A Heuristic Approach for Belief Revision in Timed Influence Nets

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Outline

- Introduction of Probabilistic Graphical Models
  - Bayesian Networks
  - Influence Nets
- Specification of Timed Influence Nets
- Belief updating in Probabilistic Graphical Models
- An algorithm for approximate belief revision in Timed Influence Nets
- Conclusions and future research directions

Important Probability Concepts

P(A, B) = P(A) P(B) (A and B are independent)

P(A | B, C) = P(A | C) (A and B are conditionally independent given C)

Bayes Theorem: $P(A \mid B) = \dfrac{P(B \mid A)\,P(A)}{P(B)}$

Advantages of Probabilistic Belief Networks

64 probability values are required to represent the joint distribution of 6 binary state variables, i.e., $2^6 = 64$

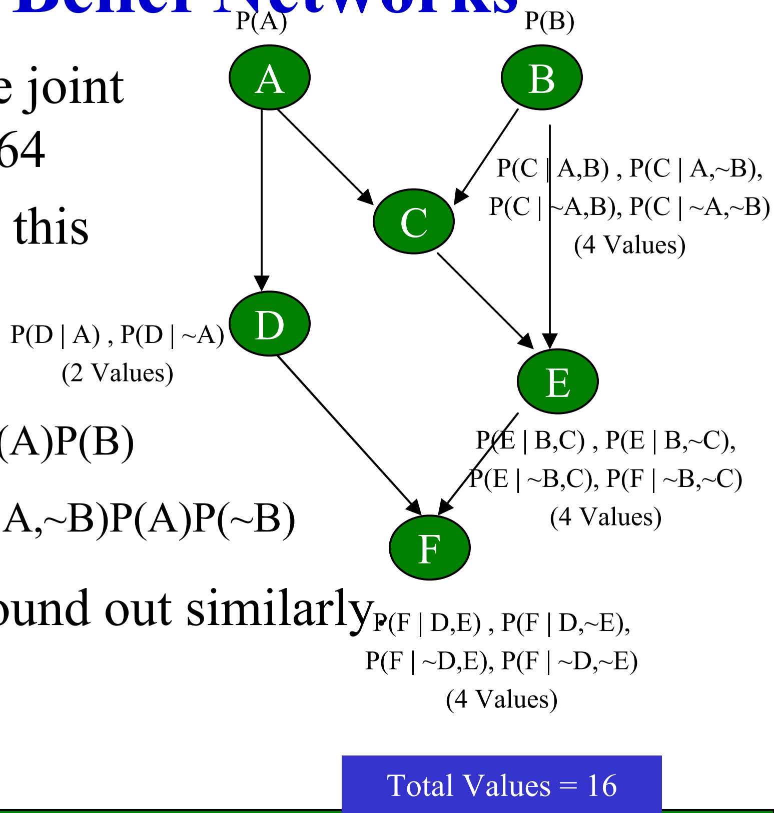
Probabilistic Network representations can reduce this number significantly

The joint distribution is computed as

P(A,B,C,D,E,F) = P(F | D,E)P(D | A)P(E | B,C)P(C | A,B)P(A)P(B)

P(A,B,~C,~D, E,F) = P(F | ~D,E)P(~D | A)P(E | ~B,C)P(C | A,~B)P(A)P(~B)

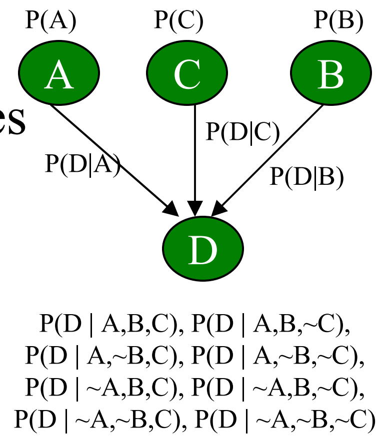
Probabilities for other 62 combinations can be found out similarly



Bayesian Network with Noisy OR (BN2O)

