An Analytical Model That Provides Insights into Various C2 Issues

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Abstract

This paper develops an analytical model that can very simply provide important insights into the consequences (in terms of combat outcomes) generated by different C2 architectures for information processing. A Lanchester-type model of force-on-force combat that reflects C2 architecture at the platform level is developed through detailed analysis of the target-engagement cycle for a single typical firer in modern tank combat. The most significant new aspect of this model is the consideration of so-called parallel acquisition of targets, i.e. new targets can be acquired while a previously acquired target is being engaged. Computational results are given that show that being able to effect parallel acquisition of targets can not only significantly increase a tank force's infliction of casualties on an enemy tank force, but also significantly reduce the number of casualties that are suffered. The model given here is developed by use of Taylor's new methodology for Lanchester attrition-rate coefficients under conditions of stochastic line of sight. This methodology allows one to play significantly more micro-combat detail than has ever been possible in Lanchester-type models. Hence, it has opened up new vistas for the mathematical modeling of force-on-force combat.

1. Introduction.

Although DoD spends literally billions of dollars on modeling and simulation (M&S) each year, essentially all of this money is devoted to simulation (including simulation technology, both hardware and software), with next to nil being spent on mathematical modeling and even less on the scientific investigation of warfare[1] (particularly, on the influence of technology on its outcome). Relatively few new developments in the mathematical modeling of warfare have appeared the last 25 years. However, Taylor [to appear] has recently developed important new methodology for developing a numerical value for a Lanchester attrition-rate coefficient (the rate at which a single, given firer type kills enemy targets of a particular type) under conditions of stochastic line of sight (LOS), with one being able to represent significantly more micro-combat detail (particularly as concerns the modeling of information flows and other cognitive processes) in such a single-weapon-system-type kill rate than ever before. Furthermore, new developments for tanks and other light-armor systems (including the Army's future combat system (FCS)) allow such systems to acquire new targets while a previously acquired target is being engaged. Taylor's methodology even allows one to model (via micro-combat detail represented in a Lanchester attrition-rate coefficient) such situation by means of so-called parallel acquisition of targets. This is the topic of the paper at hand.

A fundamental distinction made by this new methodology is whether or not new targets can be acquired by a firer while an acquired target is being attacked by this firer. The simplest conceptual model for such a distinction consists of the following two basic cases:

(1) no new target can be acquired (serial acquisition of targets),
(2) new targets can be acquired at the same rate as when no target is being attacked (parallel acquisition of targets).

Such a distinction between serial and parallel acquisition of targets is absolutely necessary because it does occur for a number of weapon system types of particular interest and results in substantially different rates for these two cases, with kill rates for parallel acquisition always being higher (sometimes substantially so). Furthermore, there are two further fundamentally different cases for parallel acquisition that must be distinguished:

(1) no preemption when higher-priority target is acquired,

[1] See, for example, Dupuy [1987]. Moreover, Bonder [2002b] has concluded that Army OR has waned dramatically in recent years, with scientific experimentation in support of it even more so.

(2) preemption when higher-priority target is acquired.
This paper will focus on the first (simpler) case. Computational results are given that show that just by changing from acquiring targets in the serial mode to the parallel mode for firers on one side can result in this side inflicting 62% more casualties and suffering 19% less attrition. Such computational results have been conveniently generated by means of an Excel spreadsheet.

The continual acquisition of targets throughout the target-engagement cycle for parallel acquisition is represented through target availability (i.e. the probability that an observer/firer has a particular target of a given type available for immediate engagement at the beginning of a new target-engagement cycle) from which one can compute the probability that one or more previously-acquired targets are available for immediate engagement at the beginning of a new target-engagement cycle. In turn, a continuous-time, three-state Markov-chain model is used to determine target availability, which then plays a critical role in computing a Lanchester attrition-rate coefficient for parallel acquisition. In other words, the state-probability vector for this three-state Markov chain must be determined in detail before the corresponding Lanchester attrition-rate coefficient can be computed. Previous work had not worked out such important details.

Moreover, this model that considers parallel acquisition of targets at the platform level can be extended from such a platform-centric force-on-force model to a network-centric one (e.g. see Alberts et al. [1999]). Such extensions are considered and insights into the greater combat effectiveness to be gained from being able to share targeting information and operating in the network-centric mode are developed (including consideration of different architectures for the U.S. Army's future combat system (FCS)). Use of such Lanchester-type models allows one to very conveniently compute differences in combat outcomes that are possible from different C2 architectures.

Finally, it must be observed here that the new attrition-rate-coefficient results given by Taylor have involved both new combat-model concepts and also new mathematics (applied probability theory) for military operations research. Specifically, Taylor's [to appear] research has led to the development of the following new combat-model concepts

(1) target-engagement policy,
(2) target-engagement cycle,
(3) target-engagement-cycle diagram.

Moreover, new mathematical results for applied probability theory (e.g. new results for the probability of one continuous, nonnegative random variable being less than another and an associated expected value) have been required in order to obtain these new analytical results for Lanchester attrition-rate coefficients. In this respect, the mathematics required for the calculation of a Lanchester attrition-rate coefficient has turned out to be more closely related to the mathematics of the theory of stochastic duels (with its emphasis on determining the probability that one nonnegative random variable will be less than another)[2] than that of queueing theory (with its emphasis on renewal theory)[3].

2. Lanchester-Type Models.

DoD extensively uses combat models for both analysis and also training. Essentially all aggregated-force models (both ITEM [a joint campaign model] and others such as CEM and VIC) currently in use for analysis, or planned for the future (e.g. JWARS, AWARS), base their attrition cal-

[2] See, for example, Ancker [1982]. The reader may also find it instructive to consult Williams and Ancker [1963] or Ancker [1967]. Moreover, Taylor's earlier work on such a probability of one continuous, nonnegative random variable being less than another (see Taylor [1983a, Appendix B]) was inspired by Ancker's [1967] work on stochastic duels, although many other combat modeling applications were considered by Taylor [1983a, Appendix B].
[3] See, for example, Saaty [1961, especially Chapter 15] or Wolff [1989, especially Chapter 2].

culations on some type of underlying Lanchester-type model[4]. The basic Lanchester-type force-on-force attrition paradigm (out of which such computer-based complex operational models have been developed by the process of model enrichment) is given by (see Fig. 1)

$$\begin{cases} \dfrac{dx}{dt} = -a\,y & \text{with } x(0) = x_0 , \\ \dfrac{dy}{dt} = -b\,x & \text{with } y(0) = y_0 , \end{cases} \qquad (1)$$

where $t = 0$ denotes the time at which the battle begins and $x(t)$ and $y(t)$ denote the numbers of X and Y at time t. Here, for example, a denotes the rate at which a single typical Y firer kills X targets and is called a Lanchester attrition-rate coefficient (single-weapon-system-type kill rate). The paper at hand investigates the explicit modeling of such a coefficient to represent the parallel acquisition of targets. Such development, however, must be done for a much more complicated model to be of practical significance.

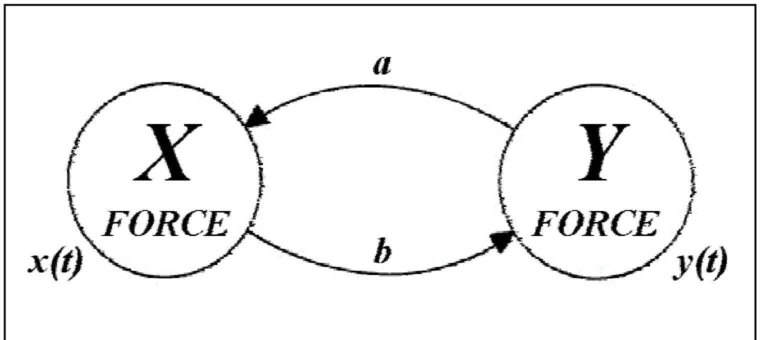

Fig. 1. Combat between two homogeneous forces.

Two enrichments of the above basic paradigm (1) that are required for practical DoD work are the following

(1) combat between heterogeneous forces,
(2) representation of line-of-sight process.

