COA Modeling with Fuzzy Information

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Abstract

We can model different Courses of Action (COAs) evaluated during the operational military planning as many different activity networks. The corresponding project scheduling model is sufficiently general to be able to represent the most part of military missions. Recently, a project scheduling mathematical model has been proposed where each activity called an action in the military context has different execution modes depending on the resource combination selected.

One way to take into account the uncertainty aspect of the COA duration within the evaluation process is to consider the fuzzy nature of planned activity durations. This fuzzy problem has not been considered within project scheduling when activities have multiple execution modes. But, criterion evaluation of COAs such as risk and cost depends on the estimated COA duration. And, in a world where resources are limited and the risk is an important factor, we must develop a mission schedule offering the best compromise between the fuzzy duration of the COAs, their cost and their risk.

This paper presents a fuzzy multiple mode resource-constrained project scheduling model for evaluating COAs duration. It describes the mathematical foundation developed to perform the project network analysis and it proposes a scheduling procedure to determine the fuzzy COA duration.

1. Introduction

Mission planning

The planning process is critical to the success of any mission. The planning process has six steps. Each step of this process begins with an input from the previous one and builds upon. Nevertheless, this process is iterative and recursive. The initiation step commences with the reception of the mission statements or in an anticipation of a new mission. During this step, the task is assigned or assumed, major combat and logistic resources and strategic transportation assets are identified for planning purposes, the intelligence process initiated, and the groundwork is laid for planning to begin. A soon as the new mission is received, the staff prepare for the mission analysis by gathering a set of tools (e.g. maps of area of operations, both own and higher headquarters' standing operating procedures (SOPs), appropriate documents, estimates). Moreover, during this step, the staff issues a warning order to other supporting and subordinate units. The orientation step is crucial to the CF OPP. It allows the Commander to begin the analysis and definition of the mission, the preparation of the planning guidance and the description of the end states of the operation. The orientation step includes the analysis of the government orders, initial intelligence, assessment of specified, implied and essential tasks, review of the available assets, estimation of the constraints, identification of the critical facts and assumptions, risk assessment, Commander's critical information requirements, initial reconnaissance, mission analysis briefing, development of initial Commander's intent, and issue of Commander's guidance.

The Course of Action (COA) development step involves the entire staff. The Commander's guidance and intent helps the staff to focus on the development of comprehensive and flexible plans within the time available. These COAs "should answer fundamental questions of when, who, what, where, why and how". Each COA should be suitable, feasible, acceptable, exclusive and complete. During the COA development step, staff should analyze the relative combat power (friendly possible actions, enemy's perspective, enemy's vulnerabilities and powers, additional resources, resources allocation, etc.), generate comprehensive COAs (defeat all feasible enemy's COAs, brainstorming, decisive point, cross-fertilization of COAs), and determine initial forces necessary to accomplish the mission.

The decision step is based on the analysis and comparison of the proposed COAs. The analysis of the COAs provides the Commander with precious information to evaluate the quality of these COAs. The main approaches used to analyze the COAs are war-gaming, advantages / disadvantages and comparison criteria. The COA comparison highlights each COA advantages and disadvantages with respects to each other. The COS decides which one he will recommend to the Commander during the Commander's Decision Brief. The COS will decide what detail is necessary to ensure that the Commander is provided with adequate information to make a decision.

COA approval consists of a choice of the best COA according to the Commander's beliefs and estimates. If the Commander rejects all the proposed COAs, then the staff should start the process over again. While the Commander chooses a COA, he may refine his intent, guidance and priorities for execution planning. By deciding on a COA, the Commander assesses what residual risk is acceptable. Based on the Commander's decision and final guidance, the staff refines the COA, complete the planning process and issue orders. The aim of the plan development step is to provide a detailed plan or orders to subordinate headquarters based on the Commander's decision. The plan should go through review and analysis processes. Orders and plans provide all necessary information to subordinates, allies and supporting units to initiate planning or execution of different operations. Finally, the Commander reviews and approves orders before any dissemination.

Project modeling of a COA

The execution of a mission requires the realization of certain activities executed by actors and using specific resources, according to some order and in a given place. The mission can be broken down to its elementary tasks. Assigning resources to each activity and deciding the order in which those activities are to executed (when) and where constitutes a COA. Following the project modeling concepts, any COA can be seen as a set of interrelated activities known as a project, cf. [Guitouni *et al.*, 2000]. Project modeling consists in breaking down generic activities into sub activities up to the point where one obtains a set of primitive activities interrelated to accomplish the mission objectives. To devise one COA as any project, one proceeds with the project decomposition technique known as work breakdown structure. This technique is illustrated in

Figure 1 with the COA *i*. It decomposes and organizes a military mission into primitive actions, called thereafter activities. The followed process results into the object of each activity, the precedence relations with others activities and the resources required for its complete execution. Each combination of resources allocated to the execution of an activity defines its execution mode. The planning process also implies the identification and the obtainment of the pool of available resources.