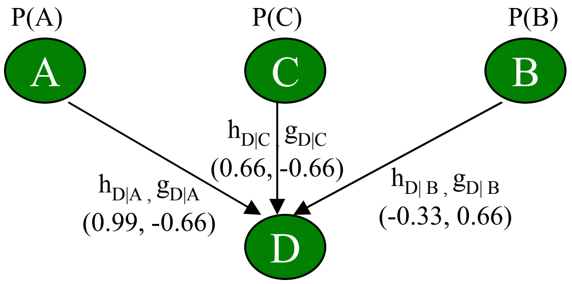
- Based on Independence of Causal Influences Assumption
- Required n parameters to estimate 2^n conditional probabilities
- Given P(D | A), P(D | B), and P(D | C)

$$P(D \mid A,B,C) = 1 - P(\sim D \mid A,B,C)$$
$$= 1 - P(\sim D \mid A)P(\sim D \mid B)P(\sim D \mid C)$$



CAusal STrength (CAST) Logic

- Extension of Bayesian Network with Noisy OR (BN2O)
- Inputs have ranges from –1 to 1.
- $h_{D \mid A}$ is analogous (but not equal) to P(D | A) while $g_{D \mid A}$ is analogous (but not equal) to P(D | ~A).
- If all g values are zero and all h values are positive then CAST Logic = BN2O



Influence Nets

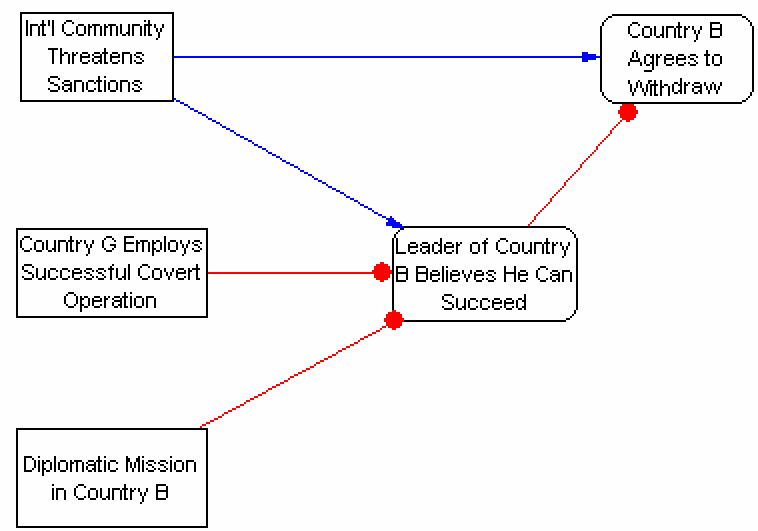
Probabilistic Belief Networks that use CAST Logic for model specification are termed as Influence Nets.

The current implementation of Influence Nets assume that the parents of a node are marginally independent.

