Application of **Network Optimization** to Design of Organizational Structures

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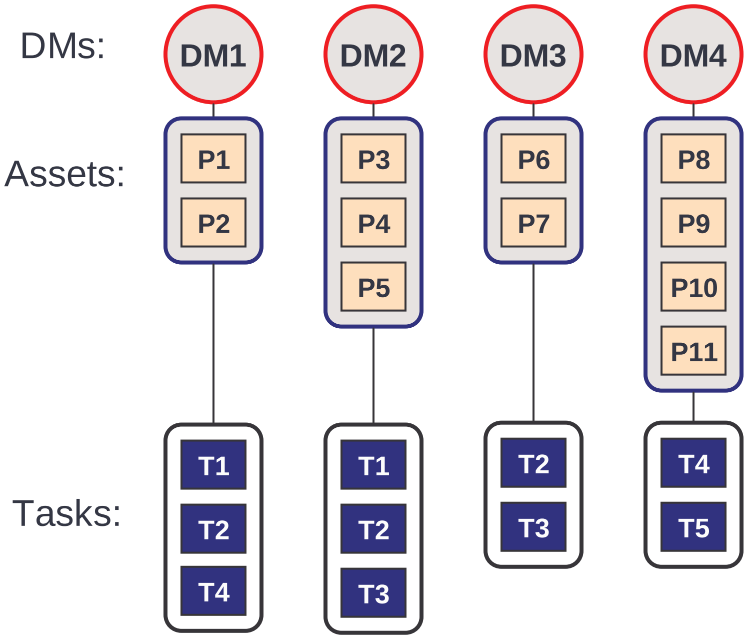
Track 3: ***Modeling and Simulation***

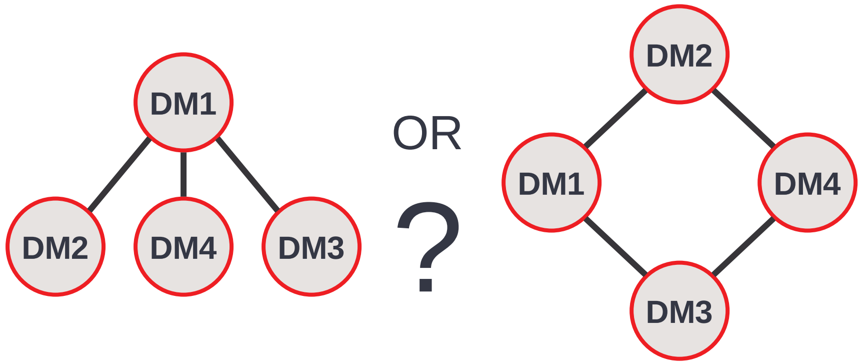
Motivation: Example

Aptima HUMAN-CENTERED ENGINEERING

<u>Organization</u>



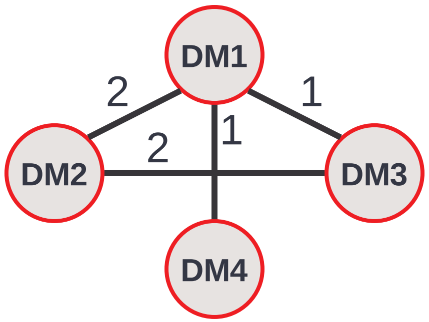
<u>Structure</u>



- Need to know the meaning of the network
- Need to optimize the processes
- **What** to communicate, and **when**

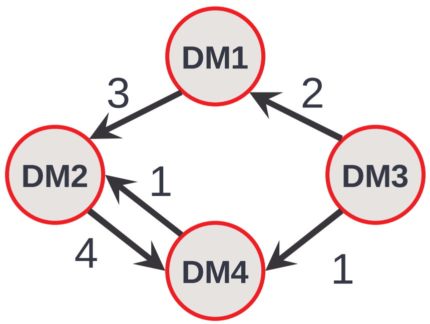
Network of communication requirements due to:

Synchronization



How to build a network to support this communication?

Task Information Flow



Overview

- Networking: examples of *structures* and *flow* types
- Information **flow**, **delay** models, and *optimal* flow ***routing***
- **Network design** problem
- **Example** of network generation

Networking?

Structure Types

- Communication structure
- Management – overhead com-n, subordinate tasking, overseeing
- Distributed optimization
- …

Communication Types

- Information flow – "what I know/found"
- Requests – "what I need"
- Decision exceptions
- …

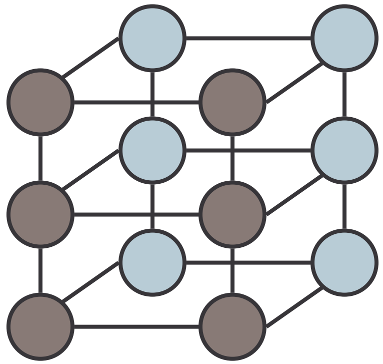
Flow Pattern

- **Stochastic**
- **Deterministic**

- **Known** agent-agent communication requirements (i.e., processes & com-n under ROE)
- **Unknown** (i.e., determined by task allocation, events, etc.)

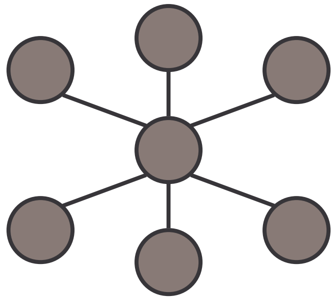
Examples of Structures

Mesh or Hypercube



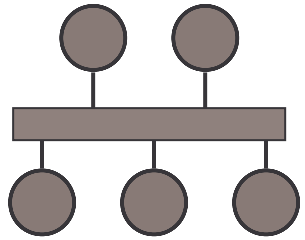
Pro: enabling multiple 2-node communication; redundancy for communication
Con: multiple connection needed; must use "smart" routing

Star



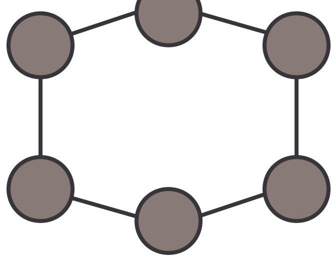
Pro: when communication with few remote nodes is needed
Con: sensitive to central node operability

Bus network



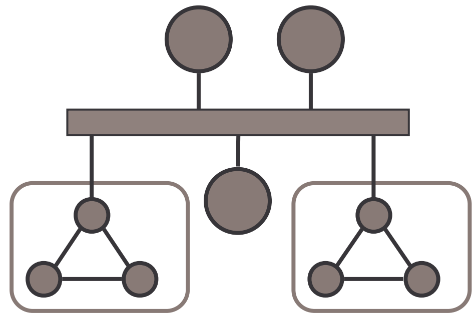
Pro: individual node failures do not affect the flow
Con: might communicate at the same time – need priority routing

Ring



Pro: tight routing rules, easy to implement
Con: high overhead on links – slow transmission

Heterarchy/Hybrid



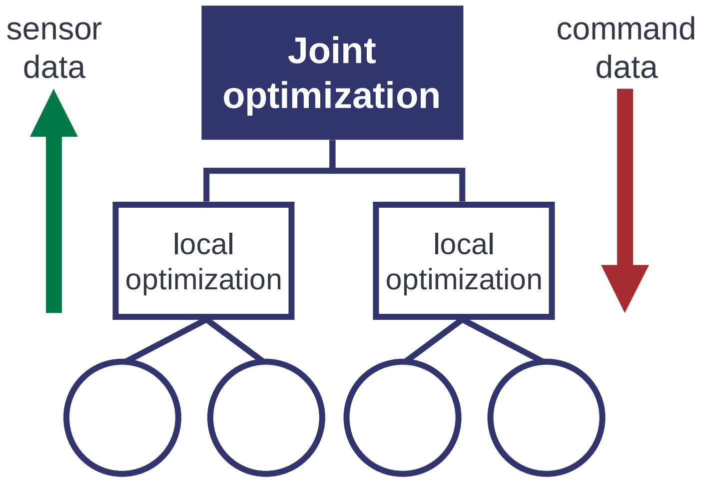
Pro: utilize good points of all structures for mission specifics

Network constraints:
- Required/available Bandwidth
- Topology limitation
- Connectivity
- Overhead
- Cost
- Throughput
- Robustness

Hierarchy versus Heterarchy

Hierarchy

- ***Distributed*** **Optimization**
- ***Hierarchical*** **Command**



Pro:

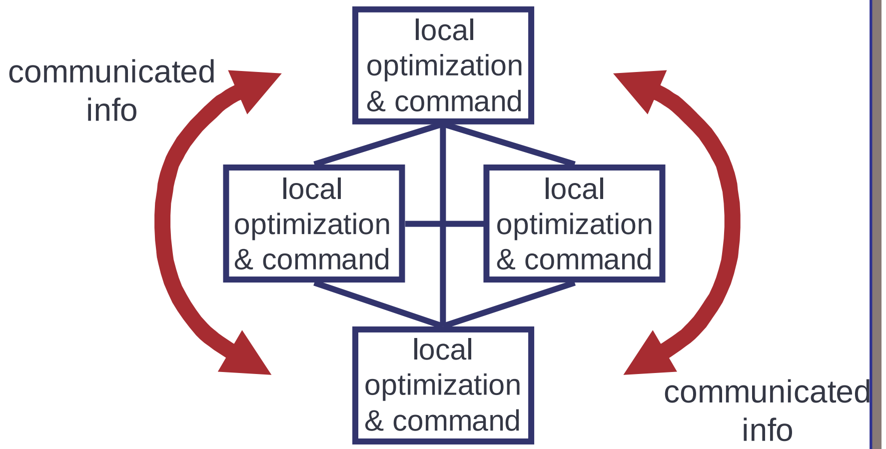
❑ **Reduce complexity**
❑ **Limited functionality of individual cells**