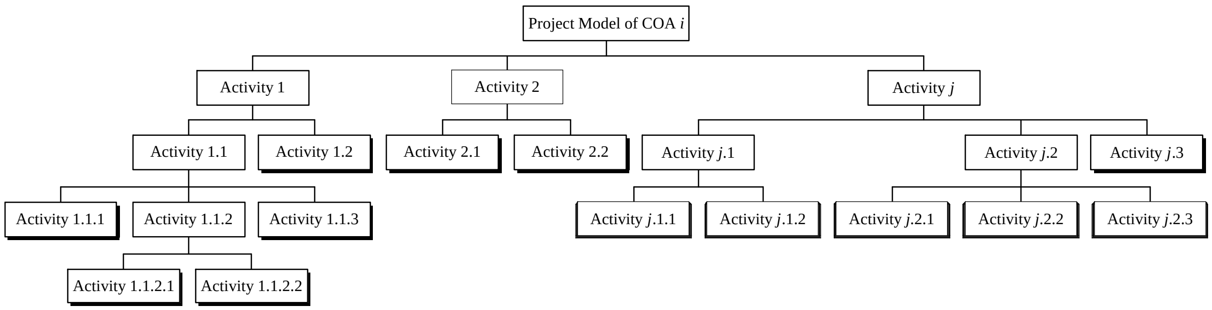

Figure 1 - Work Break-down Structure

Preparation and evaluation of different COAs during mission planning is one of the main Commander's roles. As the Commander may design different COAs to fulfill his mission, one gets as many project instances by modifying the nature of activities, each COA being obtained by creating new activities, by removing others or by changing their technological orders. This set of COAs represents as many as project alternatives A_i, as illustrated in Figure 2.

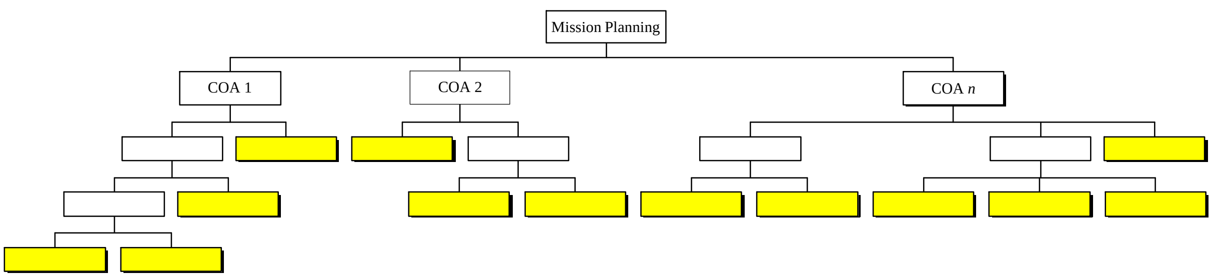

Figure 2 - Set of COA alternatives to fulfill a mission

The Commander has to select the most promising alternative with respect to available resources and to mission objectives. It is a complex and demanding task, which implies the analysis and the processing of multiple information regarding the operational zone, the capabilities of the task force and the available resources. Without appropriate tools, the analysis of different alternatives and the selection of the best COA considering mission objectives become a painful and hazardous process.

This decision problem can be symbolically represented by a decision matrix shown in Figure 3. In this type of problem, the Commander is considering a collection of predetermined alternatives $A = \{A_1, A_2, \ldots, A_n\}$, designated as COA, from which he must select the "best" one. Associated to these alternatives is a set of criteria $C = \{C_1, C_2, \ldots, C_u\}$. Then the values c_{ij} represent the payoff obtained by applying COA A_i evaluated according to criterion C_j. As in many problems meet in our daily lives, all decision problems have multiple and generally conflicting criteria.

	C_1	C_j	C_u
A_1			
A_i		c_{ij}	
A_n			

Figure 3 - Decision matrix

Familiar success criteria of the project are defined to appreciate objective accomplishment, such as the project duration and the project cost. Other criteria intervene when selecting a specific project, the most notable in a combat situation being the risk and the impact. From a practical point of view, one can consider the problem as a multi-attribute decision making when all the alternatives are known and predetermined. One can consider the problem as a multiobjective problem when alternatives have not been predetermined a priori.

Multiple Mode Resource-Constrained Project Scheduling Problem of a COA

The Multiple Mode Resource-Constrained Project Scheduling Problem (MRCPSP) modeling a COA was introduced by [Guitouni *et al.*, 2000], see also [Guitouni *et al.*, 2002]. During the activity engineering, the Commander may specify for each activity different execution modes where a mode corresponds to a specific combination of resources and a given activity duration. Again, these activities grouped according to a temporal and logical sequence define an activity network with multiple modes. Project instances are obtained by changing the resource combinations of activities and the corresponding activity durations. These project instances constitute variants of the same COA i that must be evaluated. The evaluation process of the set of alternatives of one COA with multiple modes is the object of the MRCPSP.