Root Nodes

Non-root Nodes

Roots Nodes typically represents actionable events



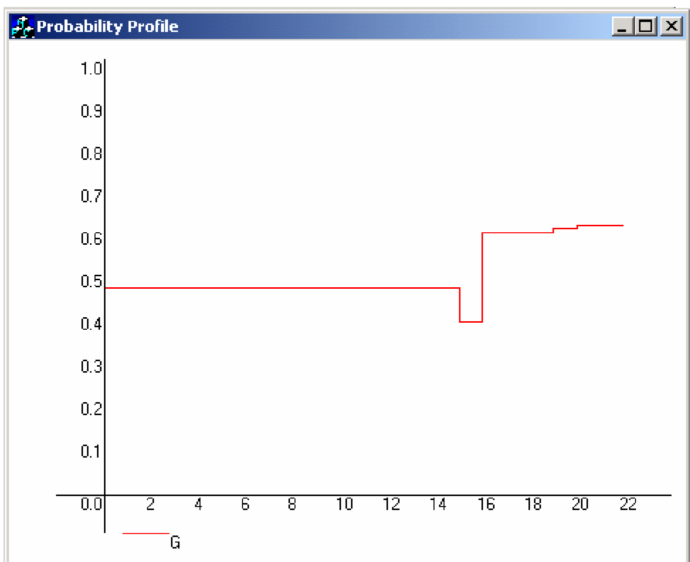
Positive Impact

Negative Impact

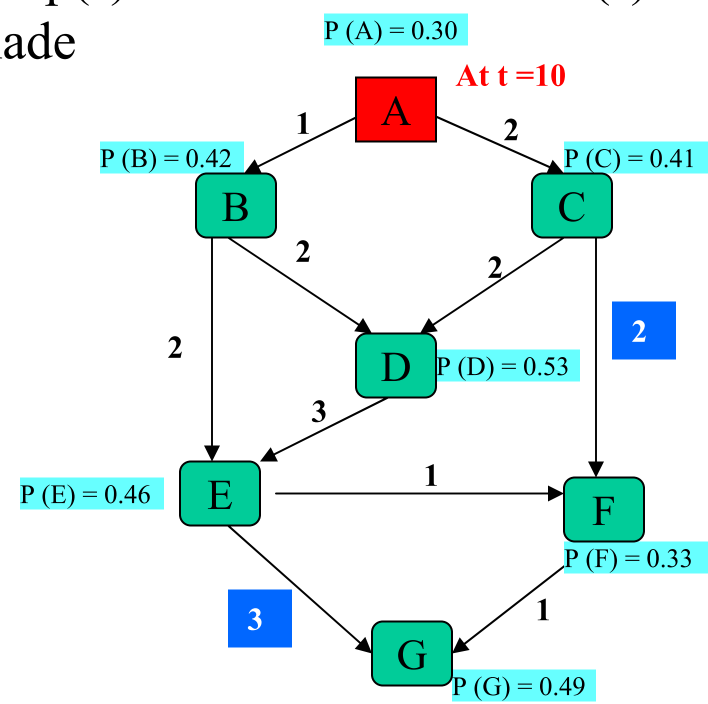
Timed Influence Nets

Timed Influence Nets have following additional parameters

- A time delay is associated with each arc.
- A time delay is associated with each node.
- Each actionable event is assigned time stamp(s) at which the decision(s) regarding the state of that action is(are) made

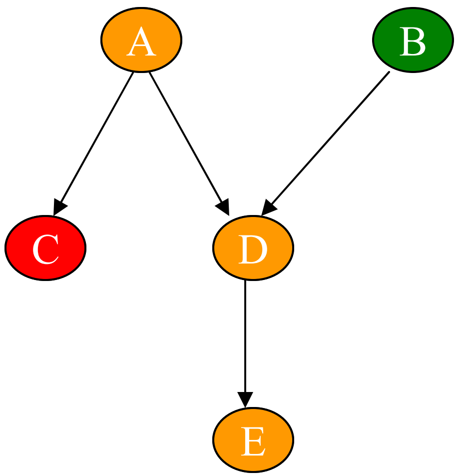


0	0.488
15	0.408
16	0.616
18	0.617
19	0.625
20	0.632

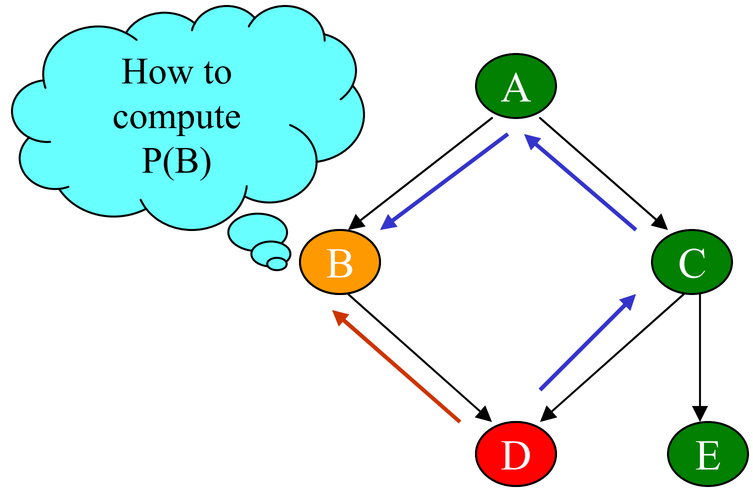


A: t = 10 B: t = 11 C: t = 12 D: t = 13, 14 E: t = 13, 16, 17
F: t = 14, 16, 17, 18 G: t = 15, 17, 18, 19

