Hunting mobile targets: probabilistic models of mission success and its implications for command and control investment

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Abstract

The development of a military capability requires the development of a system of systems (SoS) comprising of command/control, surveillance and shooter elements. This SoS may be described with an architectural approach, specifying the C2, sensor or shooter nodes, command relationships, information exchanges and an activity model of the sequence of realised events during a conflict situation. In this paper we describe a process to assess different architectures for a particular military capability, that for eliminating time sensitive surface targets through aerial attack platforms. Our assessment method is based on constructing a probabilistic metric, the joint probability that all the events of our activity model occur. This metric will be dependent on a number of performance parameters of our component systems, such as expected C2 decision time, number of available shooters, target-tracking time amongst others. As a result, we generate an effectiveness surface dependent on our system performance parameters. Analysis of this surface yields information as to where next to direct expenditure to maximise overall effectiveness in light of the non-linearities intrinsic to our SoS. We look at trade-offs between decreasing command decision times, increasing the number of shooters and increasing target-tracking time.

1. **Introduction**

A modern military force is composed of a number of systems, each tasked with some specific functionality. Examples include surveillance systems such as ground or aerial based radars, command centres and fighter squadrons. When a collection of such systems must act in unison to perform a complex task, we call the resulting organisation a system of systems because each component system is in a sense autonomous both in action and in acquisition from the other component systems [Manthorpe, 1996]. Military decision-makers realise that to improve a specific capability, we must make acquisition decisions at the SoS level because of the complex interactions between systems. We must understand the non-linearities in SoS interactions to fully exploit potential capability increases and it is this complexity that provides both the advantages and difficulties faced by military decision-makers.

Traditionally SoS interactions have been ignored, with the focus of simulations based on system performance measures as opposed to overall effectiveness. Now we are considering SoS level simulations at many levels of resolution, including low resolution models, agent based models and higher resolution models such as human in the loop distributed simulation and military experimentation [Pew *et al*, 1999]. All such approaches to simulation have trade-offs in the fidelity, reliability and time/costs required to produce results. Take for example, the push toward military experimentation. This involves exercising actual assets and organisations as opposed to virtual entities. Though the results from experimentation have high fidelity, it is difficult and expensive to do the necessary experimental replicates to understand how various performance parameters influence one another.

The approach taken in this paper is to construct a mathematical model of the SoS at a low-resolution level. Starting from an architectural description [DoD, 1997] of our SoS, specifying shooter, command and surveillance assets, information exchanges, command relationships and activities, we map the activity sequence required for a successful mission onto a success metric, which is simply the joint probability that all the events in the activity sequence occur. Our calculations are tractable because the chosen architecture is relatively simple and constructed as a model for SoS research into architecture improvement methods. We have however chosen an architecture, including an activity sequence that is relevant to one of the most difficult tactical tasks faced within a war theatre of operations, the hunting and time-sensitive surface targets (TSST's).

Elimination or interdiction of TSSTs is a systems level subject that is under current study, for two principle reasons. First, even with the sophistication of sensors, aircraft and organisation during the Persian Gulf war, allied forces failed to eliminate mobile theatre launchers before the missiles themselves where launched [Hazlegrove, 2000]. Second, mobile missile launchers are relatively inexpensive and are deployed widely throughout many nations, posing both strategic and political risks [Janes, 2000]. Time-sensitive surface targets are not restricted to the domain of missile launchers. There is an increasing trend towards the mobility of Joint Force headquarters to increase survivability. The elimination of a mobile C2 node is of primary importance.

Several systems level studies of the elimination of TSSTs are of notable mention. The first is the United States Joint Force Command's distributed exercise in hunting TSSTs [Jackson, 2000]. This military experiment exercised sensor platforms, command and control elements and attack systems in a distributed simulation environment. The general results where as follows

- The blue force killed lots of TSSTs
- The red force (commanding the TSSTs) still managed to launch many missiles, with the majority of TSSTs killed after the first launch.

The performance of various sensor and attack systems was measured, showing non-linear relationships in SoS effectiveness [Jackson, 2000]. The second study focused more on a historical analysis of the events in the Persian Gulf war, the lack of a Joint force doctrine and problems passing information to levels lower than that of brigade [Hazlegrove, 2000].

Currently, military SoSs are described using the C4ISR architectural framework, specifying operational nodes, information exchanges, command hierarchies and the activities of the SoS [DoD, 1997]. In this paper, we map the sequence of activities specified by the architecture onto a probability effectiveness metric [Mavris et al, 1999]. This metric is simply the combined probability that all the events will occur. We model target tracking times with a specified distribution, C2 decision time delays with queuing theory and the spatial distribution of attack aircraft or missiles with a two dimensional point process [Kleinrock, 1975].