Cons:

❖ **Lack of flexibility**
❖ **Slow response time**
❖ **High sensitivity**
❖ **Low fault-tolerance**

Heterarchy

- ***Distributed*** **Optimization**
- ***Heterarchical*** **Command**



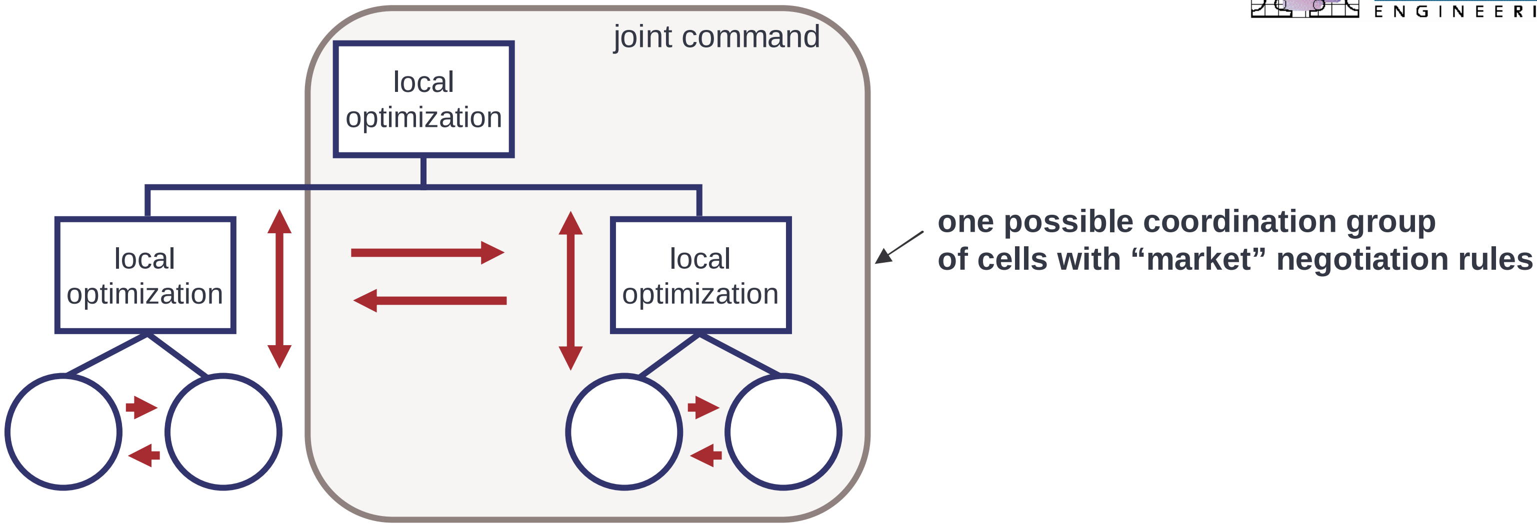
Pro:

❑ **Flexibility & adaptability**
❑ **Improved fault tolerance**
❑ **Independent operations**

Cons:

❖ **Difficulty in designing local objectives: possible mismatch with global objective**
❖ **Lack of global information**
❖ **Sensitivity of collaboration rules**

Hybrid Structures



Idea:

❑ Construct a structure that utilizes the benefits of both *hierarchy* and *heterarchy*

Result:

- Architecture with hierarchical structure which changes according to an environment
- Different communication/command/control rules that enable this change

Method/design:

❖ Overcome structural rigidity and lack of flexibility by requesting/negotiating structural change (e.g.., request supervision over rarely utilized resource)

❖ Communication among cells in the same level (horizontal)

❖ Communication among cells in different levels of current hierarchy (vertical)

Example: Task Assignment

Centralized global controller

Global controller - optimization
- Select Target & team TOT
- Select DMs/assets to prosecute targets

target request

sensor data

allocation

asset priority

DM network

DM1 DM2 DM3

Local optimization at each node for TOT

Target communication:
Controller accumulates the task information to send request to DMs for target prioritizing

DM communicates sensor data

Allocation communication:
Controller uses asset/target priorities to determine asset/target allocation

DM computes asset priorities over requested or sensed targets

- Flat structure control

Decentralized heterarchical controller

DM4

DM1
- Target request for **subteam**
- **Subteam** TOT
- **Subteam** asset selection (based on own and communicated asset set)

DM5 DM2 DM3

Each DM executes functions of the global controller over its sub-network

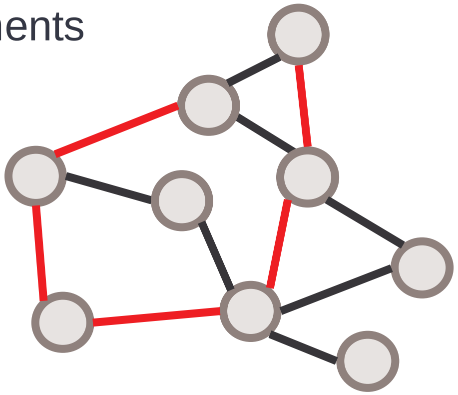
Communication:
Superior: aggregated sensor data and selected assets & their priority
Subordinates: target request, asset selection

- Communicated information is minimized
- Optimization is distributed
- Complexity is lower

Examples of Flow Problems

- **Known Communication Requirements**
- **Unknown agent network**

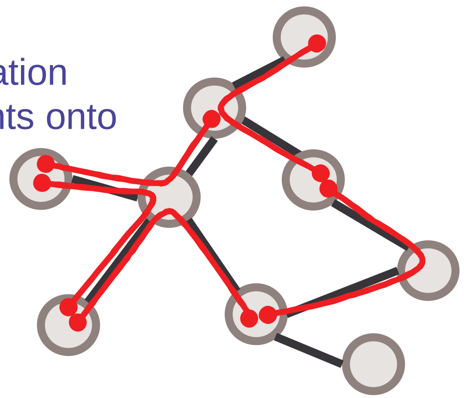
Input: network of communication requirements



Constraints: physical, topology, cognitive

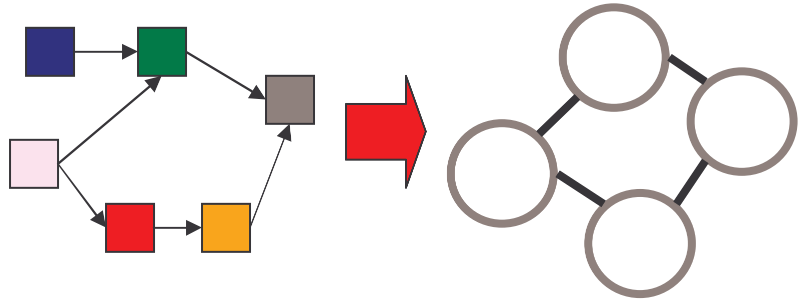
Output: agent network, information routing

Map the communication requirements onto constraints



- **Unknown Communication Requirements**
- **Known agent network**

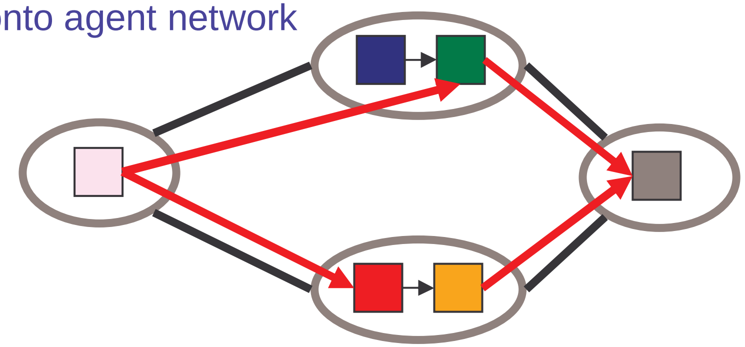
Input: task graph with flow requirements; agent communication network