One is required to consider combat between heterogeneous forces because modern combat is characterized by so-called combined-arms operations involving (for example) tanks, anti-tank weapon systems, artillery, infantry (armed with several different types of weapons), etc. The generalization of the fundamental Lanchester-type attrition paradigm (1) to such heterogeneous-force combat is then given by (for $i = 1,2,\ldots,m$ and $j = 1,2,\ldots,n$) (see Fig. 2)

$$\begin{cases} \dfrac{dx_i}{dt} = -\displaystyle\sum_{j=1}^{n} a_{ij}\,y_j & \text{with } x_i(0) = x_{0i} , \\ \dfrac{dy_j}{dt} = -\displaystyle\sum_{i=1}^{m} b_{ji}\,x_i & \text{with } y_j(0) = y_{0j} , \end{cases} \qquad (2)$$

[4] For an extensive discussion of such models, see Taylor [1983a]. Bonder [2002a] has provided an excellent overview and historical sketch of the use of such models in DoD.

where a_{ij} denotes the rate at which a single Y_j firer kills X_I targets and similarly for b_{ji}. One can also call the a_{ij}s and b_{ji}s Lanchester attrition-rate coefficients. All complex operational Lanchester-type models currently used by DoD (such as VIC[5] and JWARS) play such heterogeneous forces. However, because of the pioneering nature of the work reported here (i.e. no other work has been previously reported in the literature for parallel acquisition of targets (cf. AMSAA [2000b]), except by Taylor [to appear]), it focuses exclusively on combat between homogeneous forces (however, see Taylor [to appear] for results for combat between heterogeneous forces, but for serial acquisition of targets). Moreover, because of this preliminary nature and the fact that our work has taken the work of Bonder and Farrell on attrition-rate coefficients as its point of departure (see Taylor [to appear, especially Section 3] for details), we will only consider here the special case of exponential interfiring times[6].

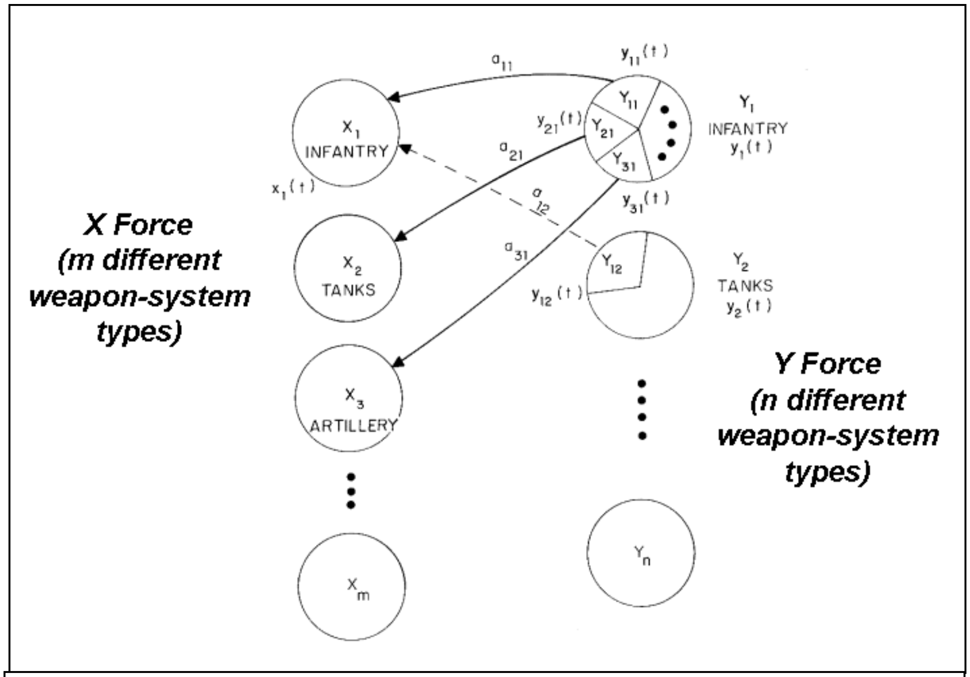

Fig. 2. Combat between two opposing combined-arms teams (heterogeneous forces).

One is required to consider the line-of-sight (LOS) process between an observer and enemy targets because of the profound impact that is has on the acquisition and engagement of targets (e.g. see Bonder [2002a] for further details). Although LOS can be played in entity-level simulations as a deterministic map[7], it is impractical for a number of important reasons to play it this way in complex operational Lanchester-type models such as VIC and JWARS[8]. Rather, LOS must be played as a

[5] See, for example, TRAC-FLVN [1992] or AMSAA [2000b].

[6] Although this assumption is at variance with the data played in essentially all U.S. Army simulations (especially training ones, see AMSAA [2000a]), it is the only one ever used in all the work by Vector Research, Inc. (VRI) (e.g. see Miller et al. [1978], CCTC [1979], or AMSAA [2000b]). Taylor et al. [2002] have reported extension of such results to more realistic lognormal (modeled by Erlang) interfiring times.

[7] The terminology used here is that of Bonder [2002a, p. 29].

[8] It should not be inferred, however, that LOS is played the same way in both VIC and JWARS. The important point to be noted here is that JWARS does not use the Bonder and Farrell stochastic LOS model (see below) as claimed by Bonder [2002a, p. 29], although it should (see Taylor [to appear, Section 2] for criticism of both the JWARS LOS model as well as its computation of single-weapon-system-type kill rates).

stochastic process[9] to represent the random occurrence over time of LOS between some pair of points representing the locations of an observer and a target. This LOS process can be modeled by a continuous-time two-state Markov-chain model (see Fig. 3). It seems appropriate to refer to this LOS model as the Bonder and Farrell stochastic LOS model (e.g. see Bonder [2002a, p. 29]). In this model $\eta = 1/E\left[T_I \right]$ denotes the rate of gaining LOS, and $\mu = 1/E\left[T_V \right]$ denotes the rate of losing LOS. Thus, for example, the rate at which LOS is lost is given by the reciprocal of the mean time that the target is visible. This stochastic LOS model is further examined in Section 6 below. The U.S. Army has developed methodology for estimating these parameters, and their numerical values for a given piece of terrain are part of the VIC database (see Taylor [2000]). As in Taylor [to appear], we will assume for simplicity that the parameters of this LOS process are independent of both the firer type and also target type, i.e. they depend only on the terrain (and the range between firer and target). This assumption can, of course, be relaxed at the expense of greater model complexity (with no additional conceptual complexity in model development, only additional notational complexity).

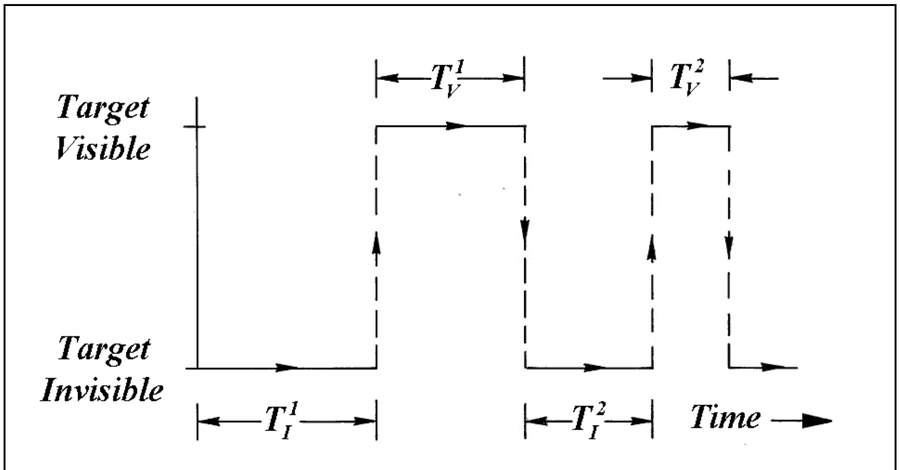

Fig. 3. Line-of-sight process in which target alternates being in either of two states. The length of time in each state is a random variable, exponentially distributed.

The practical use such differential equations in defense analysis essentially depends on one's ability to obtain realistic values for the Lanchester attrition-rate coefficients. Two general approaches that have been used to develop numerical values for Lanchester attrition-rate coefficients (i.e. single-weapon-system-type kill rates) are

(1) the freestanding-analytical-model approach (which generates these values from an analytical model, independent of any high-resolution model),

(2) the hierarchy-of-models approach (which estimates parameter values for such an attrition-rate coefficient from the output of a high-resolution Monte-Carlo combat simulation).

[9] This representation of LOS as such a stochastic process was apparently developed by Bonder and Farrell in the early 1970s (Bonder [2002a]), although statistical LOS had apparently been earlier played in Army aggregated-force models since the late 1960s (see Hawkins [1976] or AMSAA [1983]). The corresponding Lanchester attrition-rate calculations in these early Army models, however, were faulty (see Taylor [to appear] for details). Moreover, we have not been able to verify that any theoretically correct expression for a Lanchester attrition-rate coefficient was ever developed by VRI as claimed by Bonder [2002a] (see Taylor [to appear, Section 2] for details).