Project scheduling with fuzzy activity durations

Project scheduling refers to the process of defining the best sequence and the starting times of activities according to the objectives pursued. It results in a project plan defining the time t at which each activity should be accomplished with the selected resource combination. Resource-constrained project scheduling problems are NP-hard problems and they can’t be solved to optimality in polynomial time. Considering a fuzzy activity duration with each execution mode constitutes a new problem that is more complex to solve than the one where activities have deterministic durations.

This paper introduces a fuzzy Multiple Mode Resource-Constrained Project Scheduling model to evaluate a COA having activities with multiple execution modes and/or fuzzy activity durations. It describes the mathematical foundation developed to perform the project network analysis and it proposes a scheduling procedure to determine the fuzzy project duration.

Section two introduces the uncertainty modeling considered and the required mathematical operations to apply the CPM with fuzzy time parameters. Section three describes the mathematical model of the scheduling problem. Section four presents the fuzzy priority rule heuristics proposed to evaluate the project duration corresponding to the completion duration of the considered mission. These fuzzy priority rule heuristics are deduced from the scheduling theory in project scheduling. The final section gives conclusions about the advantages of modeling a military mission with uncertainty and future research works.

2. Uncertainty modeling

Commander has to devise plans and to evaluate the duration of each activity. In many military operations, activities are often executed for the first time. Moreover, activity duration is uncertain due to variations in the outside environment, such as weather, equipment failure, site preparation, team productivity level, etc. Commander may have a vague idea about activity durations that must then be estimated subjectively, cf. [Nasution 1994]. It seems natural to represent these activity durations as fuzzy intervals.

Previous attempts to consider the situation when the data are imperfectly known considered activity durations as random variables. Uncertainty is not properly modeled by probability theory. It is known that stochastic problems like PERT are complex problems and face independence problems between random variables, cf. [Lootsma, 1989]. The use of probability distributions implicitly assumes that the past performance of activities has been observed and their distribution has been modeled from these observations. Fuzzy numbers are good at describing the uncertainty about activity durations and the time these activities are planned. They are used to represent imprecise numerical such as "approximately three weeks", "about ten days".

2.1 *Definitions*

Though fuzzy numbers can take various shapes, triangular and trapezoidal fuzzy numbers are the most common in fuzzy scheduling literature. Triangular fuzzy numbers are represented by a triplet (a, m, b) and trapezoidal fuzzy numbers are represented by a quadruple (a, $\underline{m}$, $\overline{m}$, b) where a and b are the lower and the upper bounds of the left-hand and right-hand spreads, while the parameters $\underline{m},\overline{m}$ are the lower and upper modal values, respectively. And, as a generalization, a fuzzy triangular number (FTN) can be viewed as a special case of the fuzzy trapezoidal number (FTrN) for which the lower and the upper modal values are equal (e.g. $\underline{m} = \overline{m}$). Thereafter, we use the *L-R* type representation of fuzzy numbers of [Dubois and Prade, 1988], denoted by $(a, \underline{m}, \overline{m}, b)_{LR}$. A fuzzy number M is a normalized convex fuzzy subset of the real line $\Re$: M = $\{x, \mu_M(x) \mid x \in \Re\}$where $\mu_M(x)$ is the membership function taking values within [0,1] indicating the degree of appurtenance of x to M can be expressed by means of two functions L and R, with four parameters $(\underline{m}, \overline{m}) \in \Re^2$ and a, b in the form:

$$\mu_M(x) = L\left(\frac{\underline{m}-x}{a}\right) \forall\, x \le \underline{m}$$

$$= 1 \;\forall\, x \in [\underline{m},\overline{m}]$$

$$= R\left(\frac{x-\overline{m}}{b}\right) \forall\, x \ge \overline{m}$$

Note that, as a fuzzy interval, an ordinary real number t is written (0, t, t, 0). We can also characterize a FTrN by the interval of confidence at a level α. This is a useful concept to describe different groups of possible values by applying a level cut to fuzzy subset. Let $\alpha \in [0, 1]$. The α–level of set M is the set defined by :

$$I(M,\alpha) = \{x \in M \mid \mu_M(x) \ge \alpha\}.$$

As a practical way of getting suitable membership functions of fuzzy activity durations, it is proposed that the Commander, acting as an expert, specifies the prominent membership levels, see [Rommelfanger, 1990], e.g.:

- μ_M (x) = 1 means that the Commander believes that the value x certainly belongs to the subset of admitted values $[\underline{m}, \overline{m}]$,
- μ_M (x) > λ means that the Commander believes that the value x has a good possibility to belong to the subset of possible values,
- μ_M (x) > ε means that the Commander believes that the value x has a little possibility to belong to the subset of possible values.

A general graphical representation is shown in Figure 4.