In this way we can calculate

- The probability that a TSST is destroyed as a function of number of attack aircraft, expected target tracking time, expected search time, given loss of track information, expected C2 latency and the area of operations.
- Expected intercept time as a function of number of attack aircraft, loitering or base launched, the velocity of the attack vehicle and the area of operations. This metric is seen as more important in intercepting TSSTs *before* a missile launch has occurred.

With these results we turn to the question of directing the balance of expenditure towards C2, sensor or attack system comprising the SoS. This is done by doing a sensitivity analysis over the performance variables of interest such as a comparison between the number of attack vehicles and C2 latency times [Neimeier, 1999].

2. Architectural Description of our System of systems

The C4ISR architectural framework requires that a system or complex SoS be described with several viewpoints [DoD, 1997]. First is the contextual view of the SoS, its scope, intended audience and environment. Next are the operational views of node connectivity, information exchange between nodes, organisational description, activity and state transition models. Finally the system and technical aspects of the system need to be described, such as maps of the physical communications nodes and systems specification standards [DoD, 1997].

In constructing our effectiveness metric, our focus is on the use of the operational architecture rather than the technical architecture, to make the analysis independent of current technological standards. Furthermore, we do this to keep the analysis at the appropriate level to avoid over-populating the model with unnecessary parameters. First, we give a high level description of our SoS.

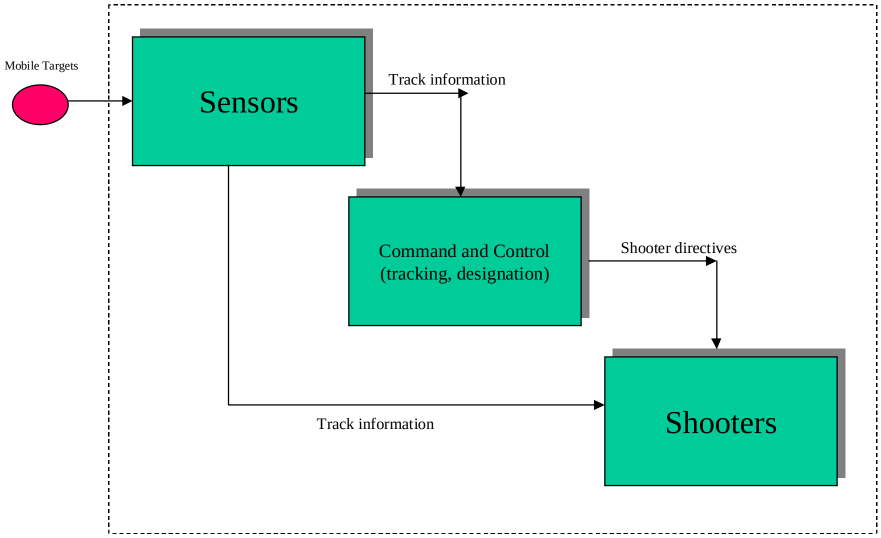

Figure 1: High level description of the SoS for eliminating TSSTs.

This description is the same as that given in many references of network-centric warfare architectures. Next we specify the command hierarchy. We have not specified in detail the command structure of the surveillance system, since our results will only depend on the expected command decision time, the internal structure of command and control being a systems level problem, not a SoS problem.

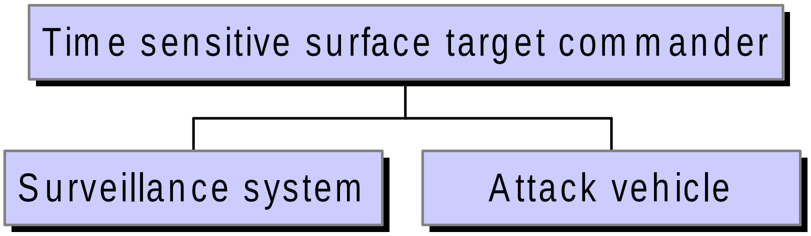

Figure 2: Command hierarchy specified in our architectural description.

In constructing our probabilistic effectiveness metric, the possible actions of the C2, sensor and attack entities will be specified by the information exchanges. The following is a high level description of the information exchanges between sensor, C2 and attack systems.