Constraints: physical, topology, cognitive

Output: agent network, information routing

Map the task graph onto agent network

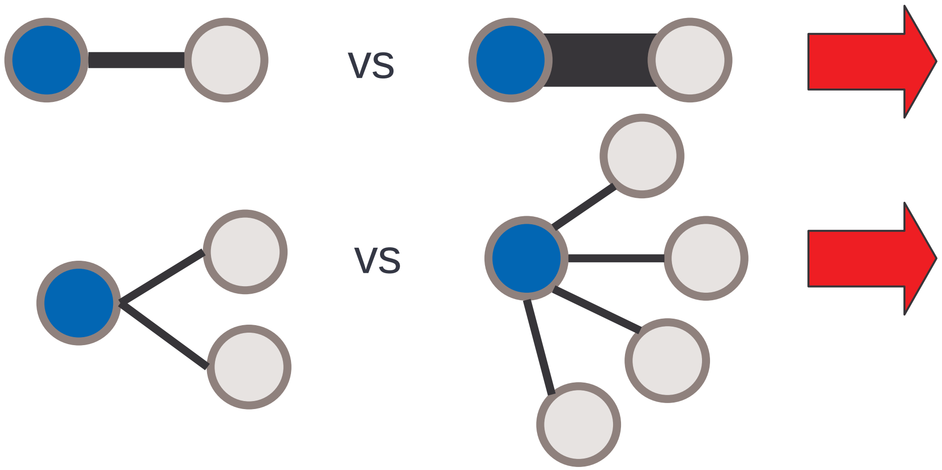


Network Constraints

Static Constraints

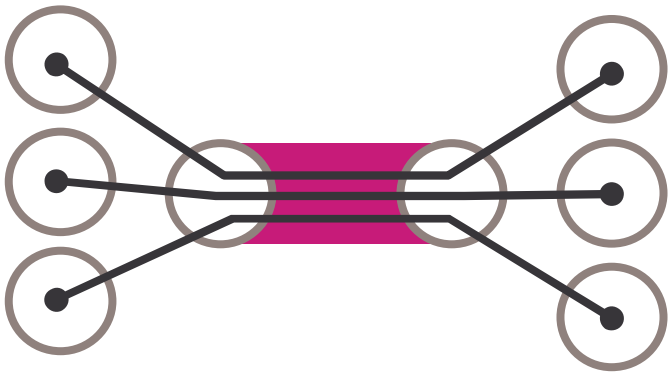
Bandwidth vs → Flow Delay

Connectivity vs → Routing Options

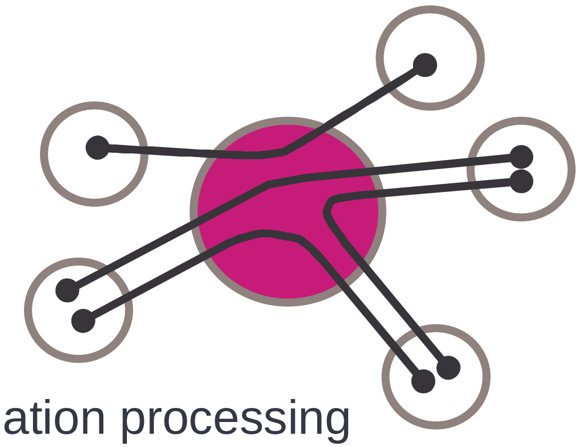


Dynamic Constraints

Link Overhead



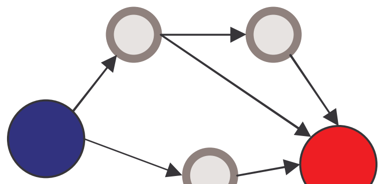
Node Overhead



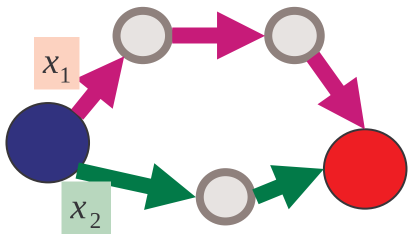
- Delays in communication and information processing

Routing & Network Architecture

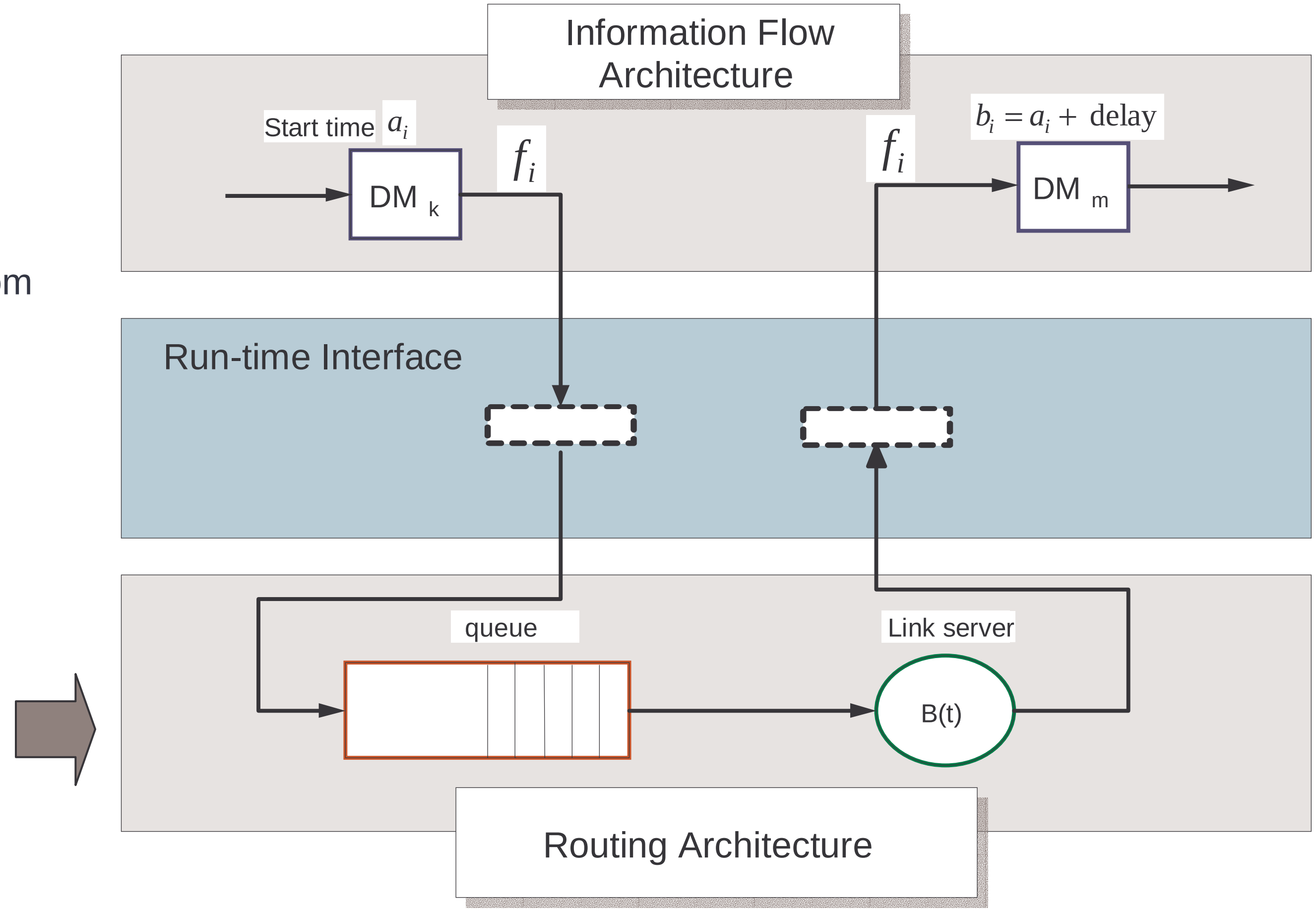
Routing:



Need to communicate from **blue** to **red** f_i units of information

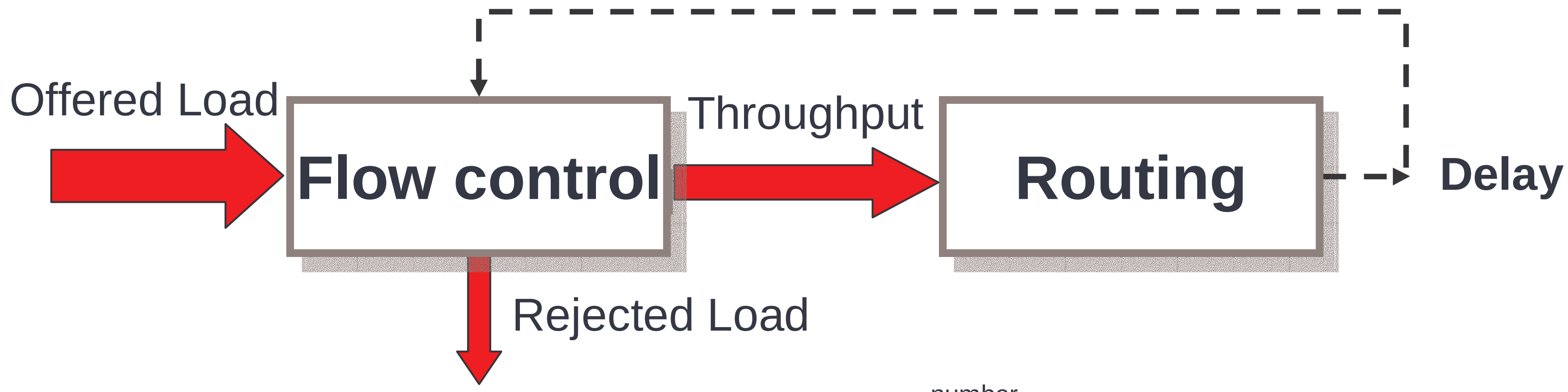


$$f_i = x_1 + x_2$$



Information is **split** to smaller portions to be communicated via different routes

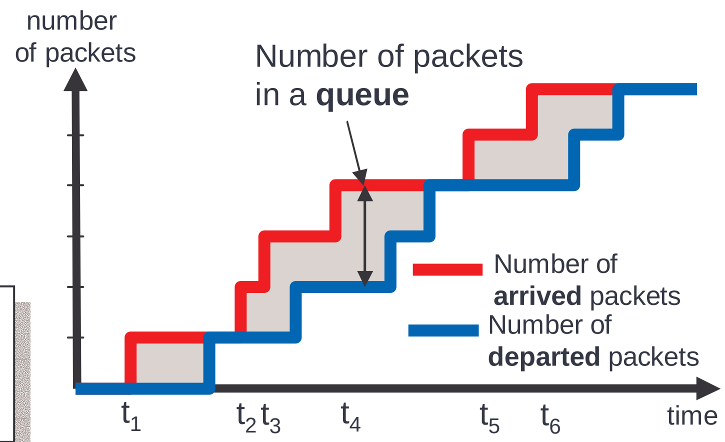
Controlling Information Flow

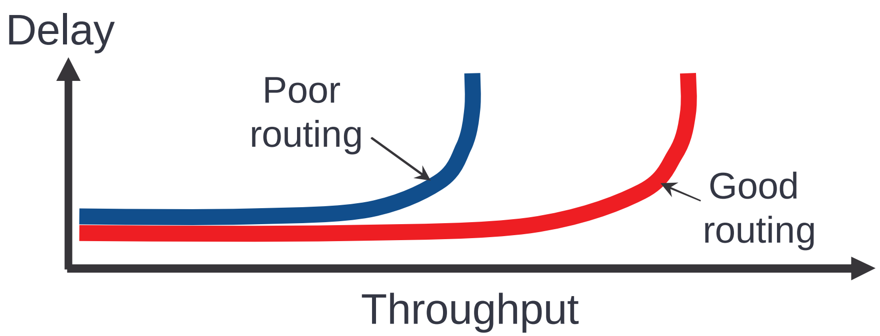


As the routing algorithm is more successful in keeping delay low, the flow control algorithm allows more traffic into the network