Belief Updating in Bayesian Networks



Singly Connected Network(SCN)

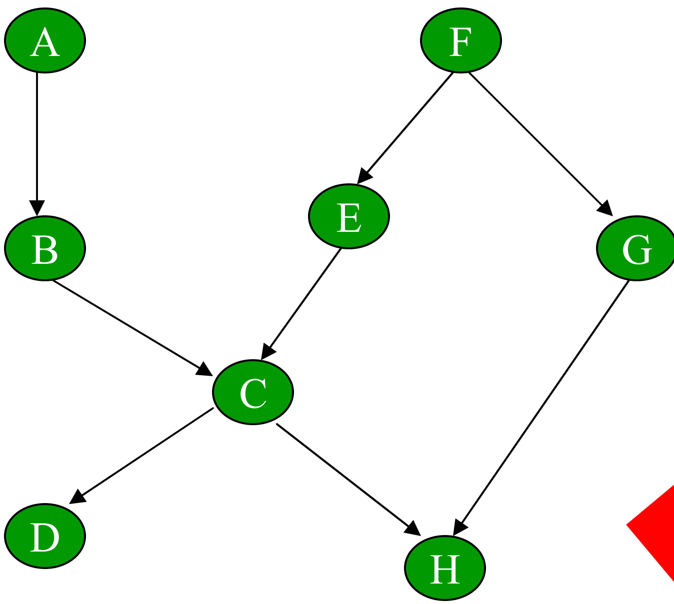


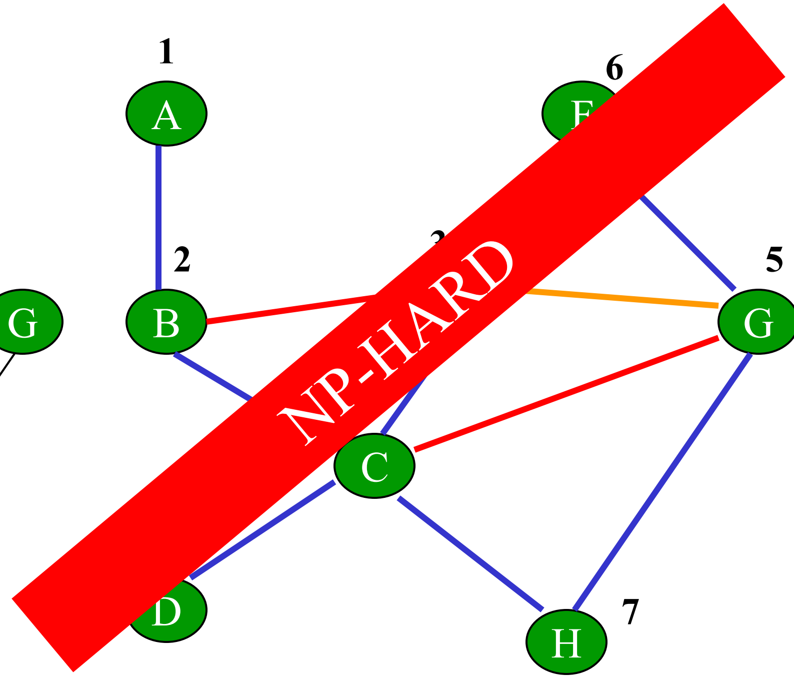
Multiply Connected Network(MCN)