	C2 system	***Sensor system***	***Attack system***
C2 system			Designation orders, coordinates.
Sensor system	Target coordinates, blue units.		Target coordinates
Attack system	Identification, battle damage assessment.	Target designation information	

Table 1: Information exchanges (from column to row systems) during the hunt for TSSTs.

. The preceding views of the SoS have been static in nature. For completeness, we must outline the state transitions of each system, as the course of the military operation occurs. This amounts to describing a state-transition diagram. Of all the operational architecture descriptions, this is the most complex to achieve as it is necessary to understand the synchronisation between systems and the timing of information flows.

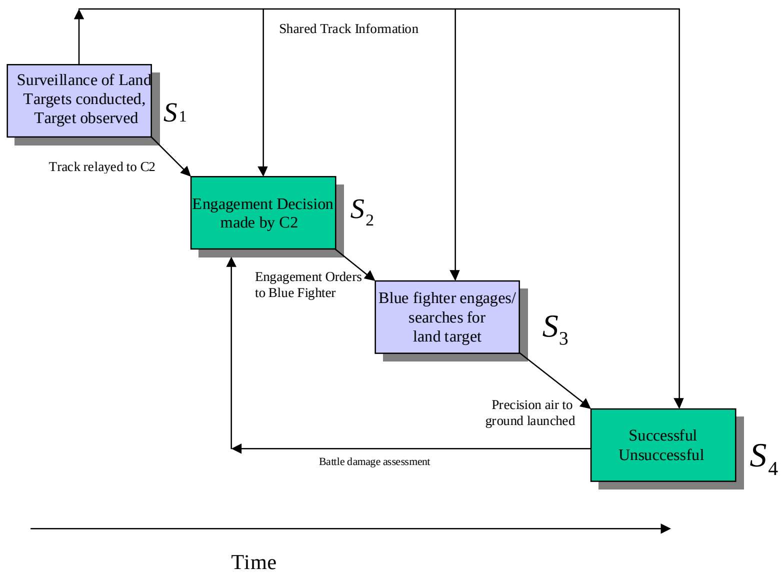

Figure 3: State transition diagram for the respective systems hunting TSSTs.

3. Effectiveness Metrics

Having established a broad low level description of our SoS architecture, we are now in a position to construct a series of effectiveness metrics. We distinguish between two such measures

- An effectiveness metric for the destruction of a missile launcher,
- An effectiveness metric of the *timely* destruction of a missile launcher, before a missile is launched.

We turn to the first problem by considering the state transition diagram in Figure 3. The states are labelled $S_1, S_2, S_{3,} S_4$. The final state, termed the absorbing state in probability texts, is the event that the attack vehicle successfully engages the TSST[1]. Our effectiveness metric is simply the combined probability that all the state transitions $S_1 \rightarrow S_2 \rightarrow S_3 \rightarrow S_4$ occur. These we define our probability of engagement to be

$$\Pr(engage) = \Pr(S_1, S_2, S_3, S_4).$$

The details of the derivation of a success probability will depend on the processes that lead to the four states. First there is the distribution of time that the sensor system is able to track the TSST. Second, the delay of the C2 system in receiving the target track, prioritising this information and dispatching the appropriate shooter to the target. Finally, we must also consider the travel time of the attack vehicle to the track site, and the search time if track information is maintained upon target engagement or a loss of track information has occurred, see [Calbert and Moon, forthcoming] for details. The following table outlines the key parameters involved in the construction of the probabilistic model.

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[1] We do not consider the effects of precision strike or battle damage assessment in this current model, with the assumption that a successful mission will be carried out if the attack vehicle is actually able to engage the target.

Symbol	Meaning
T_{C2}	Expected C2 decision time
T_{Track}	Expected tracking time of a target
T_{search}	Expected target search time, given no track information
A	Area of operations
m	Number of loitering attack vehicles available
v	Cruise velocity of attack vehicle
T_{launch}	Expected inter-launch time for TSSTs.
T_{travel}	Expected travel time from current position to TSST engagement point (a function of $A, m, v.$)

Table 2: Parameters used in the effectiveness metrics.