Routing algorithm performance measure:

1. **Throughput** = (offered load) – (rejected load) (**quantity**)
2. **Average packet delay** (**quality**)





Effects of good routing:

(a) High offered load: **increase** throughput for the same average delay

(b) Low offered load: **decrease** average delay

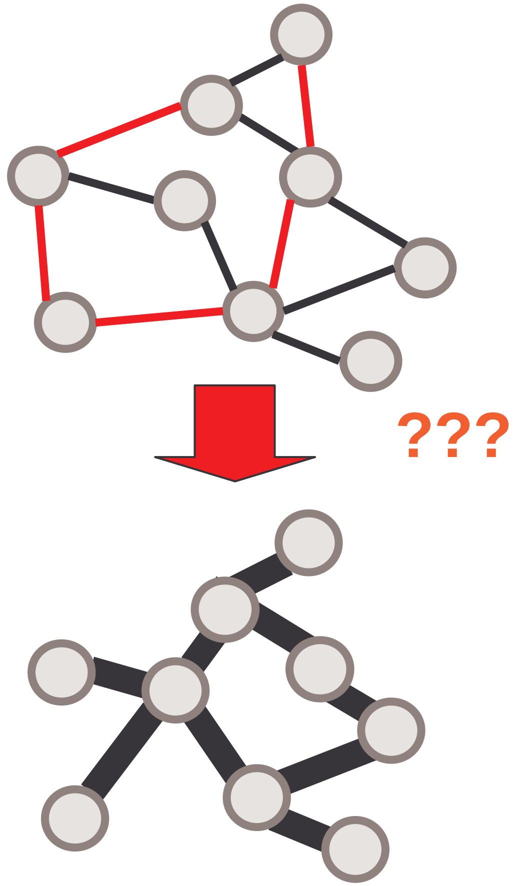
Network Design Problem

Given:

- A **traffic matrix** giving the input traffic flow from each agent to every other agent

Want to design:

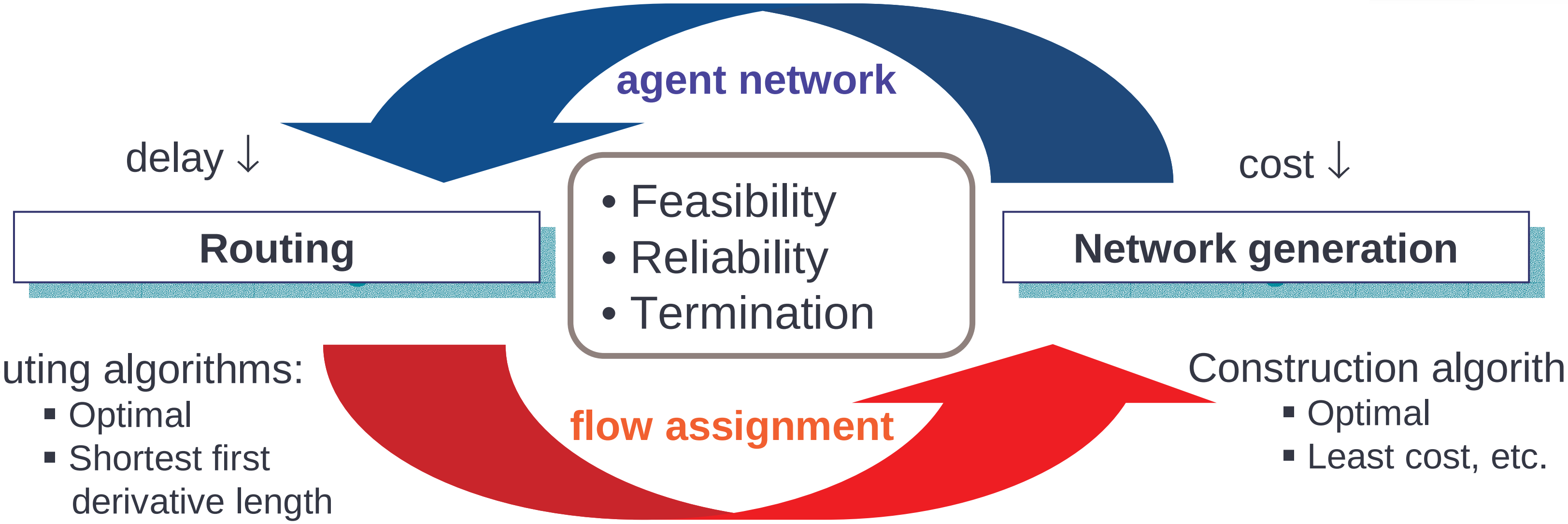
- The ***topology of communication network*** to service the communication requirements:
  - location of the nodes
  - the choice of links, and the capacity of each link
  - info routing strategy



Design objectives:

- Keep the **average delay** per message below a given level (for the given nominal traffic demands and assuming some type of routing algorithm; use M/M/1 delay model)
- Satisfy some **reliability** constraints to guarantee the integrity of the network service in the face of a number of link and node failures
- Minimize the combination of **capital investment** and **operating costs** while meeting above objectives

Heuristic for Topology Generation

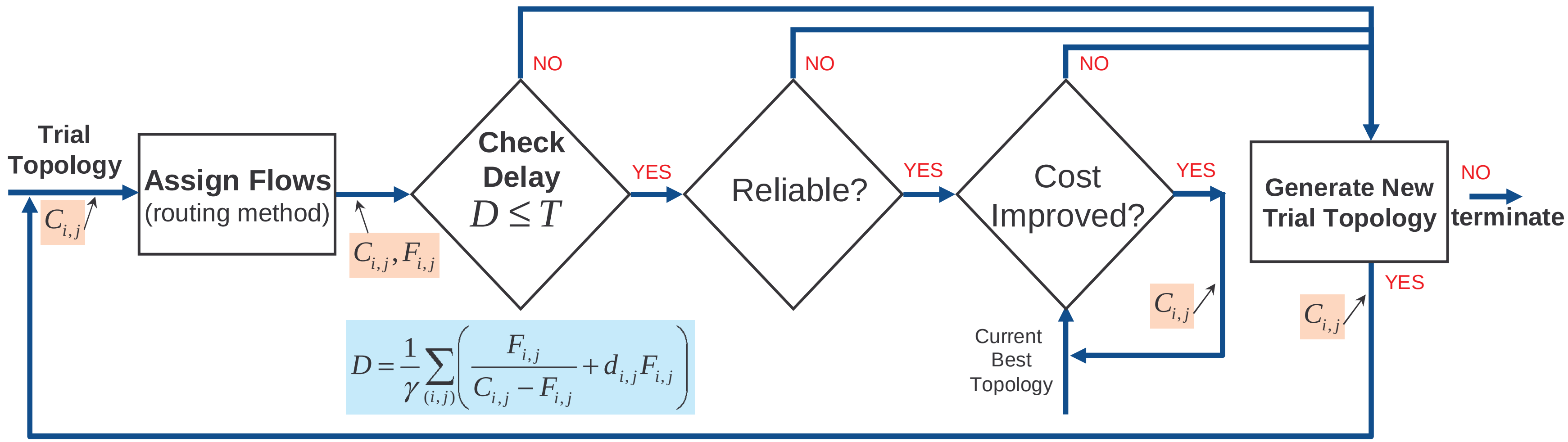


Routing algorithms:

- Optimal
- Shortest first derivative length
- Shortest path
- Shortest hops, etc.

Construction algorithms:

- Optimal
- Least cost, etc.