The first approach was pioneered by Bonder and Farrell [1970] (see, however, Bonder [1967], [1970] and Barfoot [1969]) and for this reason is frequently called the Bonder-Farrell approach. It will be the approach used in the paper at hand (see also Taylor [to appear]). It conceptually consists in considering (for the case of homogeneous forces depicted in Fig. 1 above) a single typical firer on a particular side and then computing the rate at which this firer type kills enemy targets according to a micro-combat model[10]. The original work by Bonder and Farrell cited directly above, however, only considered continuous LOS, i.e. it did not consider any type of stochastic LOS model. The second approach (sometimes also called the fitted-parameter-analytical-model approach) was pioneered by G.M. Clark [1969] and for this reason could be called the Clark approach. CAA [1983] has called it the attrition-calibration (ATCAL) approach (e.g. see Taylor et al. [1998] or Taylor et al. [2000]), and a version of it has been implemented in their concepts evaluation model (CEM) since 1983 for the assessment of ground-combat losses.

3. New Methodology for Lanchester Attrition-Rate Coefficients.

Taylor's [to appear] new methodology for computing a Lanchester attrition-rate coefficient takes the following general principle as its point of departure: **Such a single-weapon-system-type kill rate should be computed as the expected number of kills[11] in the target-engagement cycle divided by the expected length of this target-engagement cycle.** For a given firer type, say Y_j, against a particular target type, say X_i, it may be expressed in more mathematical form as

$$\boldsymbol{a}_{ij} = \frac{\bar{\boldsymbol{n}}_{k_{X_iY_j}}^{cycle}}{\bar{\boldsymbol{t}}_{cycle_{Y_j}}}, \qquad (3)$$

where $\boldsymbol{a}_{ij}$ denotes the rate a which an individual Y_j firer type kills X_i target types, $\bar{\boldsymbol{n}}_{k_{X_iY_j}}^{cycle}$ denotes the average number of X_i targets killed by a Y_j firer type in the target-engagement cycle, and $\bar{\boldsymbol{t}}_{cycle_{Y_j}}$ denotes the average length of the target-engagement cycle for this Y_j firer type.

Taylor's principle (expressed in quantitative form as (3)) extends the Barfoot/Bonder principle to cases that include the following

(1) a heterogeneous-target environment,
(2) engagement outcomes other than a kill,
(3) parallel acquisition of targets.

Thus, equation (3) is the point of departure for all attrition-rate-coefficient results given in this paper. Moreover, it applies to both serial as well as parallel acquisition of targets.

A key question concerning a firer's behavior to be asked in developing a conceptual model of the attrition process is the following, "Can new targets be acquired by the firer while an acquired target is being attacked by this firer?" The simplest conceptual model for answering this key question consists of the following two basic cases:

(1) no new target can be acquired (**serial acquisition of targets**),
(2) new targets can be acquired at the same rate as when no target is being attacked (**parallel acquisition of targets**).

In the case of serial acquisition, targets are alternately acquired and then subsequently attacked (i.e. fire directed at an acquired target), in a process that is repeated cyclically over time. This cyclical

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[10] Here micro-combat model refers to an entity-level (i.e. individual-firer) model in which all the details of the process by which this individual combatant acquires and engages an enemy target are considered.
[11] For the appropriate type of target, of course.

process has been called by Taylor the “target-engagement cycle” (see Fig. 4). Considering this target-engagement cycle, Taylor [to appear] developed a general expression for a Lanchester attrition-rate coefficient for serial acquisition, and then developed more specific results (i.e. analytical expressions) for different so-called target-engagement policies.

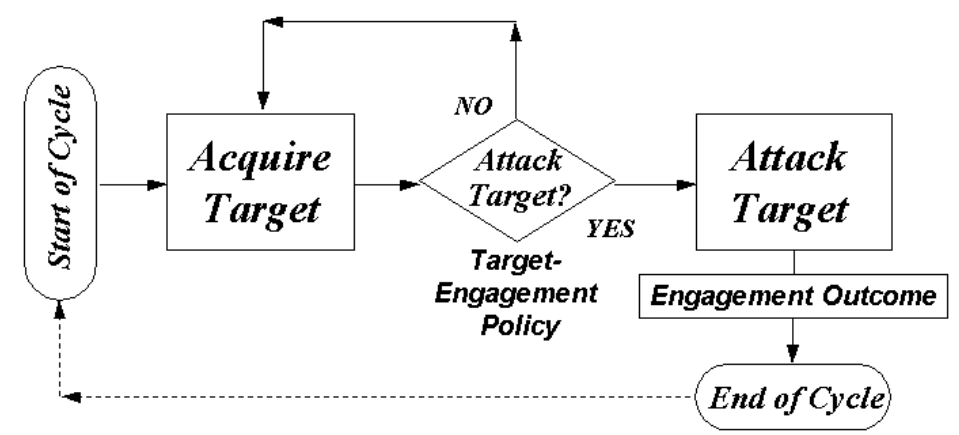

Fig. 4. Target-engagement cycle (serial acquisition of targets).

The target engagement cycle for parallel acquisition is significantly different from that for serial acquisition (see below), though, with new targets continuing to be acquired while a previously acquired target is attacked. Moreover, significantly different kill rates occur for these two different modes of acquiring targets (see Taylor [to appear] for details). This point is further investigated in the paper at hand and related to network-centric warfare.

Finally, regardless of whether one considers serial or parallel acquisition of targets, when the target-acquisition process is independent of attacking them, the expected number of kills in the target-engagement cycle is given by

$$\bar{n}^{cycle}_{k_{X_iY_j}} = P^{eng}_{X_iY_j} \, P_{K(LOS)_{X_iY_j}} , \qquad (4)$$

where $P^{eng}_{X_iY_j}$ denotes the probability that the next target type to be engaged by a Y_j firer type will be an X_i target type and $P_{K(LOS)_{X_iY_j}}$ denotes the probability that a Y_j firer type will kill an X_i target type before line of sight (LOS) is lost. It is most significant to note that although $P^{eng}_{X_iY_j}$ depends on whether serial or parallel acquisition is being played, $P_{K(LOS)_{X_iY_j}}$ does not, i.e. it is independent of exactly how targets are acquired (since it refers to the process of attacking a target that has already been acquired). Moreover, since the time to acquire any target is exponentially distributed, $P^{eng}_{X_iY_j}$ depends only on the parameters of these distributions (and the numbers of each target type); while $P_{K(LOS)_{X_iY_j}}$ depends on the interfiring-time distribution (as well as the parameter of the distribution of time to lose LOS). Thus, one may consider the following general expression for a Lanchester attrition-rate coefficient holds for serial as well as parallel acquisition of targets

$$a_{ij} = \frac{P^{eng}_{X_iY_j} P_{K(LOS)_{X_iY_j}}}{\bar{t}_{cycle_{Y_j}}}. \qquad (5)$$

Consequently, equation (5) can be taken as the point of departure for developing a single-weapon-system-type kill rate for parallel acquisition of targets.

4. Parallel Acquisition of Targets

There is, however, a fundamentally different structure for the target-engagement cycle for parallel acquisition of targets (see Fig. 5 and compare with Fig. 4). Moreover, in general (i.e. for heterogeneous forces) there are two further fundamentally different cases that must be distinguished:

(1) no preemption when higher-priority target is acquired,

(2) preemption when higher-priority target is acquired.

We will not, however, consider the latter case in the paper at hand (see Taylor [to appear] for a very brief initial examination of this important case).

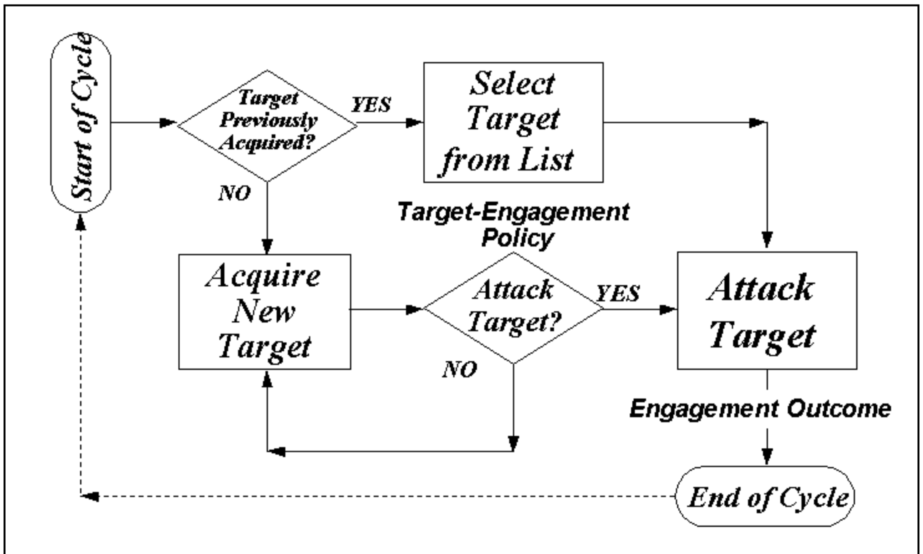

Fig. 5. Target-engagement cycle (parallel acquisition of targets).