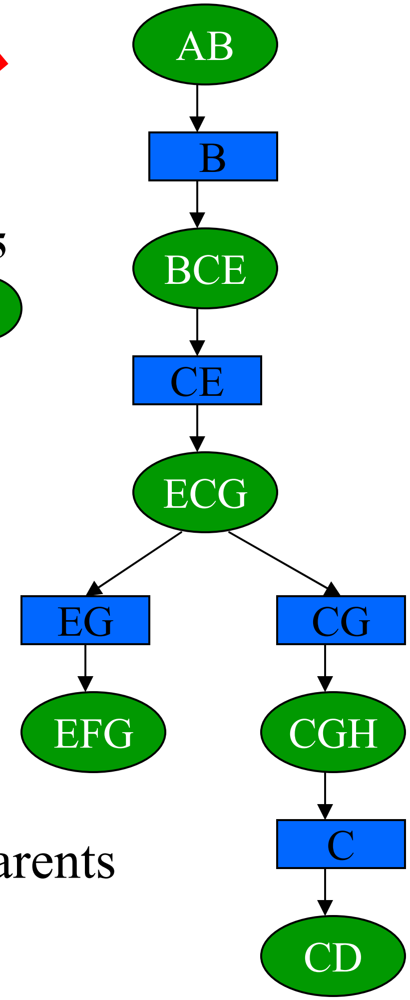
Exact Computation of Posterior Probability is

- Possible when the graph is singly-connected
- NP-Hard when the graph is multiply-connected

Computation in Multiply-Connected Networks (MCN)