Example

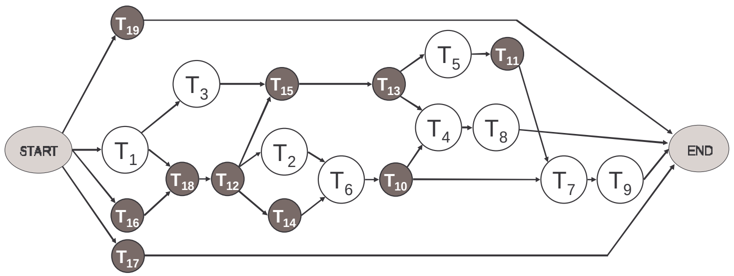
Given:

- Organization consisting of decision-makers (DMs) and assets/platforms
- Mission consisting of communicating tasks (task-graph)
- DM-task allocation and communication requirements due to:
  - ✓ task information flow
  - ✓ synchronization in multi-DM task processing

Want to design:

- The ***topology of communication network*** to service the communication requirements

Task Graph:



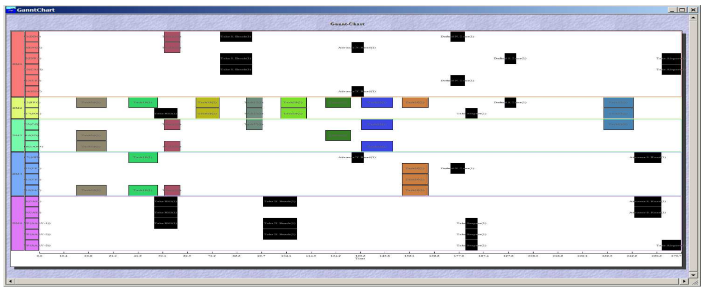
Communication due to simultaneous task processing: $\left[F^{S}_{k,m}(\hat{T})\right]$

	DM1	DM2	DM3	DM4	DM5
DM1	0	5.2	7.1	18.8	5.1
DM2	5.2	0	57.59	37	18.28
DM3	7.1	57.59	0	20.1	0
DM4	18.8	37	20.1	0	11.6
DM5	5.1	18.28	0	11.6	0

Communication due to inter-task information flow: $\left[F^{I}_{k,m}(\hat{T})\right]$

	DM1	DM2	DM3	DM4	DM5
DM1	0	15	5	5	5
DM2	20	0	20	25	15
DM3	15	20	0	15	10
DM4	10	15	0	0	15
DM5	10	10	0	5	0

Gantt-Chart:

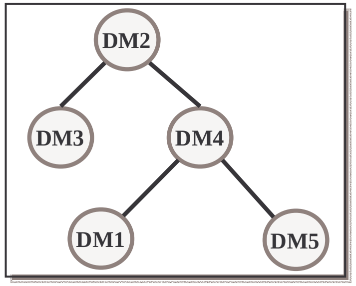


Total Communication Rate Matrix:

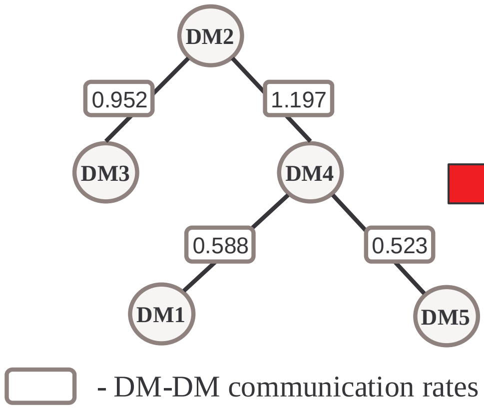
	DM1	DM2	DM3	DM4	DM5
DM1	0	0.075511	0.045232	0.088969	0.037756
DM2	0.094202	0	0.290045	0.231767	0.124407
DM3	0.082614	0.290045	0	0.13121	0.037382
DM4	0.10766	0.194385	0.075137	0	0.099436
DM5	0.056446	0.105716	0	0.062054	0

Example – Hierarchical Structure

Assume fixed network topology:



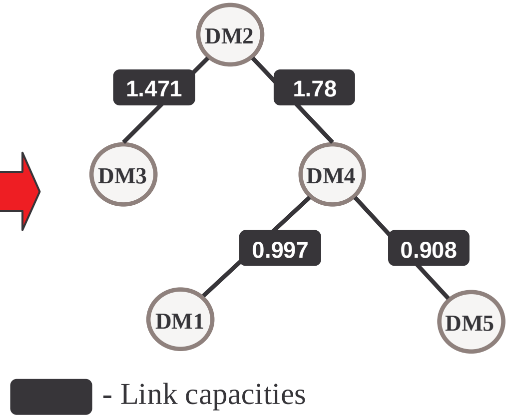
Communication Network
fixed structure ⇒ fixed routing
(unique path among nodes)



Optimal Link Capacities
found according to fixed routing:

$$C_{k,m} = \lambda_{k,m}\left(1 + \frac{1}{\hat{T}_D \gamma} \frac{\sum_{(r,u)} \sqrt{p_{r,u}\lambda_{r,u}}}{\sqrt{\lambda_{k,m} p_{k,m}}}\right)$$

Capacity Network
minimizing average packet delay



Fixed Routing (next DM ID)

Current DM	Destination DM				
	DM1	DM2	DM3	DM4	DM5
DM1	-	4	4	4	4
DM2	4	-	3	4	5
DM3	4	2	-	4	2
DM4	1	2	3	-	5
DM5	4	2	2	4	-

Network Cost:

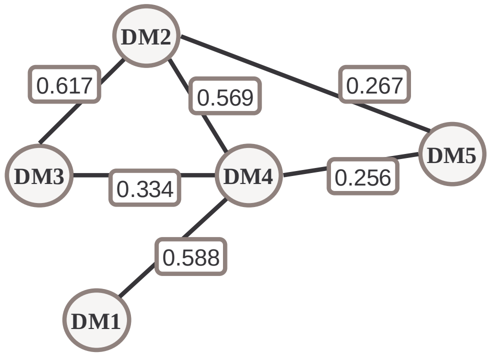
$$\sum_{k,m=1}^{D} p_{k,m} C_{k,m} = 5.156$$

Example – Networked Structure

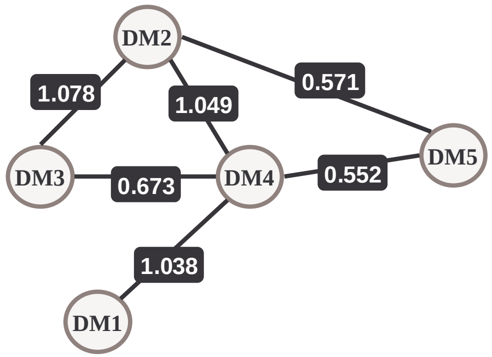
Delay function for link k→m to minimize:
(based on Little's theorem)

$$d_{k,m}(\lambda) = \frac{\lambda}{C_{k,m} - \lambda}$$

Communication Network
(with some routing)

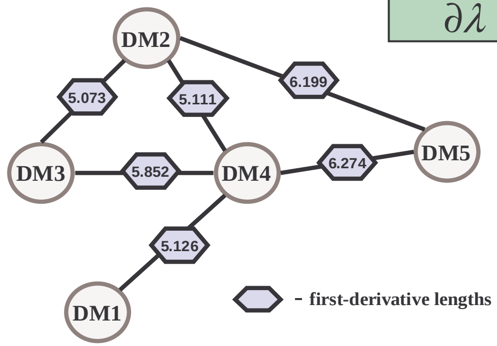


Capacity Network
minimizing average packet delay



Network of First-Derivatives

$$\frac{\partial d_{k,m}(\lambda)}{\partial \lambda}$$



Network Cost:

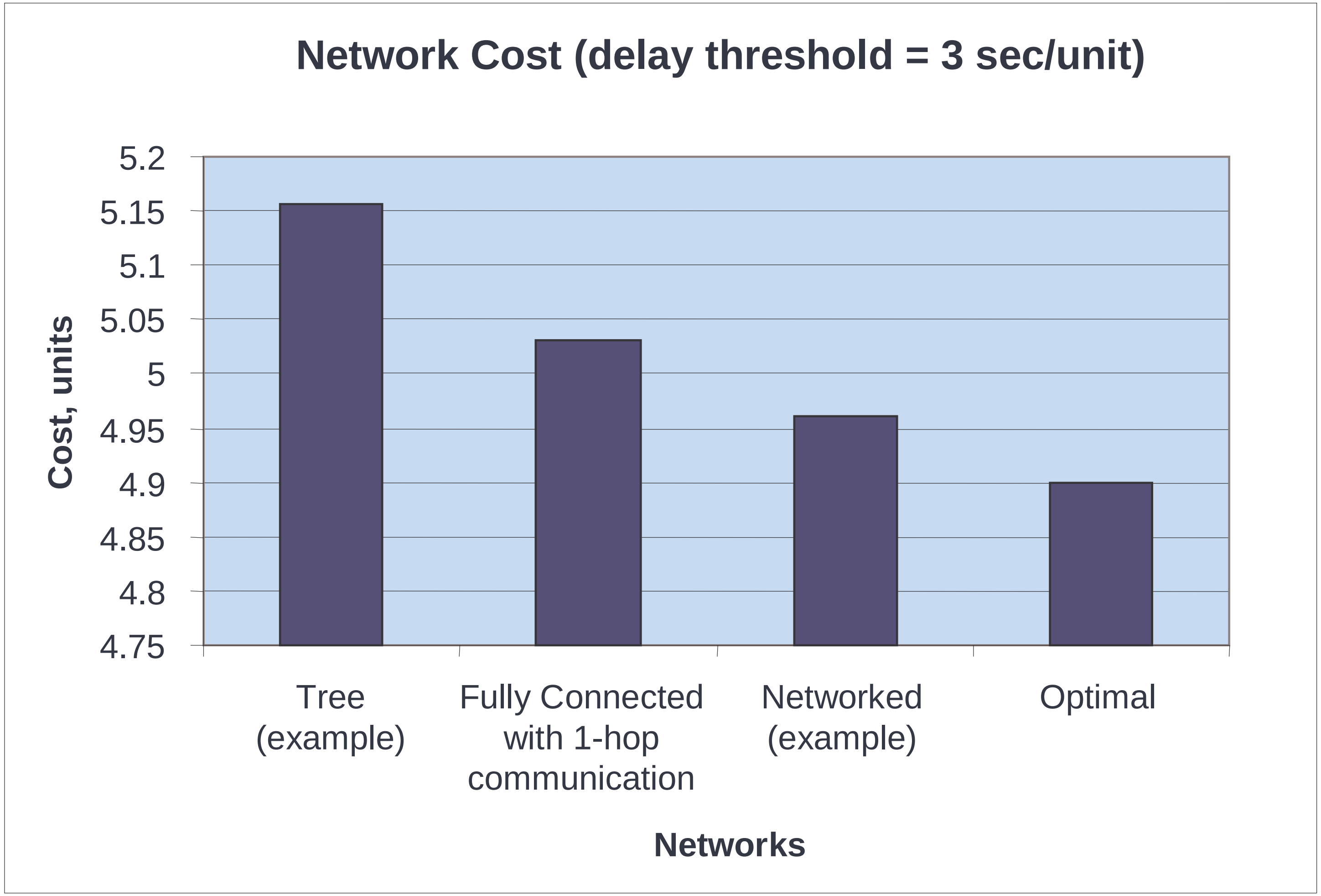
$$\sum_{k,m=1}^{D} p_{k,m} C_{k,m} = 4.961$$

Network and routing are optimal!