Using the theory of point processes, the expected travel time, given that the attack vehicles loiter within the area of operations, A, is given by [Calbert and Moon, forthcoming]

$$T_{travel} = \frac{1}{2v}\sqrt{\frac{A}{m}}.$$

Assuming that the travel distance to the target is not too great, it can be shown that the probability of hitting the target is approximated by

$$\Pr(engage) \approx \frac{\left(\frac{T_{track}{}^{2}}{(T_{track}+T_{C2})} - \frac{T_{search}{}^{2}}{(T_{search}+T_{C2})}\right)}{T_{track}-T_{search}}.$$

We may use this formula to do sensitivity analysis over our critical systems variables such as track time and C2 decision time. When travel time is significant compared to the other variables then we have

$$\Pr(engage) = \frac{\left(T_{track}\, w_{C2}(\frac{1}{T_{track}})\, w_{travel}(\frac{1}{T_{track}}) - T_{search}\, w_{C2}(\frac{1}{T_{search}})\, w_{travel}(\frac{1}{T_{search}})\right)}{T_{track}-T_{search}}$$

where,

$$w_{C2}(s) = \frac{1}{1+T_{C2}s}$$

and

$$w_{travel}(s) = 1 - sT_{travel}\exp\left(\frac{(sT_{travel})^2}{\pi}\right) erfc\left(\frac{sT_{travel}}{\sqrt{\pi}}\right)$$[2]

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[2] The function erfc is the complementary error function, see [Calbert and Moon, forthcoming]

Having considered an effectiveness metric for the destruction of a mobile target, let us turn to its timely destruction. We would like to destroy the TSST before it is able to launch a missile. The effectiveness metric is simply the probability that the total C2 time plus the total travel time is less than that of the launch time of the TSST. Our effectiveness metric can be shown to take the form

$$\Pr(engage) = T_{launch} w_{C2}\left(\frac{1}{T_{launch}}\right) w_{travel}\left(\frac{1}{T_{launch}}\right)$$

In this paper, we will only focus on the engagement probability in hunting a TSST and not on its timely hunting, this will be the subject of a forthcoming paper.

4. **Sensitivity Analysis**

Since our focus is on the choice between purchasing more attack platforms or the improvement of command and control in the engagement of TSSTs, the types of analysis that should be considered are

- sensitivity over the two variables, C2 latency time, T_{C2}, and the number of attack platforms m, when making a decision about C2 software purchases, staffing or purchasing attack vehicles.
- Sensitivity over three variables, C2 latency time, T_{C2}, the number of attack platforms, m and the velocity, v when making a decision about C2 software purchases, staffing or the number and type of attack vehicle.

The role of sensitivity analysis is to inform the decision maker about where next to invest funds, in our case in C2, more attack vehicles or another type of attack vehicle. It is clear that by decreasing the C2 decision cycle, increasing the number of attack vehicles or increasing the velocity of such vehicles we will improve the overall SoS effectiveness. This however is not sufficient, when making decisions across differing systems. We must look at the *nature of incremental effectiveness increases*, as a function of our component systems performance parameters to make our decisions. A simple way to choose between differing systems to improve upon is to look at what systems, when improved, give forth *diminishing returns* in effectiveness versus *increasing returns.* A variable with increasing returns should be improved upon first. In contrast, performance variables that exhibit diminishing returns must be improved upon in large, usually expensive quantum leaps. Thus, by examining the nature of the effectiveness increments, we may exploit the non-linearities of our SoS interactions to our advantage.

Let us first consider the SoS effectiveness as a function of area of operations. Effectiveness as a function of two variables is important for two reasons, first for a decision on the overall viability of such a mission and second area of operation restriction. Though this may not seem to be an independent variable, it may be possible to restrict an enemy’s operation area, through the use of patrols, or roadblocks.