As seen from Fig. 5, parallel acquisition is composed of two distinct modes of operation for a weapon system, depending upon whether or not there is a previously acquired target available for immediate engagement at the beginning of a new target-engagement cycle. When one or more targets have been previously acquired, one of these can be selected for immediate engagement at the beginning of a new target-engagement cycle. It seems appropriate to this case "component parallel acquisition," since for this cycle target acquisition has occurred in parallel with the attack of an acquired target in the previous target-engagement cycle. Likewise, when no target has been previously acquired, a new target must be acquired in the new target-engagement cycle. It seems appropriate to this case "component serial acquisition," since for this cycle target acquisition has occurred in series with the attack of an acquired target in the previous target-engagement cycle.

5. Homogeneous Forces

In the paper at hand, we will consider only homogeneous forces for simplicity. This choice appears to be particularly appropriate because of our paper's seminal nature: we know of no other such investigation (besides Taylor [to appear]) of the modeling and analysis of the consequences of parallel acquisition. Taylor [to appear] does consider heterogeneous forces for serial acquisition, with substantial increase in model complexity. For such heterogeneous forces, for example, one has to take into consideration target priorities and rules of engagement. Moreover, Taylor also gives some initial results for the mathematical modeling of attrition-rate coefficients for heterogeneous forces and parallel acquisition of targets, but only presents some preliminary homogeneous force numerical results for force-on-force combat, without any consideration of C2 aspects or network-centric warfare.

For homogeneous forces, making the appropriate changes in notation, we can write the general expression for the rate at which a Y firer kills X targets (cf. (5) above) as

$$a = \frac{P_{K(LOS)_{XY}}}{\bar{t}_{cycle_Y}}, \tag{6}$$

where $P_{K(LOS)_{XY}}$ denotes the probability that a Y firer will kill an acquired X target before line of sight (LOS) is lost, and $\bar{t}_{cycle_Y}$ denotes the average length of the target-engagement cycle for this Y firer. Equation (6), which is the homogeneous-force version of equation (5), is the point of departure for developing a single-weapon-system-type kill rate for parallel acquisition of targets. However, it does also apply to serial acquisition of targets as well as parallel acquisition. Thus, equation (6) is used all subsequent developments in this paper.

6. Playing Stochastic Line of Sight (LOS)

Playing intermittent visibility for a target (i.e. stochastic LOS) has the effect of slowing down target acquisition and also of terminating the attack against an acquired target before the target is killed. The first of these effects will now be further investigated here and mathematically modeled in three steps by calculating the

(1) probability that LOS exists at an arbitrary point in time,
(2) effect of the LOS process on the acquisition of a particular target,
(3) effect of multiple targets.

The second effect (i.e. that of terminating the attack against an acquired target before the target is killed) will also be further investigated and mathematically modeled by calculating the

(1) probability that the target is killed before LOS is lost,
(2) expected length of such an attack until either the target is killed or LOS is lost.

These quantities are then used in the calculation of a kill rate via equation (6) above.

6.1. Probability That LOS Exists at Arbitrary Point in Time

Thus, development of an expression for a Lanchester attrition-rate coefficient requires knowledge of a value for the probability that at any arbitrary point in time LOS exists between a particular typical firer-target pair located on the battlefield, denoted as P_{LOS}. For given locations and circumstances, this probability will be assumed to be constant over time. Because of its fundamental importance, we will give two derivations of this basic result:

(1) derivation based on calculation of the expected fraction of time that the target is visible,
(2) derivation based on consideration of a two-state Markov-chain model.

Regardless of which approach one uses, however, the same result is obtained, namely

$$P_{LOS} = \frac{\eta}{\eta + \mu}, \qquad (7)$$

where η denotes the rate of gaining LOS (given by the reciprocal of the mean time the target is in the invisible state) and μ denotes the rate of losing LOS (given by the reciprocal of the mean time the target is in the visible state) (see Section 2 above).

Moreover, for the reader's convenience, we will give again here the conceptual model for the stochastic LOS process. Consider a firer located at one point on the battlefield and a target located at another point[12]. The situation for intervisibility between these two points will be the following

(1) line of sight exists (target visible to firer),

(2) line of sight does not exist (target invisible to firer).

Moreover, both the firer and also the target can move or change posture over time. Such changes (assumed to be random) will produce changes in line of sight (LOS) (see Fig. 3 again). The preceding has been a thumbnail sketch of the basic conceptual picture that underlies any mathematical model of the LOS process.

As the reader can see, P_{LOS} depends on two independent parameters η and μ. However, one could equally as well have taken any two of these three quantities as independent parameters for expressing any derivative quantity. Moreover, in an operational model like VIC, it is more convenient and operationally relevant to use P_{LOS} and μ as the input parameters because P_{LOS} can be directly obtained by preprocessing on an electronic terrain board that involves movement of forces driven by the scenario and is very operationally meaningful to military analysts (see Taylor [2000] for further details)[13]. Moreover, one can then express any Lanchester attrition-rate coefficient in terms of P_{LOS} and μ instead of the parameters η and μ.

As noted above, the first approach for developing (7) computes the probability that LOS exists between a particular observer-target pair by calculating the expected fraction of time that the target is visible. One can do this by considering the target-visibility/invisibility cycle (referred to as simply the target-visibility cycle) in which there are alternating periods in which intervisibility exists (i.e. LOS exists) and those in which no LOS exists between the particular observer-target pair (see Fig. 3). Over the long run, the probability that LOS exists can then be computed as the fraction of time that intervisibility exists per target-visibility cycle of these alternating periods. Thus, the probability that LOS exists at some random point in time can be computed simply as the quotient of two expected values: the expected time that the target is visible per cycle divided by the total expected length of such a cycle, or

$$P_{LOS} = \frac{E\begin{bmatrix} \textit{Time Target} \\ \textit{Intervisible per Cycle} \end{bmatrix}}{E\begin{bmatrix} \textit{Time Target} \\ \textit{Intervisible per Cycle} \end{bmatrix} + E\begin{bmatrix} \textit{Time No} \\ \textit{LOS per Cycle} \end{bmatrix}},$$

which can be written in more mathematical terms as

[12] In actuality, for an aggregated-force model, one would consider something like the center of mass of a group of firers and the center of mass of a target group.

[13] This procedure (essentially undocumented), or one like it, is currently used (and apparently has been used for many years) to generate these inputs for VIC. K.J. Saeger [2002], however, has pointed out that it probably needs a more thorough investigation of its theoretical basis. This author could not agree more with this very important point.

$$P_{LOS} = \frac{E[T_V]}{E[T_V] + E[T_I]}, \tag{8}$$

where T_V (a random variable) denotes the length of time that the target is visible during a period of intervisibility, and T_I (a random variable) denotes the length of time that the target is invisible during a period when intervisibility does not exist. Furthermore, from the assumptions made concerning the LOS process (see Taylor [to appear, Section 12.1]), one has that (see also Bhat [1972, Section 3.2])

$$E[T_V] = \frac{1}{\mu}, \tag{9}$$

and

$$E[T_I] = \frac{1}{\eta}. \tag{10}$$

Substituting (9) and (10) into (8), one readily obtains (7).

Thus, (7) has been obtained by consideration of the target-visibility cycle and application of first principles concerning the relative frequency interpretation of probability. Furthermore, the reader should note the similarity between this cycle approach for determining the probability of LOS and that for determining a single-weapon-system-type kill rate, e.g. (3) or (6), based on consideration of the target-engagement cycle.

Moreover, one can also derive (7) from consideration of the evolution equations for the probability state vector (e.g. forward-Kolmogorov equations) for a two-state continuous-time Markov chain. This is, of course, just the second approach discussed above.

We consider, therefore, a continuous-time Markov chain for a target that can be in one of two system states (see Fig. 6)

(1) line of sight does not exist (target invisible),
(2) line of sight does exist (target visible).

Let us denote $\textit{Prob}[\textit{Target Invisible at Time } t]$ as $p_I(t)$, and similarly denote $\textit{Prob}[\textit{Target Visible at Time } t]$ as $p_V(t)$. Thus, the probability that at any arbitrary point in time t LOS exists between a particular typical firer-target pair, denoted as $P_{LOS}(t)$, is given in general by

$$P_{LOS}(t) = p_V(t). \tag{11}$$

We will see below that when $p_V(t)$ is approximated by its steady-state value, one again finds that (7) holds. Moreover, the usual continuous-time Markov-chain assumptions, i.e.