Step 1: Make the graph unidirectional

Step 2: Moralize the graph by adding a link between common parents

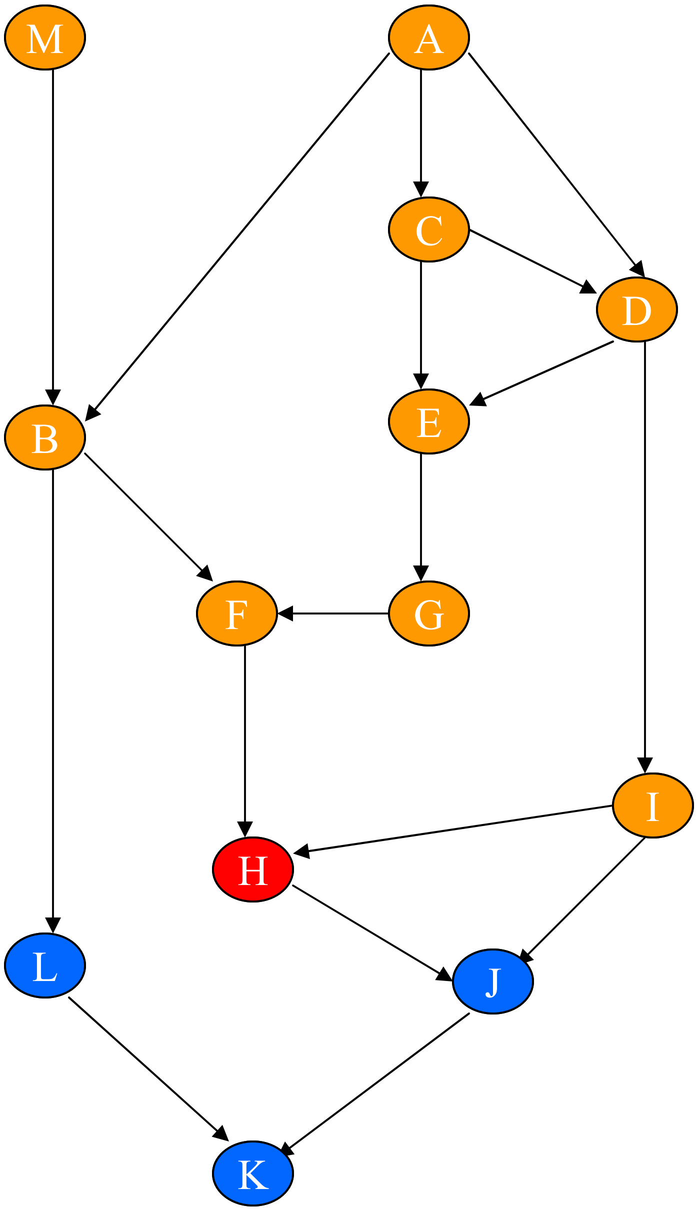
Step 3: Triangulate the graph

Step 4: Order the nodes by using Maximum Cardinality Search

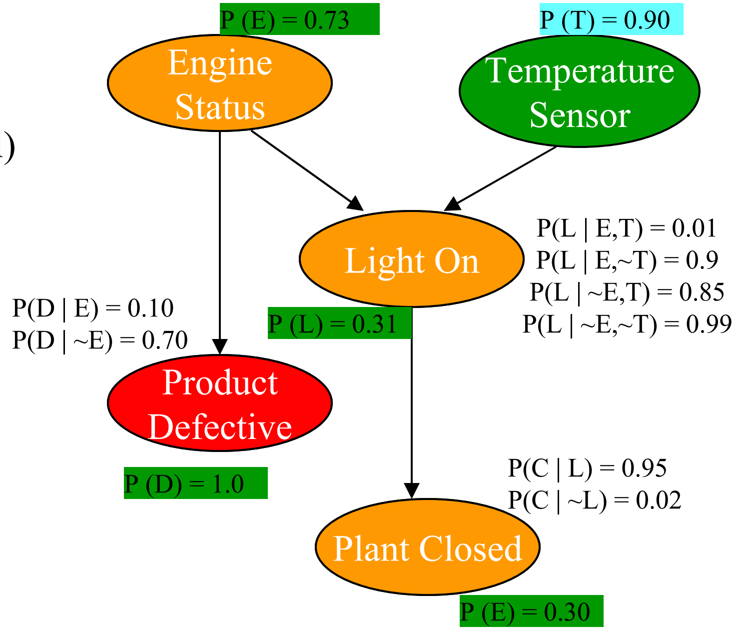
Sequencing Approach

A node will not be updated during the backward propagation until all of its descendants that are affected by the evidence are updated first.

Step1: [H, F, I]
Step2: [H, F, I, G, B]
Step3: [H, F, I, G, B, D]
Step4: [H, F, I, G, B, D, E]
Step5: [H, F, I, G, B, D, E, M, A]
Step6: [H, F, I, G, B, D, E, M, A]
D *cannot be updated as* **E** *is not updated yet.*
Step7: [H, F, I, G, B, E, M, A, D, C]
Step8: [H, F, I, G, B, E, M, A, D, C]
Step9: [H, F, I, G, B, E, M, A, D, C]
A *cannot be updated as* **C** *and* **D** *are not updated yet.*
Step10: [H, F, I, G, B, E, M, D, C, A]
Step11: [H, F, I, G, B, E, M, D, C, A]
Step12: [H, F, I, G, B, E, M, D, C, A]



Belief Propagation in Singly Connected Network



Let E = (Engine Status = Normal), T = (Temperature Status = Normal)

L = (Light On = True), D = (Product Defective = True)

C = (Plant Closed = True)

$$P(E \mid D) = \frac{P(D \mid E)\, P(E)}{P(D \mid E)\, P(E) + P(D \mid \sim E)\, P(\sim E)}$$

$$P(E \mid D) = 0.73$$

$$P(L \mid D) = P(L \mid E,T)P(E)P(T) + P(L \mid E,\sim T)P(E)P(\sim T) + P(L \mid \sim E,T)P(\sim E)P(T) + P(L \mid \sim E,\sim T)P(\sim E)P(\sim T)$$

$$P(L \mid D) = 0.31$$

$$P(C \mid D) = P(C \mid L)\, P(L) + P(C \mid \sim L)\, P(\sim L)$$

$$P(C \mid D) = 0.30$$

Belief Propagation in Multiply Connected Network

$$P(E \mid F) = \frac{P(F \mid E)\, P(E)}{P(F \mid E)\, P(E) + P(F \mid \sim E)\, P(\sim E)}$$

$$P(D') = P(D \mid E,F)P(E,F) + P(D \mid E,\sim F)P(E,\sim F) + P(D \mid \sim E,F)P(\sim E,F)$$
$$+ P(F \mid \sim D,\sim E)\, P(\sim D,\sim E)$$