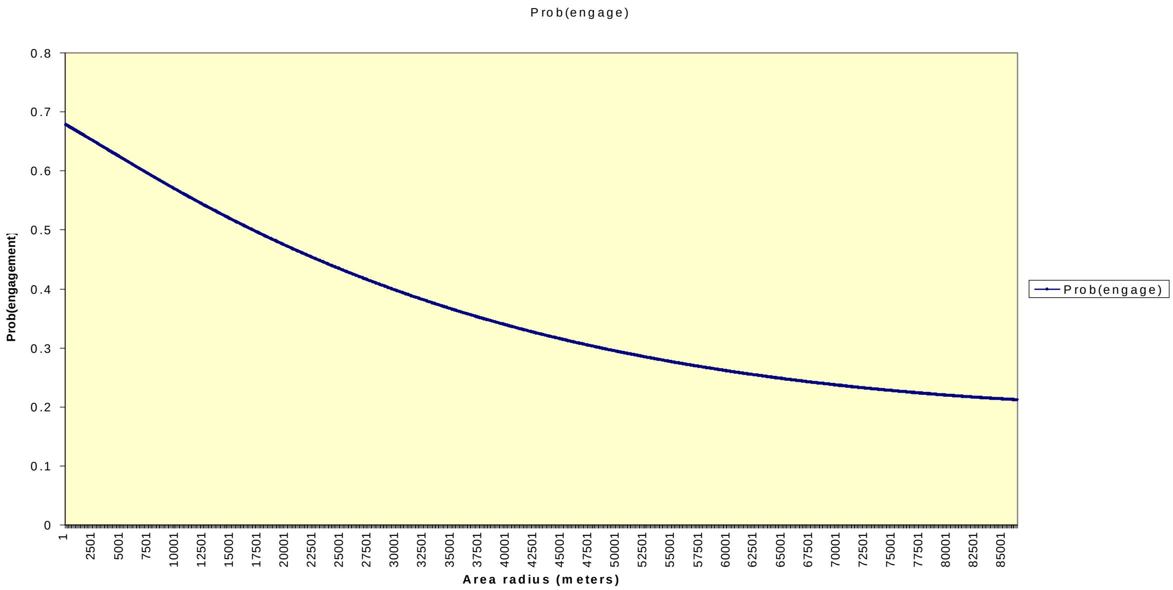

Figure 4: Sensitivity analysis of engagement probability as a function of the radius of a circular area. The parameters used in this model are $T_{C2} = 10\,\min, T_{track} = 10\,\min,$ **# of attack vehicles =20,** $v = 100m/s.$

The graph displayed above (Figure4) shows the sensitivity analysis of engagement probability as a function of the radius of the area of operations, assuming a circular area. The radius ranges from 100 metres to 8.6 kilometres. As expected, we see a decrease in the probability of engagement that is non-linear. This suggests that an incremental decrease in the area of operations disproportionately increases the probability of target engagement.

Let us turn to the number of attack vehicles allocated to the mission.

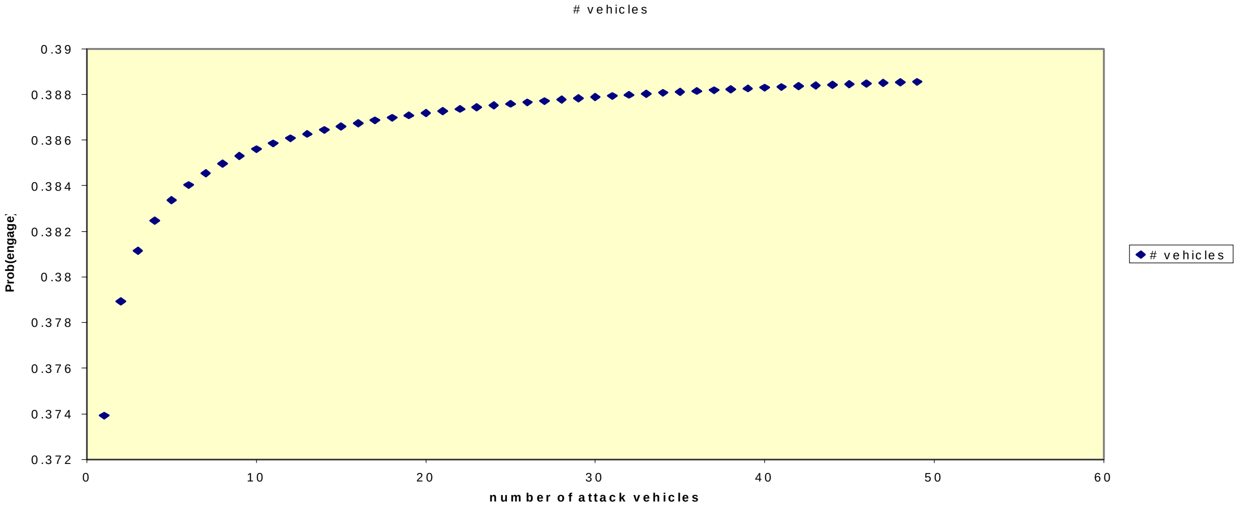

Figure 5: Sensitivity analysis of engagement probability, as a function of the number of attack vehicles. The parameters used were $A = 1000000m^2$,
$T_{track} = 10\,\min, T_{search} = 1\,\min, v = 100m/s.$