Why?

Theorem:
The routing of information is optimal ⇔ routing is performed according to shortest paths in derivative length

Example – Benefits of Networks



Network optimization allows to:

- decrease the cost of its construction while maintaining the same delay rate
- decrease delay while maintaining the same network cost

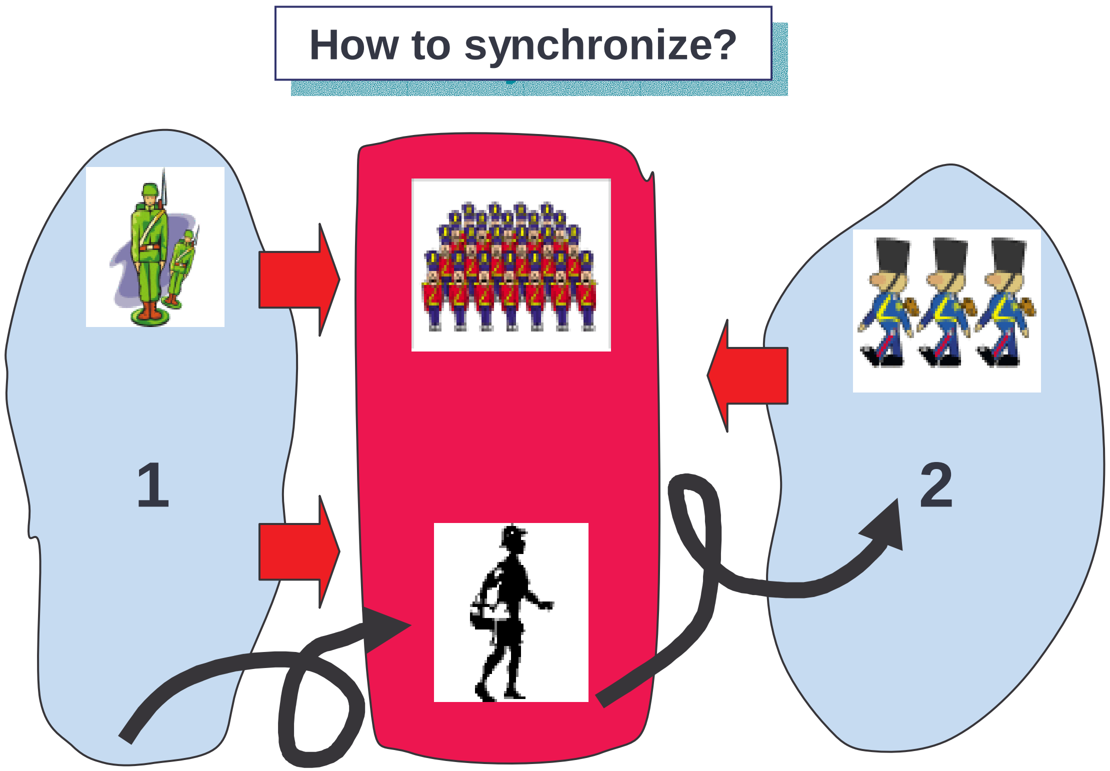
Conclusions

- Efficient ***routing of information*** among agents in the network allows to **minimize system delays**

- We presented the methodology to ***construct networks*** of communicating agents to sustain required communication while optimizing **delay/cost** and satisfying **reliability**

- Applications:
  - Information routing
  - Topology design
  - Training and adaptation applications

Distributed Algorithm Problem



Constraints:

If only **one** **blue** army attacks – **red** wins

If **both** **blue** attack – **blue** win

Communication:

Via messenger

Problem:

Messenger is either caught or goes through, but the sending army does not know that

Message:

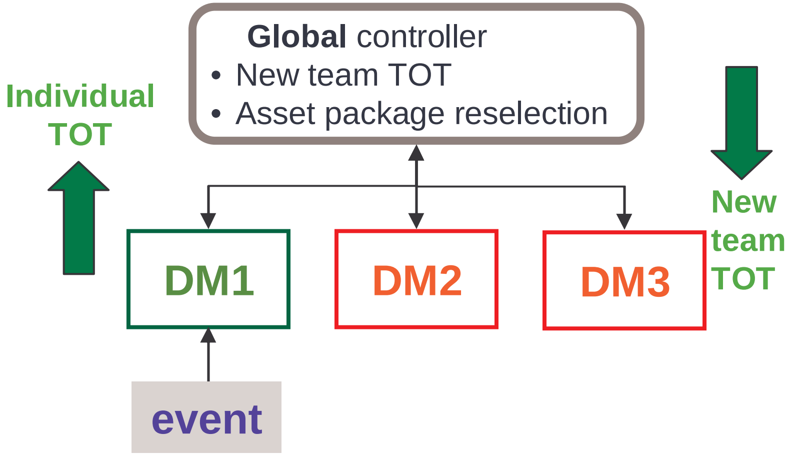
Army 1: "Let's attack at 12pm on Sunday; please acknowledge if you agree"

Army 2: "We agree; send the acknowledgement if you receive our message"

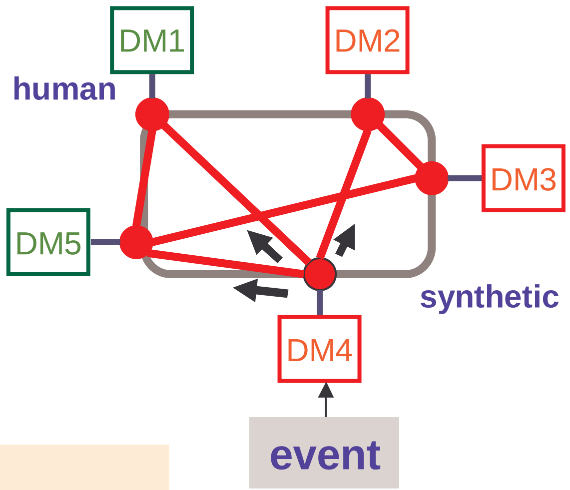
Example: Asset Synchronization

Trigger for communication:
Event, i.e.:
asset is killed, obstacle appears, new TOT is set, etc.

Centralized



Decentralized Networked



Communication:
New TOT via communication network

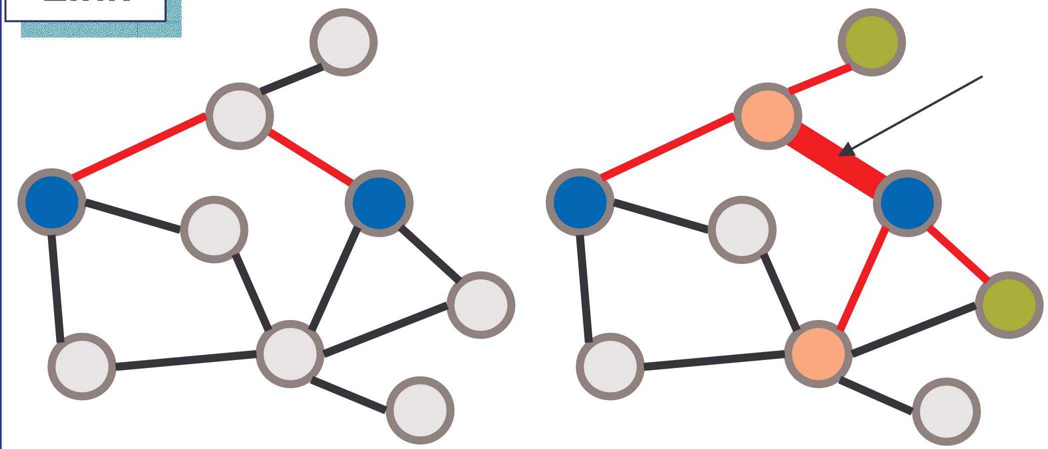
- Optimal solution
- Sensitivity of structure (to global controller functioning)

- Robustness
- Multiple cycles before converging is pair-wise communication
- Teaming ("market") reduces the cycle

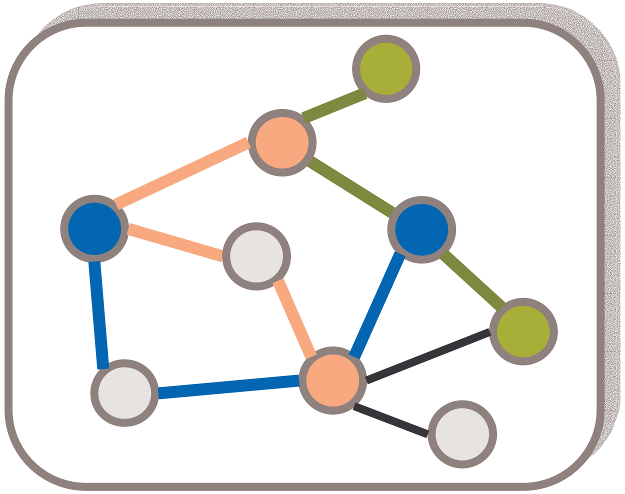
Routing: Overhead

Link

Shortest Path:



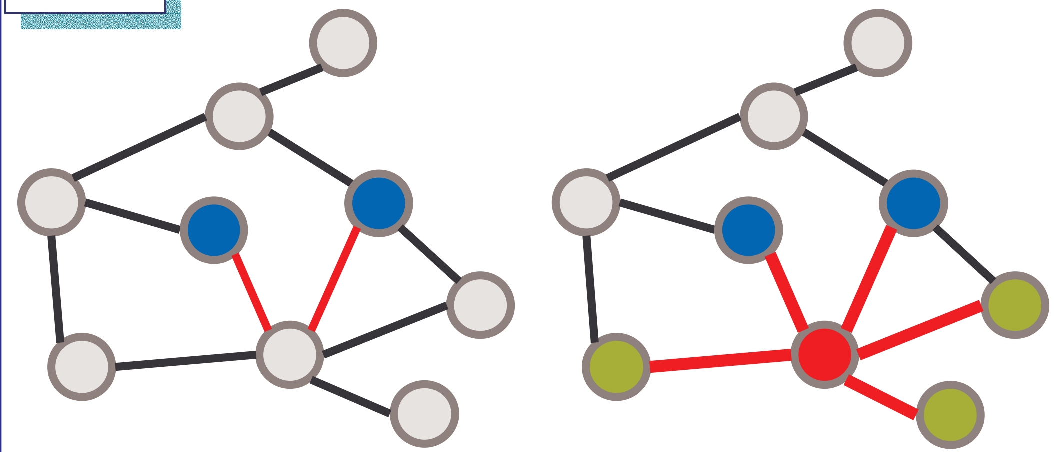
Alternative:

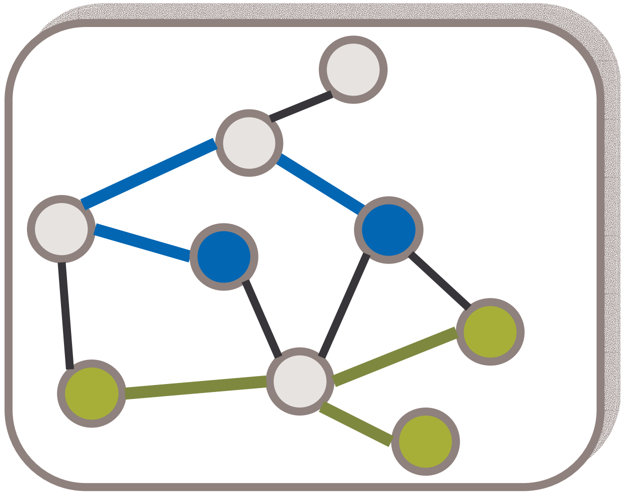


Pro: small overheads
Con: longer paths

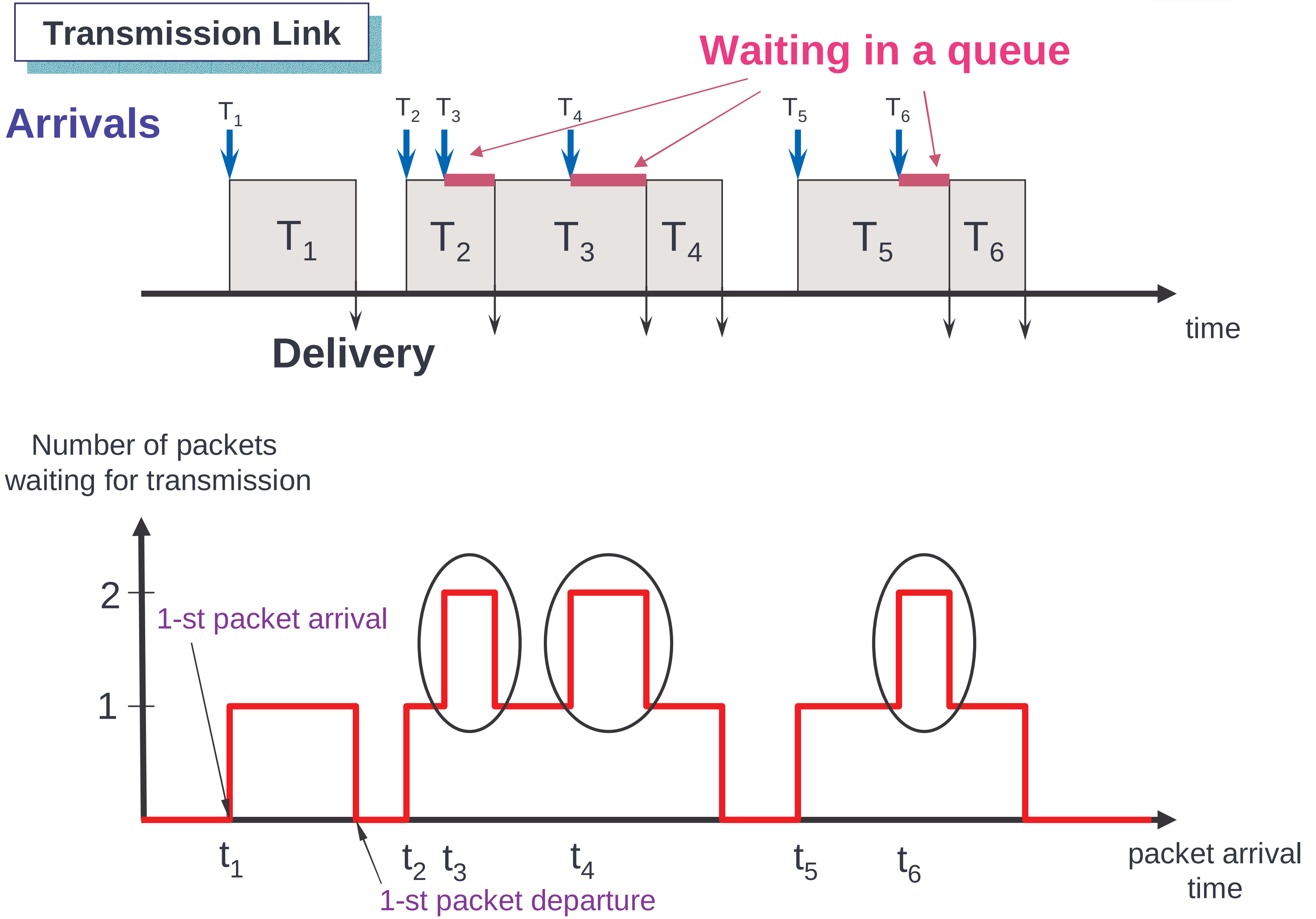
Node

Shortest Path:





Delay Model (single channel)



Delay Model – Little's Theorem

$N(t)$ Number of packets in the system at time ***t***

T_i Time to transfer ***i***-th packet

Time **average arrival rate** over [0,*t*]

$$\lambda_t = \frac{\alpha(t)}{t} \xrightarrow[t\to\infty]{} \lambda$$

Time **average of packet delay** over [0,*t*]

$$T_t = \frac{\sum_{i=0}^{\alpha(t)} T_i}{\alpha(t)} \xrightarrow[t\to\infty]{} T$$

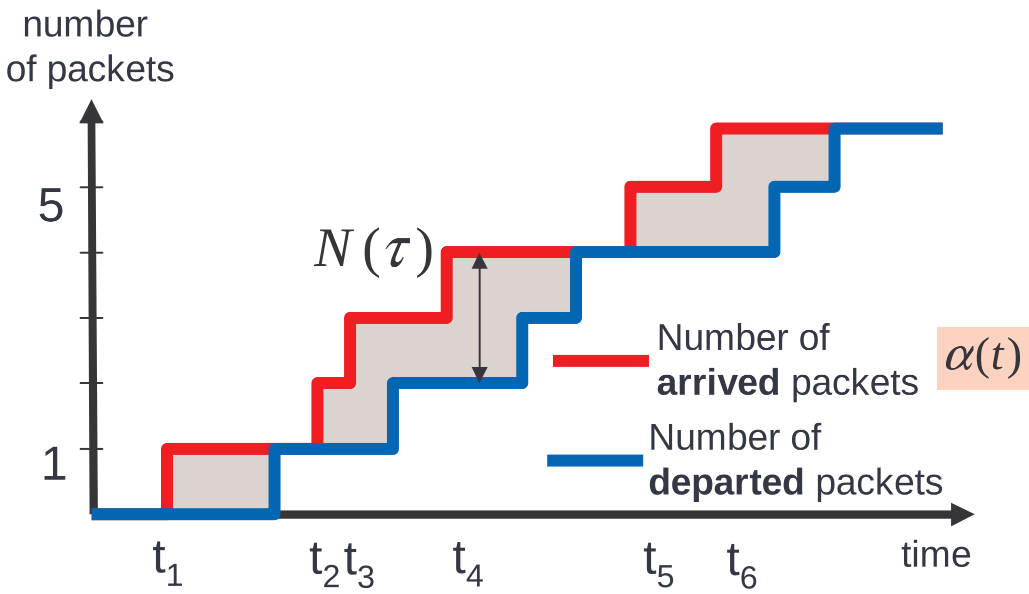
"Typical" (average) **number of packets in the system** observed up to time *t*

$$N_t = \frac{1}{t}\int_0^t N(\tau)d\tau \xrightarrow[t\to\infty]{} N$$

Little's theorem:

$$N = \lambda \cdot T$$

average number of packets in queue = (average arrival rate) • (delay time)



