	0	23	46	23	23	8	38	21	0	48	5	45	4	19	46	19	0	230	0	0	0	0	0	0	0	0
3	0	0	23	92	23	8	59	0	0	0	61	41	0	0	27	57	0	230	0	0	0	0	0	0	0	0
4	0	0	0	0	115	8	59	0	0	0	65	0	37	8	19	0	287	0	0	0	0	0	0	0	0	0
5	0	0	0	0	0	123	59	0	0	0	17	48	45	0	19	0	287	0	0	0	0	0	0	0	0	0
6	0	0	0	0	0	0	152	121	121	121	106	0	93	0	0	306	0	0	0	0	0	0	0	0	0	0
7	0	0	0	0	0	0	93	0	0	0	1	113	85	0	40	266	0	0	0	0	0	0	0	0	0	0
8	0	0	0	0	0	0	93	0	0	0	9	0	145	230	0	266	0	0	0	0	0	0	0	0	0	0
9	0	0	0	0	0	0	93	0	0	0	9	40	0	0	456	0	0	0	0	0	0	0	0	0	0	0
10	0	0	0	0	0	0	93	0	0	49	40	0	0	0	456	0	0	0	0	0	0	0	0	0	0	0
11	0	0	0	0	0	0	93	48	49	0	0	0	0	0	456	0	0	0	0	0	0	0	0	0	0	0
12	0	0	0	0	0	0	141	1	0	0	0	0	0	0	456	0	0	0	0	0	0	0	0	0	0	0
13	0	0	0	0	104	104	38	0	0	0	0	0	0	0	456	0	0	0	0	0	0	0	0	0	0	0
14	0	0	0	104	0	0	0	38	8	0	0	0	0	0	456	0	0	0	0	0	0	0	0	0	0	0
15	0	26	104	0	0	0	0	0	45	23	15	15	15	118	130	208	208	208	0	0	0	0	0	0	0	0
16	26	26	26	26	0	0	0	26	26	42	59	77	103	26	26	78	52	26	208	182	156	130	104	78	52	26

Table 2: Matrix *P* results on sample problem

A matrix *T*, of tactically significant gaps is created by performing a component wise multiplication of the Gap matrix *G* with the path counting matrix *P*, or $T(x,y) = G(x,y) \cdot P(x,y)$. Any value in *T* greater than zero represents a gap, chokepoint, or mobility corridor of a specified width, with at least one path running through it. In our working example, *T* consists of 3 significant gaps, see figure 6.

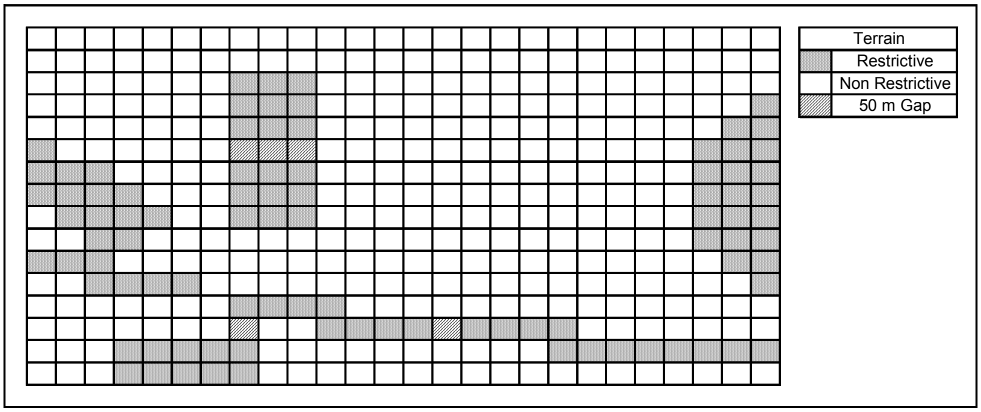


Figure 6: Tactically significant gaps as selected by automated process

The analyst then selects a lower threshold for the density of paths passing through a gap (for example, the three cell gap shown above has a gap density of (121+121+121)/3 = 121). Any gap with a density less than the threshold will be filtered out. This is done to eliminate gaps with very few paths running through them. The remaining values in *T* are then nominated as NAI's for review by the analyst. This data can then be viewed graphically in conjunction with an image of the mobility raster to cull or add NAI's.