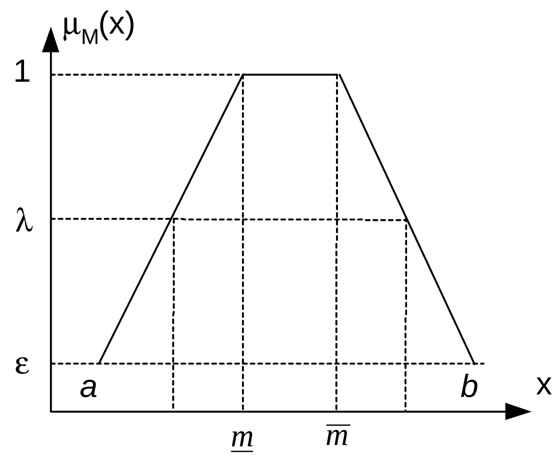

Figure 4 – Linear pieces fuzzy number

2.2 *Computation with fuzzy quantities*

In order to apply scheduling procedures processing fuzzy numbers representing time, one must first determine how to establish and how to compare the sequences of activities to be considered according to their fuzzy durations. Required operations are the addition, the subtraction, the division, the multiplication, the extended minimum, the extended maximum and the comparison. A major advantage of trapezoidal fuzzy numbers is that many operations based on the max-min convolution can be replaced by direct arithmetic operations, cf. [Dubois and Prade, 1988].

Addition of fuzzy numbers

Addition operation on two FTrN gives a FTrN.

$$M_1 \,(+)\, M_2 = (a_1,\ \underline{m}_1, \overline{m}_1,\ b_1)\,(+)\,(a_2,\ \underline{m}_2, \overline{m}_2,\ b_2) = (a_1 + a_2,\ \underline{m}_1 + \underline{m}_2,\ \overline{m}_1 + \overline{m}_2,\ b_1 + b_2)$$

Subtraction of fuzzy numbers

$$M_1 \,(-)\, M_2 = (a_1,\ \underline{m}_1, \overline{m}_1,\ b_1)\,(-)\,(a_2,\ \underline{m}_2, \overline{m}_2,\ b_2) = (a_1 + b_2,\ \underline{m}_1 - \overline{m}_2,\ \overline{m}_1 - \underline{m}_2,\ a_2 + b_1)$$

Symmetric

Symmetric of a FTrN is defined as:

$$(-)\ M_1 = (-\,a_1, -\underline{m}_1, -\overline{m}_1, -\,b_1)$$

Maximum and minimum operators

These operators are defined as follows:

$$\max(M_1, M_2) = (\max(a_1, a_2), \max(\underline{m}_1, \underline{m}_2), \max(\overline{m}_1, \overline{m}_2), \max(b_1, b_2))$$

$$\min(M_1, M_2) = (\min(a_1, a_2), \min(\underline{m}_1, \underline{m}_2), \min(\overline{m}_1, \overline{m}_2), \min(b_1, b_2))$$

Comparison of fuzzy numbers

We consider the comparison of fuzzy numbers proposed by [Roubens, 1990] in the particular case of *L-R* fuzzy numbers. It is based on the compensation of areas and it is reduced to the comparison of upper and lower bounds of α-cuts defined by the following proposition.

Proposition. Let M_1 and M_2 be *L-R* fuzzy numbers with parameters (a_1, $\underline{m}_1, \overline{m}_1$, b_1), (a_2, $\underline{m}_2, \overline{m}_2$, b_2), and reference functions (L_1, R_1), (L_2, R_2), then $M_1 \geq M_2$ *iff*

$$\sup_{xeR} \mathrm{I}(M_1, \alpha_{M_1,R}) + \inf_{xeR} \mathrm{I}(M_1, \alpha_{M_1,L}) \geq \sup_{xeR} \mathrm{I}(M_2, \alpha_{M_2,R}) + \inf_{xeR} \mathrm{I}(M_2, \alpha_{M_2,L})$$

where, if n = 1, 2,

$$\alpha_{M_n,R} = R_n\left(\int_0^1 R_n^{-1}(a)da\right) \text{ and } \alpha_{M_n,L} = L_n\left(\int_0^1 L_n^{-1}(a)da\right).$$

Then, in the case of trapezoidal fuzzy numbers, one obtains:

$$M_1 \geq M_2 \text{ iff } \underline{m}_1 + \overline{m}_1 + ½(b_1 - a_1) \geq \underline{m}_2 + \overline{m}_2 + ½(b_2 - a_2)$$

2.3 *Critical path analysis with fuzzy activity duration*

Previous work on network scheduling based on fuzzy set theory, provides methods for determining the expected fuzzy early times of each event [Chanas and Kamburowski, 1981], [McCahon, 1987], [Dubois and Prade, 1988]. Most of the priority rule heuristics rely on PERT/CPM (Critical Path Method) computation. One of the first attempts to apply the calculation of the PERT analysis with fuzzy duration estimates is by [Dubois and Prade, 1988]. These methods, however, do not support backward pass calculations in a direct manner similar to the one used in the forward pass. This is mainly due to the fact that fuzzy subtraction is not the inverse of fuzzy addition, cf. [Gazdik, 1983], [McCahon, 1993], [Nasution, 1994].

According to the scheduling literature, a project is represented by a directed acyclic activity network G (N, P) where N is the set of activities j, j = 1, …, J, and P is the set of precedence relations between activities. A primitive activity j is designated by a name. For each activity j, one denotes by d_{jm} the activity duration corresponding to the resource combination of activity j executed under mode m, m = 1, …, m_j. Without loss of generality, execution mode numbers are ordered according to the increasing activity duration.