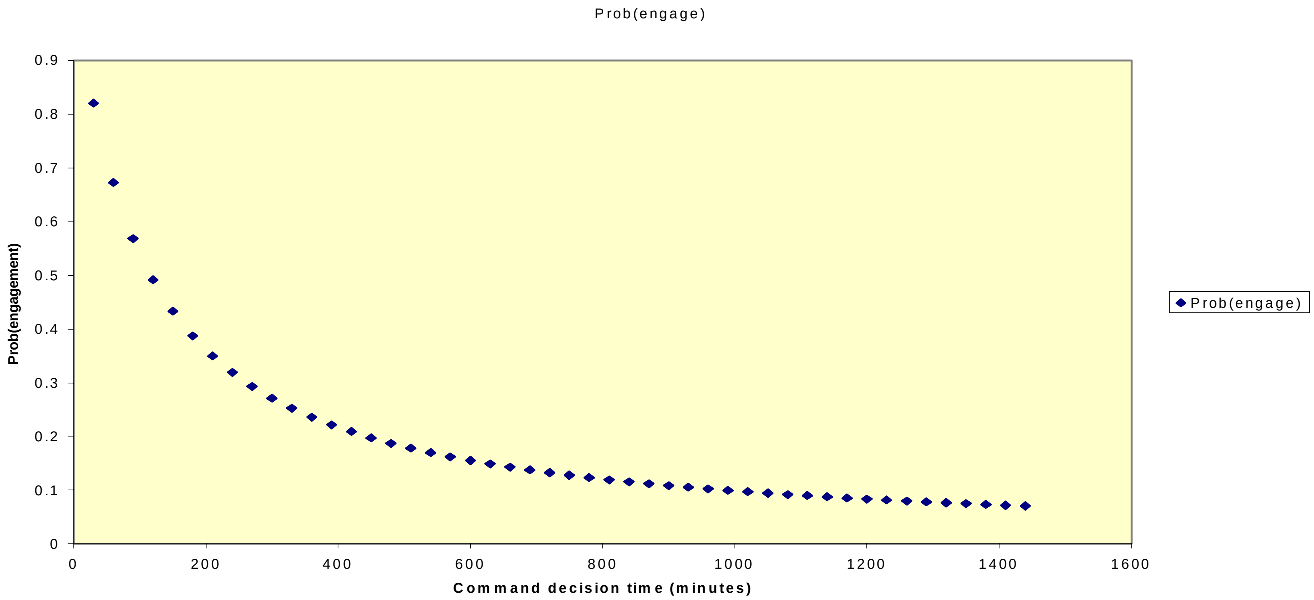

Figure 6: Sensitivity analysis of the probability of engagement as a function of command decision time, T_{C2} . Performance parameters

$$A = 1000000 sqrm, m = 20, T_{track} = 10\min, T_{search} = 10\min s, v = 100 m/s .$$

Through we see an increase in the probability of engagement, we have decreasing returns as we increase the number of attack vehicles. In investment terms, this means that there will an optimum number of vehicles to purchase before the effectiveness returns are no longer warranted, as investment costs increase linearly with the number of attack vehicles purchased.

The role of command and control has received a lot of attention, especially due to the experiences of the Persian Gulf War [Hazlegrove, 2000]. Sensitivity analysis confirms the importance of reducing the overall command decision time, as is seen in Figure 6.

A decrease in the command decision time increases the probability of engagement with *increasing returns*. Here, we have graphed the probability of engagement, moving from the traditional 24-hour air tasking order cycle (1440 minutes) to smaller time scales. This result shows that for every decrement in the command decision time we have a much larger increase in overall SoS effectiveness.

Finally, let us consider effectiveness amongst differing delivery platforms. We do not specify the exact details of the platform, as this is not the aim of an analysis at the SoS level. Instead, we may distinguish platform types simply by velocity. For example, an armed reconnaissance helicopter will generally have a cruising speed of 70m/s, a fast jet, 300m/s and a hypersonic missile will travel at velocities of Mach 5 or 6 (1500m/s) [Jane's, 2000], [Childs, 1972]. The results of sensitivity analysis over differing velocities are seen in the following Figure 7.