V. Results

The process in this paper can be run for any terrain for which there is sufficient data to run the NRMM. Size of the AO and the number of initial nodes and terminal nodes used to determine the paths are the principle factors in how long the process runs. The closing operation is computationally negligible with respect to the shortest path algorithm. As written, the closing operation has a complexity of $O(s*n)$, where s is the size of the structuring element and n is the number of nodes. The shortest path algorithm, in contrast, has a complexity of $O(c*n^2)$ but can be reduced to $O(c*n \log n)$ if a heap algorithm is used [Cherkassky et al 1996], where is c is the number of initial nodes. A final example is presented to demonstrate the results when different values for the gap size are used on the same terrain. The source code, "findchokes.cc", is contained in Appendix A. The data files, "Raster.txt", "Initial.txt" and "Terminal.txt" are Appendices B, C and D respectively.

The scenario in support of this sample problem follows. There are two possible assault positions and three possible objectives within this AO of size 5 km X 3 km. Figure 7 shows these 5 locations superimposed on the movement raster. AA1 and AA2 are the possible attack positions (initial nodes) and obj1, obj2, and obj3 are the possible objectives (terminal nodes).

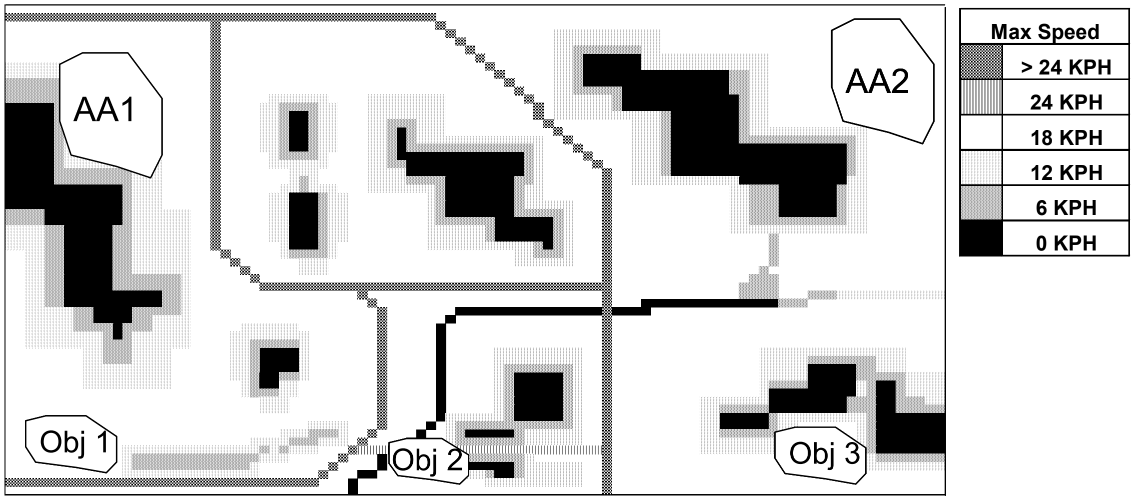


Figure 7: Raster of larger sample problem

The terrain is fictitious and has not actually been processed through the NRMM. The unit terrain size has been set at 50m by 50m. Four choke size diameters; 50m, 150m, 250m, 350m were used for this example. All terrain with maximum movement speed of 6 KPH or slower were considered restrictive terrain. The threshold for gap density was set at 1. This simulation was run on a P4 2.4 GHz computer with 2 GB of RAM running Windows XP®. The source code for this process was written in C++ and compiled on the GNU compiler (g++ version 3.2-1) and run under the Cygwin UNIX emulator for Windows®. The resulting network was 6,000 nodes with 47,044 arcs. There were 96 initial nodes and 77 terminal nodes for a total of 7,392 iterations of the shortest path problem. The process run time for each chokesize diameter was 8 minutes with all but 5 seconds allocated to the execution of the shortest path algorithm. This process with the double bucket implementation of Dijkstra's algorithm as presented in [Cherkassky et al 1996] would have been able to execute the same simulation in well under a minute.

Figure 8 shows the results of the simulation for each chokesize diameter. The black regions represent the restrictive terrain, the white regions represent the non-restrictive terrain, and the grey regions represent the non-restrictive terrain that has been identified as tactically significant.

Two significant observations can be made from these results. The first is that an artificial chokepoint or mobility corridor can be identified if restrictive terrain is sufficiently close to the boundary of the AO (see the solid grey circled gap in 8c and 8d). In true tactical scenarios, unit boundaries are rarely chosen in an arbitrary fashion that produces appealing geometric shapes. Unit boundaries are generally selected along prominent terrain to facilitate visual recognition of the AO. Therefore these false corridors may not occur. To guard against such a phenomenon however, the process should be modified to consider terrain outside of the AO or to ignore chokepoints and mobility corridors adjacent to the AO boundaries.