PERT calculation proposed by [Dubois and Prade, 1988] is adapted below to an activity-on-node representation and the network analysis method has to handle activity having multiple execution modes with uncertain duration. Denote by $\tilde{t}_0$ the time origin of the planning process of the activity network and denote by T^{lf} the latest finish time of the project. Denote by P_j the set of predecessors of activity j and denote by S_j the set of successors of activity j. The numbering of activity nodes according to the topological order is required to guarantee that $\forall\ i \in P_j,\ i < j$. To calculate the earliest and latest starting times of the activities, we proceed as follows:

The earliest starting time of an activity j, noted $\tilde{t}_j^{es}$, is given by the forward algorithm:

$$\tilde{t}_j^{es} = \begin{cases} \max\left\{ \tilde{t}_i^{es} + \tilde{d}_{j1} \mid i \in P_j \right\} \text{ if } P_j \neq \varnothing \\ 0 \text{ if } P_j = \varnothing \end{cases} \qquad \forall\ j \in N \qquad (1)$$

Then, one obtains the earliest finish time of the project $T^{ef} = \max\left\{ \tilde{t}_j^{ef} \mid j \in N \right\}$, where $\tilde{t}_j^{ef} = \tilde{t}_j^{es} + \tilde{d}_{j1}$. It corresponds to the critical path length of the project when any resource constraint applies. Denoting by T^{lf} the latest finish time of the project, the latest finish time of activity j is obtained by the following backward algorithm:

$$\tilde{t}_i^{lf} = \begin{cases} \min\limits_{j \in S_i} \left\{ \tilde{t}_j^{ls} \mid \tilde{t}_j^{ls} + \tilde{d}_{j1} = \tilde{t}_j^{lf} \right\} \text{ if } S_i \neq \varnothing \\ T^{lf} \text{ if } S_i = \varnothing \end{cases} \qquad \forall\ i \in N \qquad (2)$$

The interval $[\tilde{t}_j^{ef}, \tilde{t}_j^{lf}]$ represents the slack time of activity j, noted $\tilde{s}_j$, and it is obtained by setting $\tilde{s}_j = \tilde{t}_j^{lf} - \tilde{t}_j^{ef}$. But the criticality of activity j becomes more or less uncertain according to how the fuzzy intervals $[\tilde{t}_j^{ef}, \tilde{t}_j^{lf}]$ overlap. Within the CPM, an activity is considered critical when the interval between the earliest finish time t_j^{ef} and the latest finish time t_j^{lf} of an activity j is null. This is meaningless with imprecise durations. For this reason, [Dubois and Prade, 1988] propose to define the latest finish time of the project T^{lf} independently in an imprecise environment. But, following their approach, earlier activities would end-up being more uncertain than their successors, cf. [Lorterapong, 1995]. When the activities have imprecise durations, represented by fuzzy intervals, the traditional CPM algorithm is still good, if the operations of addition, subtraction, maximization and minimization are replaced by their extensions to fuzzy arguments.

So, representing the trapezoidal number $\tilde{t}_j^{lf}$ by $(a_j^{lf}, \underline{m}_j^{lf}, \overline{m}_j^{lf}, b_j^{lf})$, the trapezoidal number of the fuzzy duration $\tilde{d}_{j1}$ of activity j executed under mode 1 (the shortest mode must be selected for

CPM calculation) by $(a^{d}_{j1}, \underline{m}^{d}_{j1}, \overline{m}^{d}_{j1}, b^{d}_{j1})$ and applying the fuzzy subtraction operation, one obtains:

$$t^{ls}_{j} = (a^{lf}_{j} - a^{d}_{j1}, \underline{m}^{lf}_{j} - \underline{m}^{d}_{j1}, \overline{m}^{lf}_{j} - \overline{m}^{d}_{j1}, b^{lf}_{j} - b^{d}_{j1}) \quad (3)$$

This time is valid if an only if the differences in (3) are non-negative, and

$$\overline{m}^{lf}_{j} - \overline{m}^{d}_{j1} \geq \underline{m}^{lf}_{j} - \underline{m}^{d}_{j1} \quad (4)$$

Usually, these conditions are more often satisfied for those activities at the end of the project network but not for those activities at the beginning of the project network. In general, the equation $\tilde{t}^{ls}_{j} + \tilde{d}_{j1} = \tilde{t}^{lf}_{j}$ in (2) must be approximate by the fuzzy equation $\tilde{t}^{ls}_{j} + \tilde{d}_{j1} \cong \tilde{t}^{lf}_{j}$ with the additional restriction that $\tilde{t}^{ls}_{j} + \tilde{d}_{j1}$ does not exceed $\tilde{t}^{lf}_{j}$, cf. [Lorterapong, 1995], [Ramik and Rommelfanger, 1995]. Then equation of the form (3) must be used when (4) is fulfilled where the spreads are given by:

$$t^{ls}_{j} = (\max(0, a^{lf}_{j} - b^{d}_{j1}), \underline{m}^{lf}_{j} - \underline{m}^{d}_{j1}, \overline{m}^{lf}_{j} - \overline{m}^{d}_{j1}, \max(0, b^{lf}_{j} - a^{d}_{j1})) \quad (5)$$