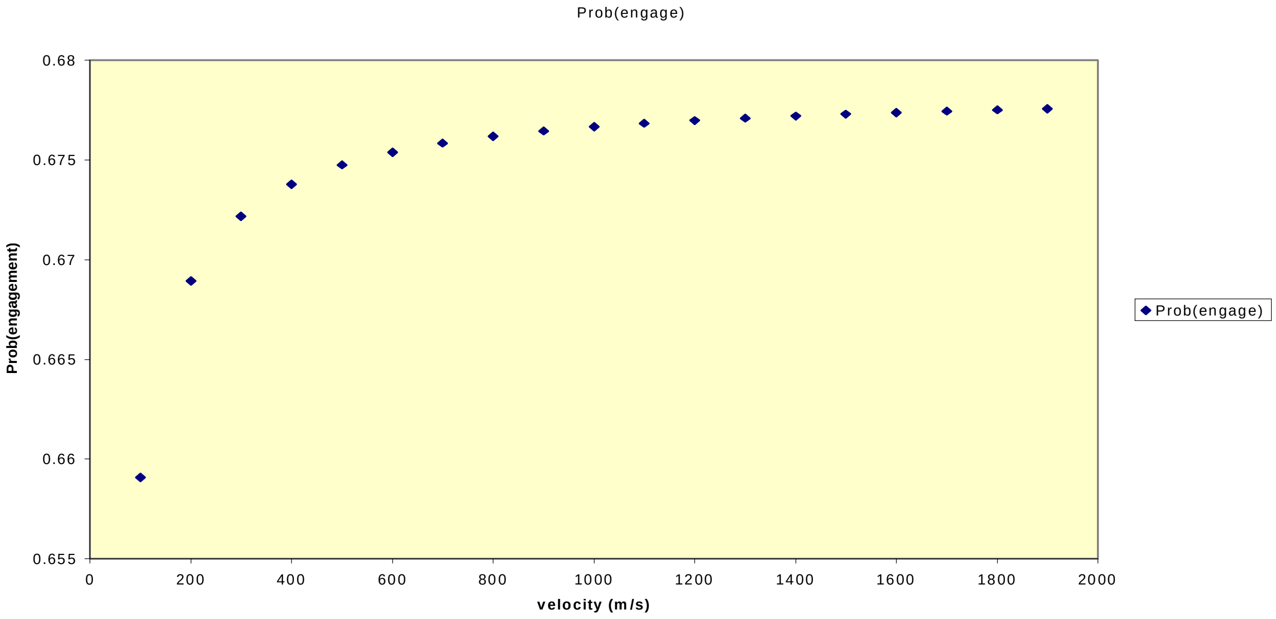

Figure 7: Sensitivity analysis over velocity. Our performance parameters are
$T_{C2} = 30 \min s, A = 1000000 sqrm, T_{track} = 10 \min s, T_{search} = 10 \min s$.

It is not surprising to see diminishing returns with velocity, the same effect as increasing the number of attack platforms. Essentially, increasing velocity decreases the travel time of loitering platforms. With more platforms, we also decrease the expected travel time, as there is a higher probability of an attack vehicle in the vicinity of the TSST. Also note the changes in engagement probability are *less than 10%*, both for both velocity and attack vehicle number.

5. What does this mean for System of system design?

Commanders have long recognised the importance of the improving command and control, through better intelligence dissemination, communications and decision-making. A quote from the Colonel Michael F. Reavey, Director of Night Operations during the Persian Gulf War encapsulates this: “Our problem was not how much air we had…Our problem started to become how much airspace we had and wedging what we had into that piece of airspace.” The decision of airspace deconflication is clearly a command and control problem.

When making decisions in regard to designing a close air support system, command and control organisational structures, streamlined through the used of automated software are essential. If our attack vehicle force structure is already in place, as is the case for smaller countries like Australia, we must make strides to improve upon command and control rather than attempt to purchase faster attack vehicles. Upon viewing Figure 6 we see that the 24-hour tasking cycle is not adequate, with engagement probabilities down to less than 10%.

Diminishing returns, in attack vehicle number and velocity is intrinsic to the nature of TSST targeting and close air support, thus we require quantum leaps in attack vehicle velocity or number. This is evidenced in the United States Joint Forces Command’s first Joint experiment, hunting time-critical surface targets [Jackson, 2000]. Over 90% of targets where killed by weapons that had either a substantial increase in velocity