(1) independent increments in time,
(2) probability of transition in short increment Δt equal to $(\textit{transition rate}) \bullet \Delta t$,
(3) more than one transition in Δt is impossible.

then yields the following forward-Kolmogorov equations (e.g. see Feller [1957], Bhat [1972], Gross and Harris [1998])

$$\begin{cases} \dfrac{dp_I}{dt} = -\eta\, p_I + \mu\, p_V \\ \dfrac{dp_V}{dt} = \eta\, p_I - \mu\, p_V \end{cases}, \qquad (12)$$

It follows from (12) that the probability that the target is visible at time t, denoted as $p_V(t)$, is given by

$$p_V(t) = \frac{\eta}{\eta+\mu} + \left\{ p_V(0) - \frac{\eta}{\eta+\mu} \right\} e^{-(\eta+\mu)t}, \qquad (13)$$

where $p_V(0)$ denotes the probability that the target is initially visible at time $t = 0$. This result (13) readily follows from the observation that $p_I(t) + p_V(t) = 1$ and use of the second of equations (12).

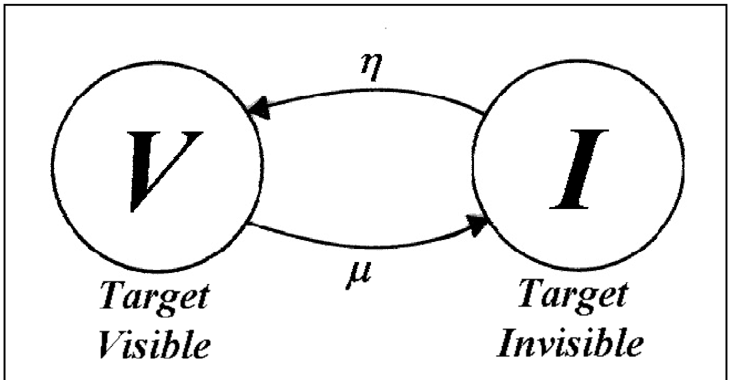

Fig. 6. Two-state Markov-chain model for determination of P_{LOS}.

From (13), one sees that

$$\lim_{t\to\infty} p_V(t) = \frac{\eta}{\eta+\mu} = p_V(\infty), \qquad (14)$$

where $p_V(\infty)$ denotes the steady-state probability that the target is visible. Also, if

$$p_V(0) = \frac{\eta}{\eta+\mu}, \qquad (15)$$

then for all $t \geq 0$

$$p_V(t) = \frac{\eta}{\eta+\mu}. \qquad (16)$$

Thus, we see that the steady-state probability $p_V(\infty)$ is dominant feature of the probability that a target is visible at time t. We will assume that this probability provides a good approximation to the probability that the target is visible given by (13). Considering (11) above, we see that we are again led back to (7).

6.2. Effect of LOS Process on Acquisition of Particular Target

Here we will show that the effect of the LOS process on acquiring a particular target is to produce a net rate of target acquisition (in an exponential distribution for the time to acquire) for this single particular target equal to $P_{LOS}\,\lambda_{XY}$. It will be convenient to express this as

$$\lambda_{XY}^{net(1)} = P_{LOS}\,\lambda_{XY}, \qquad (17)$$

where $\lambda_{XY}^{net(1)}$ denotes the net rate of acquiring a target when only one X target is present for a Y observer/firer and the other symbols are as defined above. If $T_{a_{XY}}^{cont}$ denotes the time for a Y observer/firer to acquire an X target under conditions of continuous LOS and it is exponentially distributed with parameter equal to λ_{XY}, then for intermittent LOS one would have

$$Prob\left[\, T_{a_{XY}}^{int} \le t \,\right] = 1 - e^{-\lambda_{XY}^{net(1)}\,t}, \qquad (18)$$

where $T_{a_{XY}}^{int}$ denotes the time for a Y observer/firer to acquire an X target under conditions of intermittent LOS. However, one can write (18) as

$$Prob\left[\, T_{a_{XY}}^{int} \le t \,\right] = 1 - e^{-\lambda_{XY}\,t_{effective}}, \qquad (19)$$

where $t_{effective}$ denotes the net time spent on acquiring the target (i.e. the total time that the target is visible) and is given by $P_{LOS}\,t$. Consideration of a filtered exponential distribution for the time to acquire the target (cf. Parzen's [1962, pp. 47-49] concept of preservation of the Poisson process under random selection) immediately leads to the same conclusion.

Likewise, the following considerations for a continuous-time Markov chain lead to the same conclusion. For simplicity we will omit the subscripts denoting firer and target type. Assume that (7) holds and denote the cumulative distribution function for the time to acquire by $F(t)$. Assuming that the target has not been acquired by time t, one finds that the probability of acquiring it in the next time increment of length dt, denoted as dF, is given by

$$dF = \left\{1 - F(t)\right\} P_{LOS}\,\lambda\, dt, \qquad (20)$$

since $P_{LOS}\,\lambda\, dt$ gives the probability that acquisition occurs in the time increment of length dt. In other words, underlying (20) is the following probability statement

$$Prob\begin{bmatrix} First\ Acquire\ Target \\ between\ t\ and\ t+dt \end{bmatrix} = Prob\begin{bmatrix} Target\ Not \\ Acquired\ by\ t \end{bmatrix} Prob\begin{bmatrix} Target \\ Visible\ in\ dt \end{bmatrix} Prob\begin{bmatrix} Target \\ Acquired\ in\ dt \end{bmatrix}.$$

Separating variables in (20) and integrating, one readily finds that

$$F(t) = 1 - e^{-P_{LOS}\lambda t}, \qquad (21)$$

whence follows (17) (see Taylor [1982b, Appendix E] for further details).

6.3. Effect of Multiple Targets

Similarly to the developments of the last section, here we will show that the effect of having m independently behaving targets available for acquisition (each with an exponential distribution of time to acquire with rate equal to $P_{LOS}\,\lambda_{XY}$) is to produce a net rate of target acquisition (in an exponential distribution for the time to acquire) equal to $m\,P_{LOS}\,\lambda_{XY}$. It will be convenient to express this as

$$\lambda_{XY}^{net(m)} = P_{LOS}\,\lambda_{XY}\,m, \qquad (22)$$

where $\lambda_{XY}^{net(m)}$ denotes the net rate of acquiring a target when m X targets are present. Equation (22) states that when there are multiple targets of a particular type available for acquisition, the rate for acquiring the next target is simply the sums of the rates for these targets. Furthermore, the time to acquire the next target is exponentially distributed with this rate. Moreover, for the determination of a kill rate for use in a Lanchester-type differential-equation model, this net rate is approximated as $P_{LOS}\,\lambda_{XY}\,x$.

For simplicity in the proof of (22), let us drop subscripts and write

$$\lambda^{net} = \lambda\, m, \qquad (23)$$

where the factor P_{LOS} has been observed into the rate λ. Thus, it suffices to consider m identical and independently behaving targets, each with an exponential time to acquire with rate equal to λ. In the case of two targets, the probability that no target has been acquired by time t is given by

$$Prob[No\ Acquisition\ by\ Time\ t] = e^{-\lambda t}\,e^{-\lambda t} = e^{-2\lambda t}, \qquad (24)$$

whence it is clear that for m targets

$$Prob[No\ Acquisition\ by\ Time\ t] = e^{-m\lambda t}. \qquad (25)$$

Hence, the cumulative distribution function for the time to acquire (here denoted simply as T, a random variable) is given by

$$F_T(t) = Prob\left[T \le t\right] = 1 - e^{-m\lambda t}, \qquad (26)$$

whence follow the above assertions, i.e. when there are m identical and independent targets present (each of which has an exponential distribution for the time to acquire it), the time to acquire the next target is exponentially distributed with rate equal to m times the single target acquisition rate.

More significantly, the argument just given can be carried over virtually unchanged to the determination of the distribution of the minimum of n independent exponentially distributed random variables. We state this result here for future reference as Theorem 1.

Theorem 1. **Consider n independent exponentially distributed random variables, denoted as T_1 through T_n, with rates denoted as λ_1 through λ_n. The minimum of these n independent random variables, denoted as T_{min}, is again exponentially distributed with rate, denoted as λ_{min}, given by**

$$\lambda_{min} = \sum_{k=1}^{n} \lambda_k . \qquad (27)$$

Furthermore, from (26) one readily concludes that the expected time to acquire the next target is given by

$$E\left[T\right] = \frac{1}{m\lambda}. \qquad (28)$$

Returning to our earlier notation, when there are only m identical and independent X targets present, the expected time for a Y observer to acquire the next target would be given by[14]

$$E\left[T_{a_Y}\right] = \frac{1}{P_{LOS}\,\lambda_{XY}\,m}. \qquad (29)$$

Again, for the determination of a kill rate for use in a Lanchester-type differential-equation model, this expected time would be approximated by

$$E\left[T_{a_Y}\right] = \frac{1}{P_{LOS}\,\lambda_{XY}\,x}. \qquad (30)$$

6.4. Probability That Target Is Killed before LOS Is Lost and Expected Length of Attack

Modeling competition and conflict (especially combat) frequently leads to the requirement of determining which of two competitors will win a race in time[15]. Mathematically, this problem is

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[14] In unpublished work that apparently has not been incorporated into VIC, Thompson [1990] shows that one must take into account that a target may not be initially visible. Here and in the sequel, for serial acquisition we will assume that all targets are initially visible (see Taylor [to appear, Section 13.3] for further details).