In the case

$$\overline{m}^{lf}_{i} - \overline{m}^{d}_{i1} < \underline{m}^{lf}_{i} - \underline{m}^{d}_{i1} \quad (6)$$

we adopt the approximation proposed by [Ramik and Rommelfanger, 1995]. Equation (3) is then evaluated by the following formula:

$$t^{ls}_{j} = (\max(0, a^{lf}_{j} - a^{d}_{j1} + (\overline{m}^{lf}_{j} - \overline{m}^{d}_{j1}) - (\underline{m}^{lf}_{j} - \underline{m}m^{d}_{j1})),$$
$$\overline{m}^{lf}_{j} - \overline{m}^{d}_{j1}, \overline{m}^{lf}_{j} - \overline{m}^{d}_{j1}, \max(0, b^{lf}_{j} - b^{d}_{j1})) \quad (7)$$

When condition (6) prevails and additionally $b^{lf}_{j} - b^{d}_{j1} < 0$, then the extended sum $\tilde{t}^{ls}_{j} + \tilde{d}_{j1}$ exceeds $\tilde{t}^{lf}_{j}$ on all membership levels α smaller than 1, if t^{ls}_{j} is calculated according to (7). If we want to avoid this optimism, [Ramik and Rommelfanger, 1995] propose the following formula to calculate the latest starting times:

$$\tilde{t}^{ls}_{j} = (a^{ls}_{j}, \underline{m}^{ls}_{j}, \overline{m}^{ls}_{j}, b^{ls}_{j}) \quad (8)$$

where

$$a^{ls}_{j} = \max(0, a^{lf}_{j} - a^{d}_{j1}) - \max(0, (\underline{m}^{lf}_{j} - \underline{m}^{d}_{j1}) - (\overline{m}^{lf}_{j} - \overline{m}^{d}_{j1})) - \max(0, b^{d}_{j1} - b^{lf}_{j}) \quad (9)$$

$$\underline{m}^{ls}_{j} = \min(\underline{m}^{lf}_{j} - \underline{m}^{d}_{j1}, \overline{m}^{lf}_{j} - \overline{m}^{d}_{j1}) - \max(0, b^{d}_{j1} - b^{ls}_{j}) \quad (10)$$

$$\overline{m}_j^{ls} = \overline{m}_j^{lf} - \overline{m}_{j1}^{d} - \max(0, b_{j1}^{d} - b_j^{lf}) \qquad (11)$$

$$b_j^{ls} = \max(0, b_j^{lf} - b_{j1}^{d}) \qquad (12)$$

Note that the processing of availability time and due-date of each activity j can be introduced within the previous algorithms (1) and (2).

3. Fuzzy Multimode Resource-Constrained Project Scheduling Problem

Suppose the project (e.g. a COA) may require a set R of K renewable resources where each resource type $k \in R$ has a variable resource availability over the time horizon, denoted by Q_{kt}. Defining the zero-one decision variables x_{jmt} equal to one if the activity j executed according to the mode m is completed at the end of period t and zero otherwise, then Figure 5 formulates by zero-one programming the Fuzzy Multimode Resource-Constrained Project Scheduling Problem (FMRCPSP) with fuzzy activity durations. The corresponding notation is given in Table 1.

N	Project activity set, $N = \{1, \ldots, j, \ldots, J\}$
P_j	Predecessor set of activity j within the project graph G (N, P)
x_{jmt}	Decision variable equal to 1 if activity j executed in mode m is completed at period t and o otherwise
d_{jm}	Duration of mode m for activity j
Q_{kt}	Constraint given the amount of resources of type k available at period t
q_{jmk}	Amount of resources of type k required by activity j executed under mode m

Table 1 - Notation of the FMRCPSP model

$$\textit{Minimize} \sum_{m=1}^{m_J} \sum_{t=\tilde{t}_J^{ef}}^{\tilde{t}_J^{lf}} t x_{Jmt} \qquad (13)$$

Subject to the constraint:

$$\sum_{m=1}^{m_j} \sum_{t=\tilde{t}_j^{ef}}^{\tilde{t}_j^{lf}} x_{jmt} = 1 \qquad j = 1, 2, \ldots, J \qquad (14)$$

$$\sum_{m=1}^{m_i} \sum_{t=\tilde{t}_i^{ef}}^{\tilde{t}_i^{lf}} x_{imt} \le \sum_{m=1}^{m_j} \sum_{t=\tilde{t}_j^{ef}}^{\tilde{t}_j^{lf}} (t - \tilde{d}_{jm}) x_{jmt} \qquad \forall\, i \in P_j,\ j = 2, 3, \ldots, J \qquad (15)$$

$$\sum_{j=1}^{J} \sum_{m=1}^{m_j} \sum_{t=t}^{t+\tilde{d}_{jm}-1} q_{jmk} x_{jmt} \le Q_{kt} \qquad k = 1, 2, \ldots, K,\ t = 1, 2, \ldots, T \qquad (16)$$

$$x_{jmt} \in \{0,1\} \qquad j = 1, 2, \ldots, J,\ m = 1, \ldots, m_j,\ t = \tilde{t}_j^{ef}, \ldots, \tilde{t}_j^{lf} \qquad (17)$$