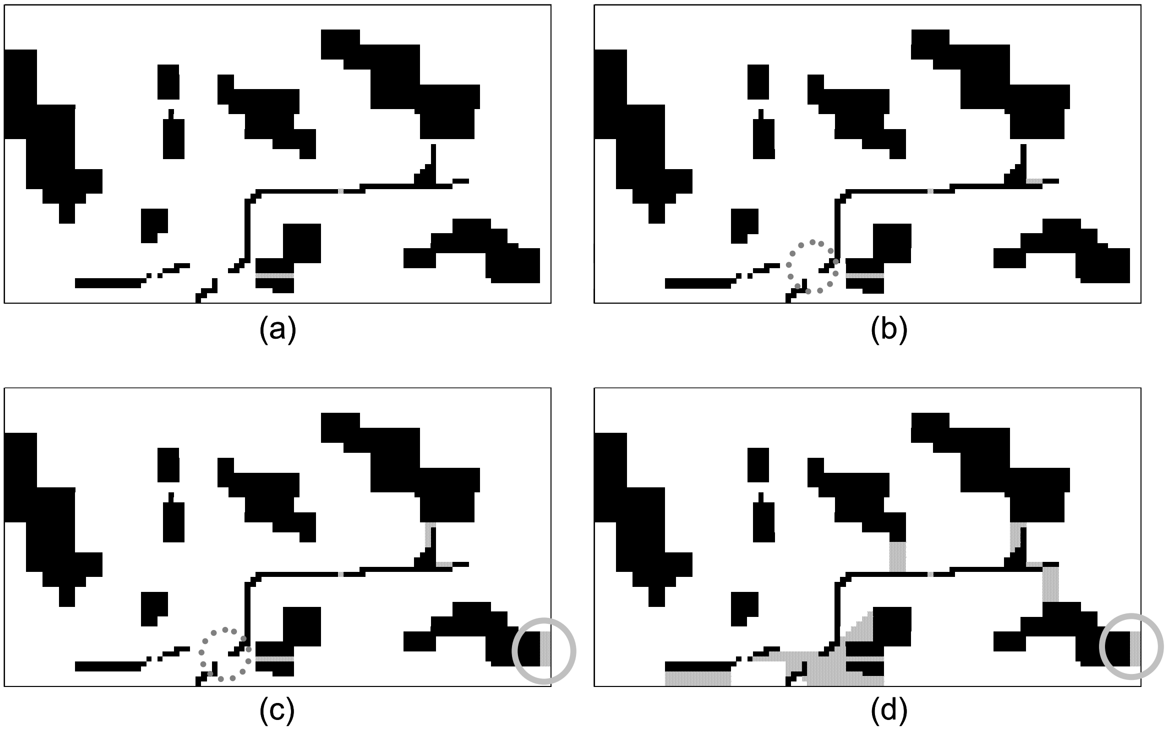


Figure 8: Results of the simulation on the same terrain with varying chokesize diameters: (a) = 50 meters, (b) = 150 meters, (c) = 250 meters, (d) = 350 meters

The second observation is the closing operation as implemented in this research is incapable of identifying gaps created by narrow obstacles along a diagonal trajectory (see the dotted grey circle in 8b and 8c). This chokepoint is identified in 8d, but only because it is within 350 meters of other restrictive terrain. While the features in this example that caused this condition (a bridge running parallel to and on top of a river) are artificial, such a condition could arise. A geometry based implementation of the closing operation, such as the internal and external buffering tools available in GIS packages, should rectify the problem.

VI. Future Research

All of the terrain data required to run this algorithm resides in a Geographical Information System (GIS). Therefore significant gains in speed and detail could be realized by embedding this process in a GIS in order to exploit its inherent terrain and geometry based spatial analysis capabilities. Such an implementation, if successful, would lend this algorithm to serve as a graphical, tactical decision aid for the intelligence analyst.

This methodology could be extended through path analysis to identify decision points. The gaps currently located by this process actually identify terrain which confirms the target vehicle has been committed to a decision. By analyzing the actual paths, it may be possible to locate enemy decision points for route selection. Knowledge of these enemy decision points can provide the commander more time to make his decisions.

By placing sensors clusters on each of the tactically significant gaps identified with the algorithm, all likely paths through the AO would be covered. In effect, this would provide coverage of the AO with a significantly reduced set of sensors than would be required by total area coverage. Research must be conducted to determine if the benefits gained by reducing the logistical burden associated with such a sensor employment plan outweighs the costs associated with the risk of not covering the entire AO with a sensor network.

VI. Conclusion

In this paper we have demonstrated an automated process that can identify tactically significant terrain in a short period of time, thus freeing the intelligence analyst to perform other tasks associated with the IPB process. This method is not designed to replace the intelligence analyst's role in terrain analysis, but is instead designed to enhance the terrain analysis process. We also posit that complete AO coverage could be assumed by using the nominated NAI's generated by this process as a basis for a sensor employment plan.

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