[15] John Boyd (e.g. see Hammond [2001] or Coram [2002]) has stressed the importance of such problems in military affairs. The OR Community has taken little note of Boyd's ideas (except for his conceptualization of decision making by means of the OODA loop, e.g. see Hammond [2001, pp. 188-191]), although they lead to new problems in probability

equivalent to determining which of two independent, nonnegative continuous random variables will be realized first. Therefore, let $\boldsymbol{S}$ and $\boldsymbol{T}$ be two such random variables, with probability density functions denoted as $f_S(s)$ and $f_T(t)$ respectively. We will denote the cumulative distribution function for S as $F_S(s)$, and hence $F_S(t) = \int_0^t f_S(s)\,ds$, with the complementary distribution function being denoted as $\bar{F}_S(s)$. Thus, $\bar{F}_S(s) = 1 - F_S(s)$. Also, similarly for the random variable T.

The probability that S is less than[16] T, denoted as $Prob[\,S < T\,]$, is then given by

$$Prob[\,S < T\,] = \int_0^{\infty} \bar{F}_T(t)\, f_S(t)\, dt. \tag{31}$$

A probability such as that given by (31) applies in many problems of practical interest in military OR (e.g. see Taylor [1983a, Appendix B]). Moreover, it is the basic result for development of the theory of stochastic duels (e.g. see Ancker [1967], [1982]). When both random variables are exponential, equation (31) takes a particularly simple form, which is so important that we will state it here as Theorem 2 (see also Taylor [to appear], in which it yields many important results for target acquisition in a heterogeneous target field that are then used to develop an analytical expression for a Lanchester attrition-rate coefficient for combat between heterogeneous forces).

<u>Theorem 2.</u> Let S and T denote two independent random variables, exponentially distributed with rates denoted as λ_S and λ_T. It follows that

$$Prob[\,S \le T\,] = \frac{\lambda_S}{\lambda_S + \lambda_T}. \tag{32}$$

Moreover, for exponential interfiring times[17] (all assumed to be identical) and independent rounds with constant single-shot kill probability, the distribution of time to kill an acquired target is well known (e.g. see Williams and Ancker [1963, p. 804)) to be again exponential, however, with rate (denoted simply as α) given by

$$\alpha = P_{SSK_{XY}}\, \nu_{XY}, \tag{33}$$

where P_{SSK} denotes single-shot kill probability, ν denotes firing rate, and the first of the double subscripts denotes who the target is, while the second denotes the firer type. Since the time to lose LOS has also been assumed to be exponential with rate (of occurrence) = μ, it follows via Theorem 2 that

$$P_{K(LOS)_{XY}} = \frac{\alpha}{\alpha + \mu}, \tag{34}$$

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theory of great importance in military affairs (see Taylor [2003] for further details). A leading military theorist (C. Gray [1999, p. 91]) says, "The OODA loop may appear too humble to merit categorization as grand theory but that is what it is. It has an elegant simplicity, an extensive domain of applicability; and contains a high quality of insight about strategic essentials, such that its author (John Boyd) well merits honourable mention as an outstanding general theorist of strategy." Moreover, Hammond [2003] considers Boyd to be "one of the great military minds of the twentieth century."

[16] This is the same probability as that for $S \le T$, since there is zero probability that they are equal under the assumption of continuous density functions.

[17] The reader should recall that this assumption was made in Section 2 above.

and, similarly, that

$$P_{K(LOS)_{YX}} = \frac{\beta}{\beta + \mu}. \qquad (35)$$

Finally, one can now invoke Theorem 1 to find that the expected time for a Y firer to engage (i.e. attack) an X target (after it has been acquired), denoted as $E\left[T_{atk|acq_{XY}} \right]$, is given by

$$E\left[T_{atk|acq_{XY}} \right] = \frac{1}{\alpha + \mu}, \qquad (36)$$

and similarly for an X firer against a Y target. Moreover, the above expression holds, however, only for exponential interfiring times.

Finally, we should note that the above results can be extended to any interfiring-time distribution that has a Laplace transform and a mean value. Since this is true for the vast majority of probability density functions, we can use any such density function to obtain useful results for a Lanchester attrition-rate coefficient. Bonder and Farrell only obtained results for the very restrictive case of exponential interfiring times.

7. Operational Motivation for Development of Model with Parallel Acquisition

Modern tank combat provides the operational motivation for developing a model with parallel acquisition of targets, since the typical modern tank (e.g. T-62 or any subsequent Soviet/Russian tank, M1A2) operates this way. In such a modern tank weapon system, the tank commander and gunner act independently part of the time as regards the acquisition of targets in the following manner. The tank commander will designate a target to be attacked by the tank and the gunner will then implement such an attack until either the tank is killed or LOS is lost. Meantime, the tank commander will look for new targets to attack (including refreshing his memory as to where previously-acquired targets are now located). In fact, in such systems the commander has his own view port (that can be moved independently from the firing of the main armament on the tank) for such target acquisition while the tank is attacking a previously acquired target. When the gunner finishes attacking the last target, a new target is given to him by the commander (given that he has acquired one or more new targets), with automatic slewing of the gun to the direction of the commander's view port (which should be aimed at the new target). To a first approximation, this situation corresponds to that of parallel acquisition described above.

Moreover, many modern tanks have an automatic loader the produces either almost deterministic interfiring times or interfiring times that can be modeled with an Erlang distribution with a relatively large shape parameter (i.e. the interfiring times have relatively low variability). For these reasons, the exponential distribution provides a very poor model for such interfiring times. However, the results given in this paper are readily extended to such Erlang interfiring times (even three-parameter Erlang times).

Tanks based on Soviet/Russian design have both of these characteristics. Tanks produced China, Iran, Ukraine, and many other countries share this heritage. It has been demonstrated (using CASTFOREM) that not accounting for these differences (as compared to classical tank design), results in a misrepresentation of such modern weapon systems.

8. The Lanchester Attrition-Rate Coefficient for Parallel Acquisition of Targets

One can develop an analytical expression for a Lanchester attrition-rate coefficient for parallel acquisition of targets by considering the appropriate target-engagement cycle (see Fig. 5) and developing mathematical expressions for the three quantities occurring on the right-hand side of equation (5): namely, the probability of the next target type to be engaged, the probability of killing such a target before losing LOS, and the expected length of the target-engagement cycle for the parallel acquisition of such a target. As seen above (see Fig. 5), parallel acquisition consists of the above two components. It therefore seems appropriate to call this overall target-engagement process "composite parallel acquisition." Thus, this composite parallel acquisition consists of

(1) component parallel acquisition (when one or more previously-acquired targets are available for immediate engagement at the beginning of a new target-engagement cycle),
(2) component serial acquisition (when no target is available for immediate engagement at the beginning of a new target-engagement cycle).

Therefore, to determine a numerical value for a Lanchester attrition-rate coefficient for parallel acquisition, one must compute the probability that one or more previously acquired targets are available for immediate engagement at the beginning of a new target-engagement cycle. Under the assumption that all targets behave independently, however, this latter probability involves the probability that an observer/firer has a particular target (of a given type) available for immediate engagement at the beginning of a new target-engagement cycle. For simplicity, however, we will refer to this probability just as "target availability." All other necessary probabilities for playing parallel acquisition of targets can be built up from this basic building block. It is the subject of the next section.

8.1. Target Availability for Particular Target

Thus, for (composite) parallel acquisition, target availability plays a key role. What is target availability? It is the probability that an observer/firer has a particular target (of a given type) available for immediate engagement at the beginning of a new target-engagement cycle. For simplicity, however, we will refer to this probability just as "target availability." How does one go about determining target availability? Considering the interaction of the LOS and target-acquisition processes, one should easily see that target availability is equal to the probability that a particular target (of a given type) is visible (i.e. LOS exists) to an observer/firer and has been acquired by this observer. This situation suggests modeling target availability with a three-state continuous-time Markov-chain model (see Fig. 7).