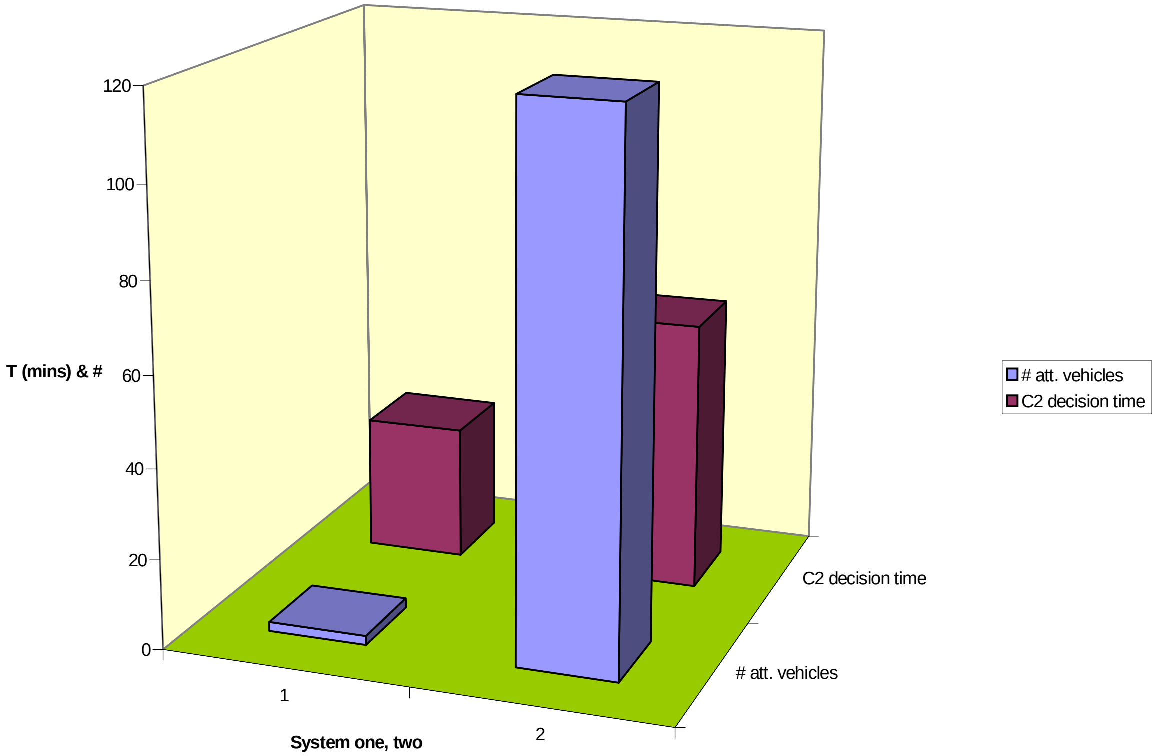

Figure 8: Systems of systems of equivalent performance, in terms of command decision time and attack vehicle number. The other performance parameters that characterise this result are $T_{track} = 10\min s, T_{search} = 10\min s, A = 1000000 sqrm, v = 100 m/s.$

(the future hypersonic missile) or number (the future army missile system, a loitering missile, which dispenses bomblets equipped with IR). These solutions are available to countries like the United States, but not to a country like Australia, where command and control development must come first.

To appreciate the importance of command and control, let us consider two “equivalent”- in the sense of probability of engagement, systems of systems. For an 80% probability of mission success, the sensitivity analysis shows that we need two attack vehicles and a command decision time of approximately 30 minutes. Equivalently, we require for a command decision time of 1 hour approximately 120 attack vehicles. At three hours, any number of attack vehicles will be insufficient to achieve an 80% probability of success. These results are displayed in Figure 8.

6. **Conclusions**

Tempo has always been a crucial, if not deciding factor in winning wars. With modern conflicts being fought with complex systems of systems, it is no longer sufficient just to talk of the importance of tempo as it may be decomposed into complex processes such as intelligence collection, command decision time and ordinance delivery. Performance increments in each of these areas will improve SoS effectiveness. We must however focus our attention on those component systems that give the greatest effectiveness returns.

In this paper we have analysed, through low-resolution modelling, SoS effectiveness in the hunting of time-sensitive surface targets. Three simple conclusions can be drawn,

- increasing the number of attack assets or attack asset velocity improves SoS effectiveness, with *diminishing returns,*
- area restriction increases effectiveness with increasing returns,
- by far the greatest effectiveness improvement comes from decrements in the command decision cycle.

These observations were drawn from an analysis that mapped performance parameters of our SoS architecture onto overall effectiveness, as measured by a probabilistic metric. In essence, this enabled the "decoupling" of performance parameter effects, whilst keeping the non-linear system interactions intact.

Presently, there is a thrust towards "military experimentation", combining extensive distributed simulations with real assets, command posts and hypothetical weapons of the future. This low-resolution simulation may be thought of as a preliminary testing of the parameter space, before the costly and timely military experiment is carried out. In particular, this paper suggests that high-resolution experiments should be carried out, comparing mission effectiveness across different loitering platform numbers, compared to restrictions on decision times, through automated software in decision aids.

7. **References**

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