Figure 5 - FMRCPSP model

The objective function (13) minimizes the project duration. This is obtains by scheduling the last activity *J* as soon as possible. Constraint set (14) verifies that each activity $j \in N$ is executed according to only one execution mode *m*. Constraint set (15) specifies the precedence relation between activities. Availability of renewable resources is verified by constraint set (16). The sum of resources of each type *k* required by activities executed at period *t* must not exceed the amount available Q_{kt} . Constraint set (17) specifies the binary values of decision variables x_{jmt}.

The solution of the FMRCPSP is a schedule determining the finish time, noted $\tilde{F}_j$, and the selected mode *m* of each project activity *j*. The problem can be solved by one of the fuzzy zero-one programming methods. However, because of the computational complexity of the problem in real-world mission, these exact methods are restricted to small scheduling problems. So one adopts approximate methods for solving these problems. Approximate methods subdivide mainly between truncated exact methods, construction methods and improvement methods like Tabu Search and Simulated Annealing. Up to now, most of the researches have been dedicated to construction methods based on priority rule heuristics because of their low computational complexity, cf. [Boctor, 1990], [Alvarez-Valdés, 1989], [Kolisch, 1995].

4. Fuzzy project scheduling procedures

4.1 *Scheduling schemes*

Construction methods are characterized according to a scheduling scheme often referred as the serial and the parallel approaches. The serial approach ranks all the project activities according to their priority rule and it schedules each eligible activity *j* at a starting time, noted $\tilde{S}_j$, where required resources are available. The finish time $\tilde{F}_j$ of each activity *j* is then determined by the relation:

$$\tilde{F}_j = \tilde{S}_j + \tilde{d}_{jm_j^s}, \forall j$$

Here, an activity is considered eligible as soon as all its predecessors have been scheduled. The starting time $\tilde{S}_j$ of *j* is such than $\tilde{S}_j \geq \tilde{F}_{j'}, \forall j' \in P_j$. Within the parallel method, eligible activities considered at time $\tilde{t}$ according to their respective priorities, but if an activity having a higher priority can't be scheduled due to resources unavailability, the next prioritized activity is considered. If no more activity can be scheduled, the time *t* is advanced to the next period where an activity in progress will terminate and release resources or where there is enough available resources to schedule an eligible activity. Within the serial approach, the activity priorities remain the same during the whole scheduling process while, within the parallel approach, eligible activity priorities are reevaluated at the beginning of each stage.

4.2 *Formulation of fuzzy priority rules*

A lot of studies have been conducted to determine the relative efficiency of the priority rules used within construction methods mainly for the single mode RCPS, see [Alvarez-Valdés, 1989], [Boctor, 1990], [Davis and Patterson, 1975], [Kolisch, 1995]. For the multiple mode case, fewer results are available. These researches try to identify the best priority rule according to the privileged objective function. Priority rule definitions depend on the scheduling scheme retained. And, depending on the priority rule calculation method, activity priorities may remain the same even if the priority rule is applied within the parallel approach. But then, the resulting quality with the application of parallel approach may be less. [Boctor, 1993] studied different priority rule combinations for the MRCPSP. Selection of activities according to priority rules where experimented with three mode selection rules: the shortest feasible mode (SFM), the least criticality ratio or least critical resource (LCR) and the least resource proportion (LRP). Now, we define activity selection priority rules on the basis of traditional ones and we applied them within the construction methods considering fuzzy times. They are presented in Table 2.

In these definitions, an immediate candidate is an activity *j* which is schedulable if activity *j'* is scheduled to start at the considered period while the remaining work is defined as the sum of the shortest possible durations of the activity *j* considered and all its successors

Priority name	**Priority symbol**	**Priority Value**
Minimum SLacK (SLK) time	MIN SLK	MIN SLK- $\tilde{t}$
Latest Finish Time (LFT)	MIN LFT	MIN LFT- $\tilde{t}$
Maximum number of immediate successors	MAX NIS	MAX $\|S_j\|$
Maximum remaining work	MAX RWK	MAX RWK j
Maximum processing time	MAX PTM	MAX $\tilde{d}_{jm}$
Minimum processing time	MIN PTM	MIN $\tilde{d}_{j1}$
Maximum number of immediate candidates	MAX CAN	MAX CAN

Table 2 - Activity selection priority rules

4.3 *Parallel procedure with SFM heuristic*

We propose to schedule activities according to the parallel scheme. Figure 6 gives the detailed fuzzy parallel procedure proposed to schedule activities with fuzzy durations and multiple execution modes. Table 3 gives the corresponding notation and definition.