Where

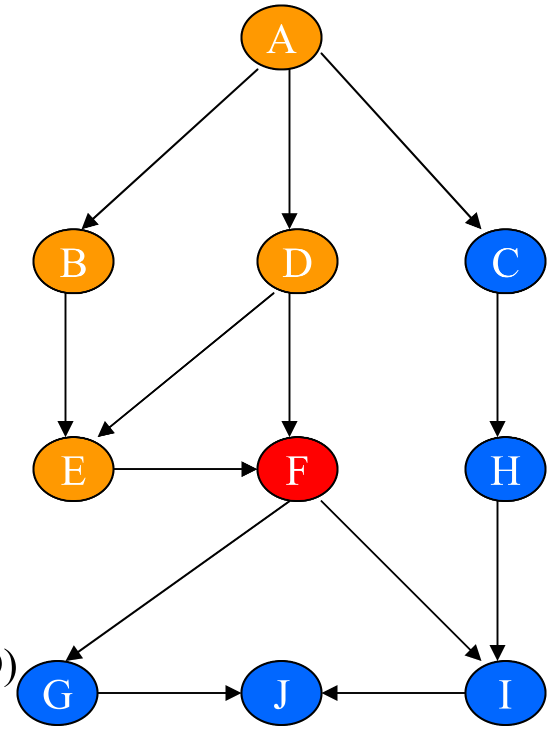
$$P(D \mid E,F) = \frac{P(F \mid E,D)\, P(E)P(D)}{P(F \mid E,D)\, P(E)P(D) + P(F \mid E,\sim D)\, P(E)P(\sim D)}$$

Similarly,

$$P(A') = P(A \mid B, D)P(B,D) + P(A \mid B,\sim D)P(B,\sim D) + P(A \mid \sim B,D)P(\sim B,D)$$
$$+ P(A \mid \sim B,\sim D)P(\sim B,\sim D)$$