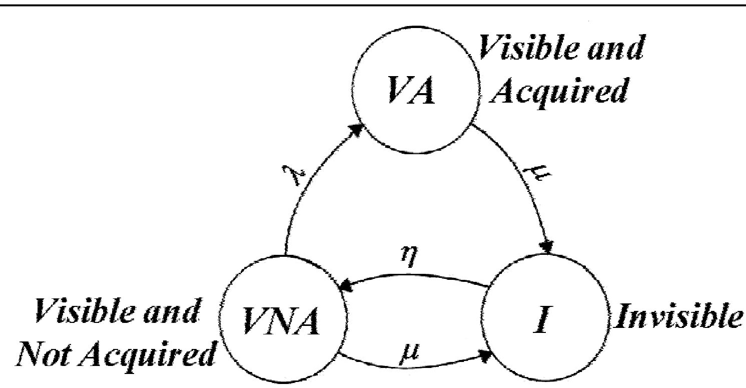

Fig. 7. Three-state Markov-chain model of target availability.

Hence, we consider a continuous-time Markov chain for a target that can be in one of three states

(1) target invisible to observer,
(2) target visible to observer but not acquired,
(3) target visible to observer and acquired.

Let us introduce the following notation

$$p_I(t) = \boldsymbol{Prob}\left[\boldsymbol{Target\ Invisible\ at\ Time\ t}\right],$$

$$p_{VNA}(t) = \boldsymbol{Prob}\left[\boldsymbol{Target\ Visible\ and\ Not\ Acquired\ at\ Time\ t}\right],$$

$$p_{VA}(t) = \boldsymbol{Prob}\left[\boldsymbol{Target\ Visible\ and\ Acquired\ at\ Time\ t}\right].$$

Making the usual Markov-chain assumptions, one can easily derive the following forward-Kolmogorov equations to describe the evolution of the above components of the system's state (cf. Section 6.1 above)

$$\begin{cases} \dfrac{dp_I}{dt} = -\eta\, p_I + \mu\, p_{VNA} + \mu\, p_{VA} \\ \dfrac{dp_{VNA}}{dt} = \eta\, p_I - (\lambda + \mu)\, p_{VNA} \\ \dfrac{dp_{VA}}{dt} = \lambda\, p_{VNA} - \mu\, p_{VA} \end{cases}, \qquad (37)$$

In the above model for target availability, time is measured from the beginning of the battle, not the beginning of the target-engagement cycle. Since the target must be in one of the three system states, it follows that

$$p_I(t) + p_{VNA}(t) + p_{VA}(t) = 1, \qquad (38)$$

which will be useful in the sequel. Mathematically, equation (32) also follows from (31) by adding the three equations together and integrating. Moreover, let us observe that only two of the three state variables (namely, p_I, p_{VNA}, and p_{VA}) are independent. Hence, one need only specify only two initial conditions to make the system (37) a determinate system (i.e. a "well-posed problem").

Solving (37) for target availability, i.e. the probability that the target is visible and acquired, one finds that

$$p_{VA}(t) = \left\{\left(\frac{\lambda}{\lambda-\eta}\right)p_I(0) + p_{VA}(0) - \left(\frac{\lambda}{\lambda-\eta}\right)\left[\frac{(\lambda-\eta)+\mu}{\lambda+\mu}\right]\right\}e^{-(\lambda+\mu)t}$$
$$\left\{-p_I(0) + \frac{\mu}{\lambda+\mu}\right\}e^{-(\eta+\mu)t} + \frac{\eta\lambda}{(\eta+\mu)(\lambda+\mu)}. \qquad (39)$$

We will see that there really is no problem when $\eta = \lambda$. The two initial conditions appearing in (39), i.e. $p_I(0)$ and $p_{VA}(0)$, are the appropriate ones from the standpoint of mathematical modeling. If one assumes that the LOS process has reached its steady state, then

$$p_I(0) = \frac{\mu}{\eta + \mu} = P_{LOS}, \qquad (40)$$

and (39) reduces rather dramatically to

$$p_{VA}(t) = p_{VA}(0)e^{-(\lambda+\mu)t} + P_{LOS}\left(\frac{\lambda}{\lambda+\mu}\right)\left\{1 - e^{-(\lambda+\mu)t}\right\}, \qquad (41)$$

where we must have that

$$0 \le p_{VA}(0) \le \frac{\eta}{\eta+\mu} = P_{LOS}, \qquad (42)$$

since we have all the initial conditions are probabilities, they sum to one, and we have assumed that the steady state has been reached, i.e. (40) holds.

For some circumstances (e.g. meeting engagement) it seems appropriate to assume that no enemy targets are initially acquired, equivalently

$$p_{VA}(0) = 0, \qquad (43)$$

and hence

$$p_{VA}(t) = P_{LOS}\left(\frac{\lambda}{\lambda+\mu}\right)\left\{1 - e^{-(\lambda+\mu)t}\right\}, \qquad (44)$$

which is a remarkably simple expression for target availability.

Again, for simplicity, we will assume that the simplest case holds in the paper at hand. We will then denote the availability of a particular X target to a Y firer as ***A(t)***, with that of a particular Y target to an X firer being denoted as ***B(t)***. It follows that

$$A(t) = P_{LOS}\left(\frac{\lambda_{XY}}{\lambda_{XY}+\mu}\right)\left\{1 - e^{-(\lambda_{XY}+\mu)t}\right\} = p_{VA_{XY}}(t), \qquad (45)$$

and similarly

$$B(t) = P_{LOS}\left(\frac{\lambda_{YX}}{\lambda_{YX}+\mu}\right)\left\{1 - e^{-(\lambda_{YX}+\mu)t}\right\} = p_{VA_{YX}}(t), \qquad (46)$$

where $p_{VA_{XY}}(t)$ denotes the probability that a Y firer has a particular X target available to fire at when the attack of a previously acquired target is over (through either the target being killed or LOS being lost) and a new target engagement cycle begins (again, see Fig. 5). In this expression t denotes elapsed time measured from the beginning of the force-on-force engagement, not the beginning of the target-engagement cycle.

8.2. Target Availability in General (Any Target)

Assuming that all targets act independently of each other, one readily computes the probability that a Y firer has no previously acquired target available for immediate engagement at the beginning of a target-engagement cycle, denoted as P_Y^0, as follows

$$P_Y^0 = \left[1 - A(t) \right]^x . \tag{47}$$

Similarly, the probability that a Y firer has one or more previously acquired targets available for immediate engagement at the beginning of a target-engagement cycle, denoted as P_Y^{1+}, may then be computed by

$$P_Y^{1+} = 1 - P_Y^0 , \tag{48}$$

or

$$P_Y^{1+} = 1 - \left[1 - A(t) \right]^x . \tag{49}$$

8.3. Expected Length of Target-Engagement Cycle

When one or more X targets are immediately available for engagement by a Y firer at the beginning of the target-engagement cycle, the expected length of this target-engagement cycle, denoted as $\bar{t}_{cycle_Y}^{1+}$, is given by

$$\bar{t}_{cycle_Y}^{1+} = E\left[T_{atk|acq_{XY}} \right], \tag{50}$$

where $E\left[T_{atk|acq_{XY}} \right]$ denotes the expected time that a Y firer will spend attacking an X target (that has been acquired) until either the target is killed or LOS is lost. However, when no such X target is available for immediate engagement by a Y firer at the beginning of the target-engagement cycle, then a new one must first be acquired (with $E\left[T_{acq_{XY}} \right] = E\left[T_{a_Y} \right]$ denoting the expected time for the Y firer to acquire this new X target) so that the expected length of this target-engagement cycle when no previously acquired target is immediately available, denoted as $\bar{t}_{cycle_Y}^{0}$, is given by

$$\bar{t}_{cycle_Y}^{0} = E\left[T_{acq_{XY}} \right] + E\left[T_{atk|acq_{XY}} \right]. \tag{51}$$

It follows by the theorem of total probability that the expected length of the target-engagement cycle for parallel acquisition (see Fig. 8), denoted as $\bar{t}_{cycle_Y}^{par}$, is given by

$$\bar{t}_{cycle_Y}^{par} = P_Y^0 \left\{ E\left[T_{acq_{XY}} \right] + E\left[T_{atk|acq_{XY}} \right] \right\} + P_Y^{1+} E\left[T_{atk|acq_{XY}} \right], \tag{52}$$

or

$$\bar{t}^{\,par}_{cycle_Y} = \left[1 - A(t) \right]^x E\left[T_{a_Y} \right] + E\left[T_{atk|acq_{XY}} \right]. \tag{53}$$

Substituting (30) and (36) into (53), we find that

$$\bar{t}^{\,par}_{cycle_Y} = \frac{\left[1 - A(t) \right]^x}{P_{LOS}\, \lambda_{XY}\, x} + \frac{1}{\alpha + \mu}. \tag{54}$$