N	Project activity set, $N = \{1, \ldots, j, \ldots, J\}$
P_j	Predecessor set of activity j within the project graph $G\,(N, P)$
t	Actual calendar time, $t \in [0, \ldots, T]$ where T is the planning project horizon
C	Set of completed activities at the actual calendar time
E	Set of eligible activities. An activity j is eligible if at time t, all its predecessors are completed and all the required resources are available to its execution for at least one of its execution mode m for the duration $\tilde{d}_{jm}$
X	Set of active activities, e.g. those scheduled but not competed at the actual time
$\tilde{F}_j$	Fuzzy finish time of activity j corresponding to the end of the period where its is completed
M_j	Mode set of activity j
φ	Function giving the activity priority ($\varphi_j = \varphi(j), \forall j \in E$)
Q_{kt}	Renewable resources of type k available at period t

Table 3 - Notation and definition of the fuzzy parallel procedure

1. $\tilde{t} := 0,\ C := \emptyset,\ X := \emptyset$
2. do
3. $E := \{ j \mid j \in N, j \notin C \cup X, P_j \subseteq C, \exists\, m \in M_j \mid q_{jmk} \le Q_{kt}, \forall\, k, t = \tilde{t} + 1, \ldots, \tilde{t} + \tilde{d}_{jm} \}$
4. Compute the priority $\varphi(j), \forall\, j \in E$
5. Order E according to priorities $\varphi_j, j \in E$
6. for each activity $j \in E$ in sequence, do
7. if $\exists\, m \in M_j \mid (q_{jmk} \le Q_{kt}, \forall\, k, t = \tilde{t} + 1, \ldots, \tilde{t} + \tilde{d}_{jm})$ then
8. find $m \in M_j \mid \tilde{F}_j = \min \{ \tilde{t} + \tilde{d}_{jm} \mid q \le t, q_{jmk} \le Q_{kt}, t = \tilde{t} + 1, \ldots, t + \tilde{d}_{jm}, \forall\, k\}$
9. $Q_{kt} := Q_{kt} - q_{jmk},\ t = \tilde{t} + 1, \ldots, \tilde{F}_j, \forall\, k$
10. $X := X \cup \{j\}$
11. $E := E \setminus \{j\}$
12. end if
13. end do
14. $\tilde{q} := \min_{j \in E} \{ t \mid q_{jmk} \le Q_{kt}, t = \tilde{t} + 1, \ldots, \tilde{t} + \tilde{d}_{jm}, \forall\, k, \forall m \in M_j \}$
15. $\tilde{t} := \min\{\min\{\tilde{F}_j \mid j \in X\}, \tilde{q}\}$
16. $C := C \cup \{ j \mid j \in X, \tilde{F}_j = \tilde{t} \}$
17. $X := X \setminus \{ j \mid j \in X, \tilde{F}_j = \tilde{t} \}$
18. while $|C| < |N|$

Figure 6 - Fuzzy parallel procedure with multiple modes

The procedure begins with the empty sets C and X at line 1. It iterates from lines 3 to 18 until all activities are in the completed activity set C. The eligible activity set E is builds at each stage at line 3. The priorities of eligible activities are computed at line 4 and these activities are sorted according to their priorities at line 5. Then, the set E is processed at lines 6 to 13. Here, the rule SFM (Shortest Feasible Mode) is applied at line 8 if there still exists at least one feasible mode. If there is still a feasible mode at line 7 then line 8 determines the shortest mode and line 9 updates the resource availability according to the selected mode m. The scheduled activity is added to the set of active activities X at line 10 and it is removed from the set of eligible activities E at line 11. At line 14, one determines the nearest period $\tilde{q}$ as the beginning period where there are enough available resources to execute at least one of the remaining eligible activities. At line 15, the next scheduling period is set to the minimum fuzzy time $\tilde{t}$ between $\tilde{q}$ and the nearest finish time of active activities. Completed activities at time $\tilde{t}$ are added to the set C at line 16 and they are removed from the set of active activities X at line 17. Line 18 applies the ending criterion, e.g. when all activities have been scheduled.

5. Summary and conclusions

In this paper, we recall the approach used to model a COA according to the project decomposition method. A COA having multiple execution modes and fuzzy activity durations is then modeled as a Fuzzy Multiple Mode Resource-Constrained Project Scheduling Problem. To take into account the uncertainty of activity durations encountered during mission execution, these activity durations are represented by trapezoidal numbers of *L*-*R*-type. The Commander can define different multiple mode COAs and the preferred COA can then be selected according to a multi-criteria method. To evaluate the duration of a given multiple mode COA, a fuzzy parallel construction method based on the parallel scheme and the SFM heuristic rule as activity selection priority rule is proposed. The aim of the fuzzy parallel scheduling procedure proposed is to evaluate the efficiency of different resource combinations to execute a specific COA. A statistical experiment must be designed to assess the performance of the suggested fuzzy parallel method and the priority rules described.

The overall evaluation methodology can be integrated into an interactive decision support system where the Commander can select one COA on the basis of selected performance criteria and his preferences.

The proposed fuzzy multiple mode resource-constrained project scheduling model may be extended to consider fuzzy multiple objective functions simultaneously.

6. References

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