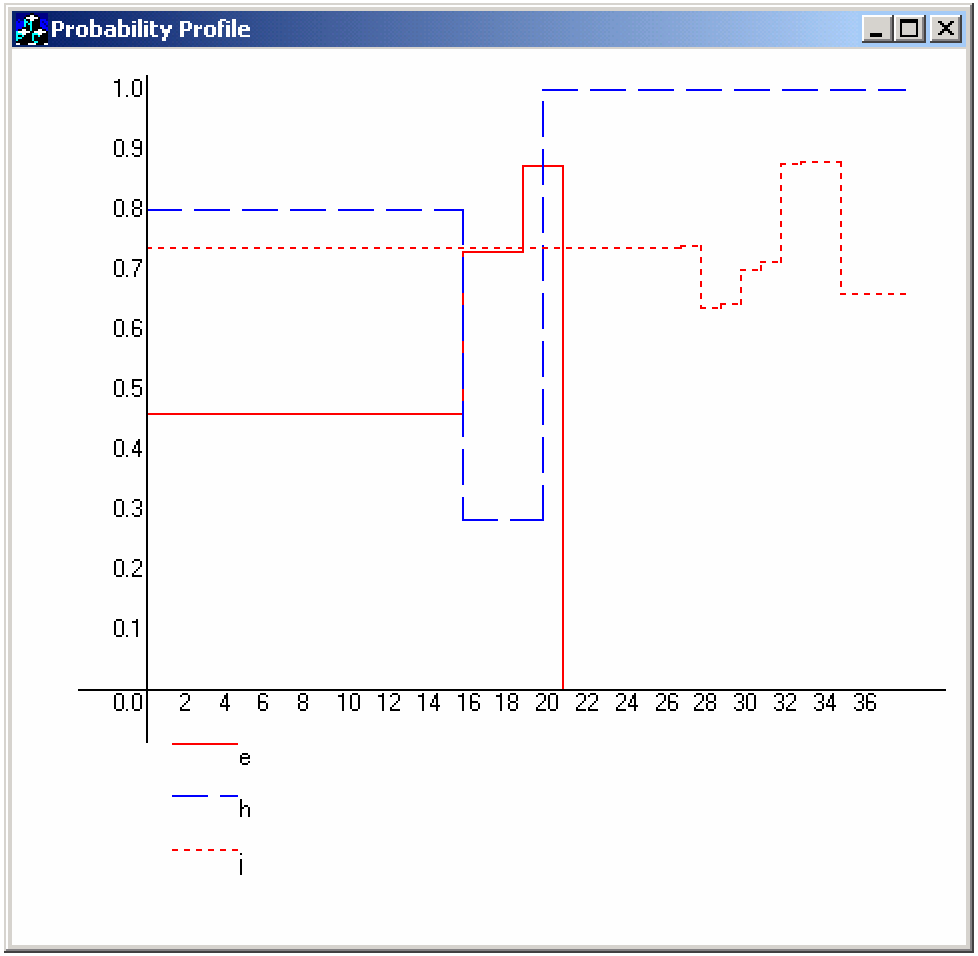
Where

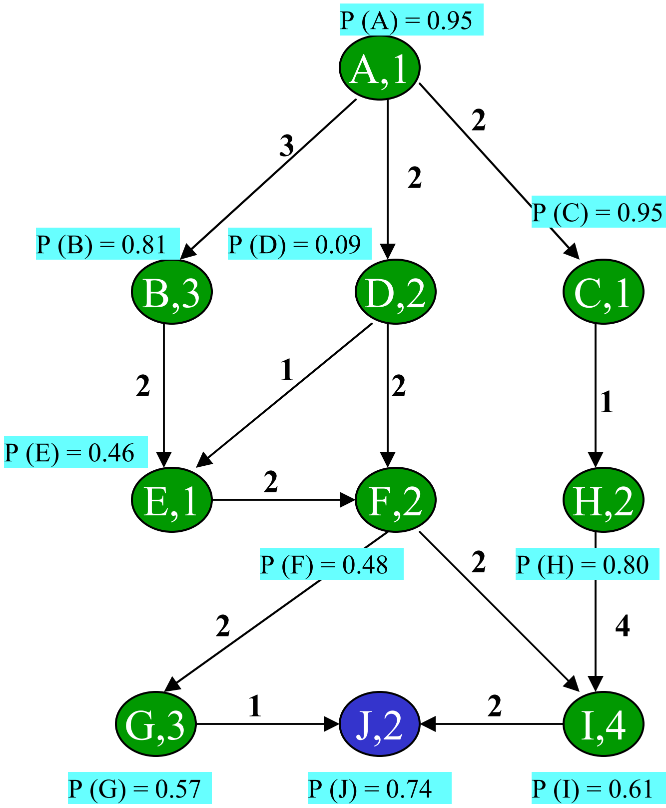
$$P(A \mid B,D) = \frac{P(D \mid A)\, P(B \mid A)P(A)}{P(D \mid A)\, P(B \mid A)P(A) + P(D \mid \sim A)\, P(B \mid \sim A)P(\sim A)}$$



Belief Updating in Timed Influence Net

P(E) = 0 at time t = 20





Conclusions

- A heuristic approach of belief revision for Timed Influence Nets is presented.
- The approach updates the nodes in the sequential manner during the backward propagation.
- Limitations: The algorithm works only if the time stamp of the evidence is later than the time stamp of the last update of the evidence node caused by the forward propagation of the effects from all of the action nodes.
- One alternative approach is to convert a Timed Influence Net into a Time Sliced Bayesian Network.