Delay Model – M/M/1

Utilize **M/M/1** queuing model:

- single server (one transmission channel)
- "memoryless" arrival (Poisson) – average number of arrivals per unit time (packets/sec) = λ
- "memoryless" transfer (Poisson) – average number of packets transferred per unit time = μ

<u>Note:</u> definition of Poisson transfer rate only mean that probability to transfer **n**-th packet in ***s*** units of time or less is equal to $P\{s_n \le s\} = 1 - e^{-\mu s}$

Also, average number of packets is equal to

$$N = \frac{\lambda}{\mu - \lambda}$$

<u>Application to information flow and routing:</u>

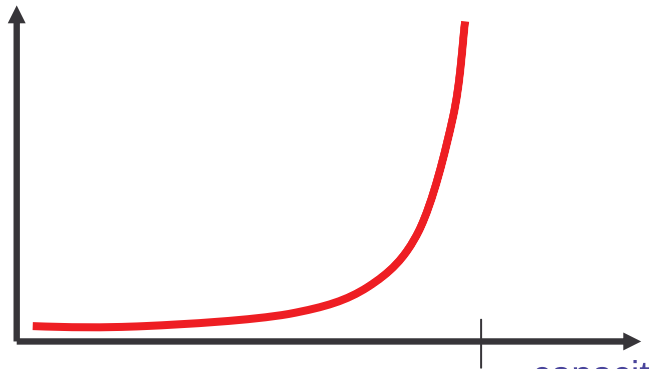
channel capacity = C (bits/sec)

information arrival rate = λ (packets/sec)

average packet length = L (bits)

average delay per packet (wait + transfer) is **proportional** to average packet length and **inversely proportional** to marginal capacity

$$T = \frac{L}{C - \lambda L}$$



Optimal Routing Problem

Input: agent communication requirement network

Need: communication routing (who talks **through** whom)

Objective: minimize overall communication delays

Network communication delay modeling:

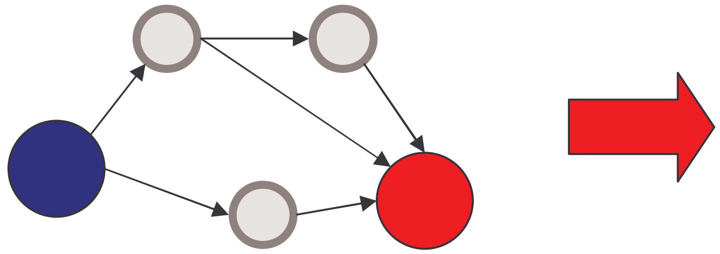
Accumulated (over all packets) delay at a single channel $(i \rightarrow j)$ with flow $F_{i,j}$ and capacity $C_{i,j}$ is equal to

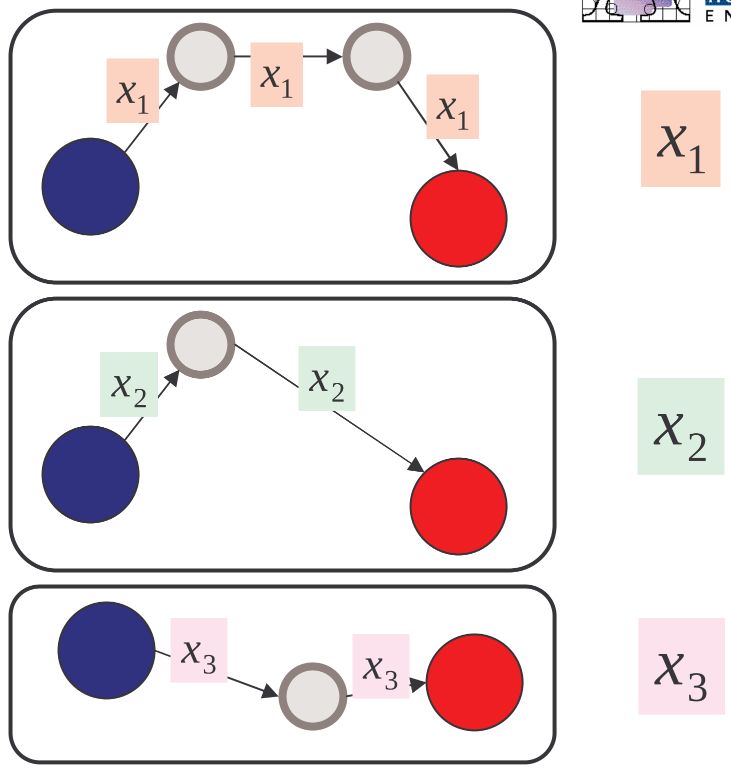
$$D_{i,j}(F_{i,j}) = \frac{F_{i,j}}{C_{i,j} - F_{i,j}}$$

Aggregated network delay:

$$D = \sum_{(i,j)} D_{i,j}(F_{i,j})$$

Optimal Routing Idea





Need to communicate from **blue** to **red** f_w units of information

Information is **split** to smaller portions to be communicated via different routes

$$f_w = x_1 + x_2 + x_3$$

$$F_{i,j} = \sum_{\substack{\text{all paths } p \\ \text{containing link } (i,j)}} x_p$$

$$\min \sum_{(i,j)} D_{i,j}\left[F_{i,j}\right]$$

$$\text{subject to } \begin{cases} \sum_{p \in P_w} x_p = f_w, \text{ for } \forall w \in W \\ x_p \geq 0, \text{ for } \forall p \in P_w, w \in W \end{cases}$$

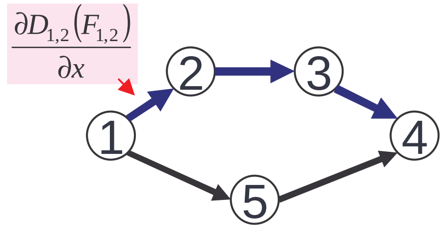
Optimal Routing Solution

Using Lagrangian relaxation, we find:

1. Optimal path flow is positive only on paths with **minimum first derivative flow**

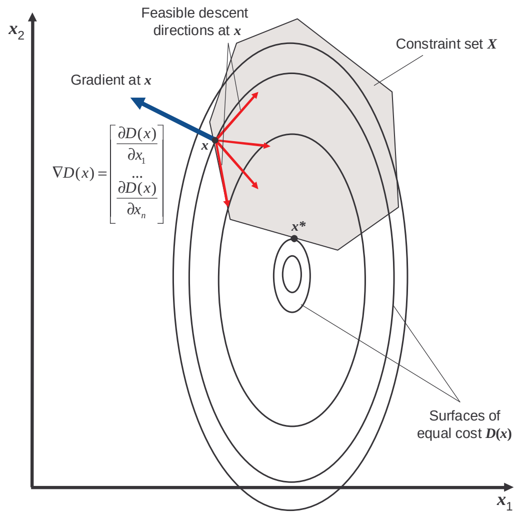
$$\frac{\partial D_{i,j}\left(F_{i,j}\right)}{\partial x}, \quad F_{i,j} = \sum_{\substack{\text{all paths } p \\ \text{containing link } (i,j)}} x_p$$

$$\frac{\partial D_{1,2}\left(F_{1,2}\right)}{\partial x} + \frac{\partial D_{2,3}\left(F_{2,3}\right)}{\partial x} + \frac{\partial D_{3,4}\left(F_{3,4}\right)}{\partial x} \le \frac{\partial D_{1,5}\left(F_{1,5}\right)}{\partial x} + \frac{\partial D_{5,4}\left(F_{5,4}\right)}{\partial x}$$



2. The optimal paths between any two communicating agents, among which the information flow is split, must have **equal first derivative length**

Consequence:
Can use optimization methods, such as
- Feasible directions
- Frank-Wolfe (flow deviation) method
- Projection methods



Problem Formulation

Aptima HUMAN-CENTERED ENGINEERING

Routing

$$\min \sum_{(i,j)} \frac{F_{i,j}}{C_{i,j} - F_{i,j}}$$

$$\text{subject to} \begin{cases} \sum_{p \in P_w} x_p = f_w, \text{ for } \forall w \in W \\ x_p \geq 0, \text{ for } \forall p \in P_w, w \in W \end{cases}$$

- **Objective:** minimize **delays**
- **Constraints:** flow conservation

- **Fixed:** capacities
- **Manipulate:** flow routing

Capacity Assignment

$$\min \sum_{(i,j)} p_{i,j} C_{i,j}$$

$$\text{subject to} \begin{cases} \frac{1}{\gamma} \sum_{(i,j)} \frac{F_{i,j}}{C_{i,j} - F_{i,j}} \leq T \\ C_{i,j} \geq 0, \text{ for } \forall i, j \end{cases}$$

- **Objective:** minimize **network cost**
- **Constraints:** flow conservation, delay below threshold

- **Manipulate:** capacities, flows

Solution to *capacity assignment* problem

Optimal cost:

$$\min_{F_{i,j}} \sum_{(i,j)} p_{i,j} F_{i,j} + \frac{1}{T\gamma} \left(\sum_{(i,j)} \sqrt{p_{i,j} F_{i,j}} \right)^2$$

Optimal capacity:

$$C_{i,j} = F_{i,j} \left(1 + \frac{1}{T\gamma} \frac{\sum_{(m,n)} \sqrt{p_{m,n} F_{m,n}}}{\sqrt{F_{i,j} p_{i,j}}} \right)$$

PROBLEM!!!!!!!!!
Tend to get networks with **low connectivity** (few links with large capacities) – violate **reliability**