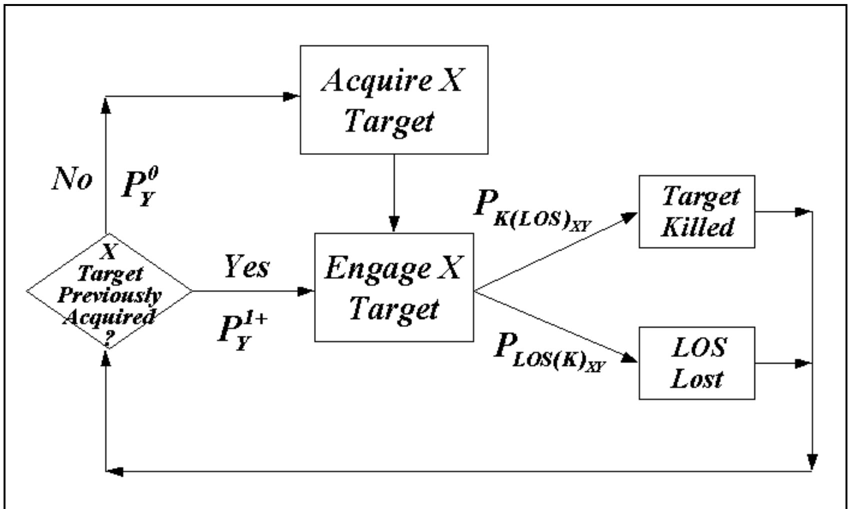

Fig. 8. Target-engagement cycle (parallel acquisition of targets) with branching and probabilities shown.

8.4. Expression for Kill Rate for Exponential Interfiring Times

It is convenient to rewrite equation (6) here specifically for the case of parallel acquisition of targets. Thus,

$$a^{par} = \frac{P_{K(LOS)_{XY}}}{\bar{t}^{\,par}_{cycle_Y}}, \tag{55}$$

where a^{par} denotes a single-weapon-system-type kill rate for a Y firer against X targets under conditions of parallel acquisition of targets and homogeneous forces, $\bar{t}^{\,par}_{cycle_Y}$ denotes the expected length of the target-engagement cycle for a Y firer (against X targets) under conditions of parallel acquisition, and $P_{K(LOS)_{XY}}$ denotes the probability that a Y firer will kill an acquired X target that it is attacking before LOS is lost. This latter probability is independent of the target-acquisition process, i.e. it does not depend on whether one is assuming serial or parallel acquisition of targets. Recalling that for exponential interfiring times, one readily sees that the probability of a Y firer killing an X target before LOS is lost is given by

$$P_{K(LOS)_{XY}} = \frac{\alpha}{\alpha + \mu}, \qquad (56)$$

and the expected length of the target-engagement cycle for parallel acquisition of targets is given by

$$\bar{t}^{par}_{cycle_Y} = \frac{\left[1 - A(t)\right]^x}{P_{LOS}\,\lambda_{XY}\,x} + \frac{1}{\alpha + \mu}. \qquad (57)$$

Consequently, the a single-weapon-system-type kill rate for a Y firer against X targets under conditions of parallel acquisition of targets and exponential interfiring times is given by

$$a^{par} = \frac{\left\{\dfrac{\alpha}{\alpha + \mu}\right\}}{\dfrac{\left[1 - A(t)\right]^x}{P_{LOS}\,\lambda_{XY}\,x} + \dfrac{1}{\alpha + \mu}}. \qquad (58)$$

9. Summary of Assumptions Made for Model

Basically we have assumed that all targets act independently of one another and that all inter-event times are exponential (including all acquisition and interfiring times). Furthermore, the simplest model for conditional kill rates (e.g. for α) has been assumed, although this assumption can be relaxed somewhat. A more thorough and organized discussion of these underlying assumptions (but for the case of heterogeneous forces on each side) has been given by Taylor [to appear, Section 16].

10. The Lanchester Attrition-Rate Coefficient for Serial Acquisition of Targets

With the results given above for parallel acquisition, it is now a simple matter to develop an expression for a Lanchester attrition-rate coefficient for serial acquisition of targets (at least for homogeneous forces). These results are necessary for the sequel, since we will want to compare combat outcomes for parallel and serial acquisition of targets.

Thus, it is also convenient to rewrite equation (6) here specifically for the case of serial acquisition of targets. Hence,

$$a^{ser} = \frac{P_{K(LOS)_{XY}}}{\bar{t}^{ser}_{cycle_Y}}, \qquad (59)$$

where a^{ser} denotes a single-weapon-system-type kill rate for a Y firer against X targets under conditions of parallel acquisition of targets and homogeneous forces. However, consideration of the target-engagement cycle for serial acquisition (see Fig. 4 above) yields that (cf. expression (51) for $\bar{t}^{0}_{cycle_Y}$ above)

$$\bar{t}^{ser}_{cycle_Y} = E\left[T_{a_Y}\right] + E\left[T_{atk|acq_{XY}}\right]. \qquad (60)$$

Consequently, the single-weapon-system-type kill rate for a Y firer against X targets under conditions of parallel acquisition of targets and exponential interfiring times is given by

$$a^{ser} = \frac{\left\{ \dfrac{\alpha}{\alpha + \mu} \right\}}{\dfrac{1}{P_{LOS}\, \lambda_{XY}\, x} + \dfrac{1}{\alpha + \mu}}. \tag{61}$$

Furthermore, it is worthwhile to note that for $A(t) > 0$ one has that

$$a^{ser} < a^{ser}, \tag{62}$$

the only real question being how significant an effect on battle outcome can occur when a force (by some combination of operating procedures and technology) can change from serial to parallel acquisition of targets. However, computational experiments have revealed that such a change can produce a rather profound change in battle outcome. This point will now be further investigated in the balance of the paper at hand.

11. Some Computational Results

In this section we consider two different (but yet closely related) battles in order to show the great advantage of having a system that uses parallel acquisition (as opposed to serial acquisition). More specifically, we will consider combat between two homogeneous forces in two slightly different battles, the only difference being that one side uses serial acquisition in one battle and parallel acquisition in the other (with all other aspects[18] for both sides being the same in these two battles).

Accordingly, let us consider combat between two homogeneous forces as depicted in Fig. 1. Furthermore, we will consider two slightly different battles as follows. The Y force will always use serial acquisition. Consequently, the Lanchester attrition-rate coefficient for the Y force (using serial acquisition) is always given by (59). The X force can change from serial acquisition of enemy targets to parallel acquisition. Computations will be done for the following two cases

(1) serial acquisition by X,

(2) parallel acquisition by X.

Thus, we will consider two slightly different battles. In the first battle, both sides use serial acquisition, with the battle dynamics being given by

$$\begin{cases} \dfrac{dx}{dt} = -\dfrac{\big(\alpha/(\alpha+\mu)\big)\, y}{\left\{ \dfrac{1}{P_{LOS}\, \lambda_{XY}\, x} + \dfrac{1}{(\alpha+\mu)} \right\}} \\[2ex] \dfrac{dy}{dt} = -\dfrac{\big(\beta/(\beta+\mu)\big)\, x}{\left\{ \dfrac{1}{P_{LOS}\, \lambda_{YX}\, y} + \dfrac{1}{(\beta+\mu)} \right\}} \end{cases}. \tag{63}$$

In this first battle, the numerical value for the Lanchester attrition-rate coefficient for the X force (using serial acquisition) is computed from a formula analogous to (59). Numerical results for a set

[18] In particular, all input parameters to each battle are exactly the same except that one side uses serial acquisition in one battle and parallel acquisition in the other.

of model parameters (see Table I) are shown in Fig. 9. In order to generate these numerical results, the first-order system of differential equations (63) was converted to a system of approximating difference equations by consideration of Euler's method (e.g. see Kreyszig [1999, Chapter 19] or Nagle and Saff [1993, Section 1.3]). Such a system of difference equations can easily be implemented on a spreadsheet (see Fig. 10). Furthermore, such a numerical approximation must always be used when the submodel for a Lanchester attrition-rate coefficient (even for the simplest models) involves either the numbers of targets or firers, which leads to nonlinear differential equations (that are inherently intractable by analytical methods (e.g. see Taylor [1978], [1982a], or [1983a, Chapter 6])).

<table>
<tr><th>(All Rates in Hours)</th><th>X (Blue)</th><th>Y (Red)</th></tr>
<tr><td>Single-Target Acquisition Rate against Enemy Target</td><td>0.1</td><td>0.07</td></tr>
<tr><td>Conditional Kill Rate against Enemy Target</td><td>8.0</td><td>1.5</td></tr>
<tr><td>Probability of LOS</td><td colspan="2">0.9</td></tr>
<tr><td>Rate of Losing LOS</td><td colspan="2">0.01</td></tr>
</table>

Table I. Inputs for the two battles discussed in the text.

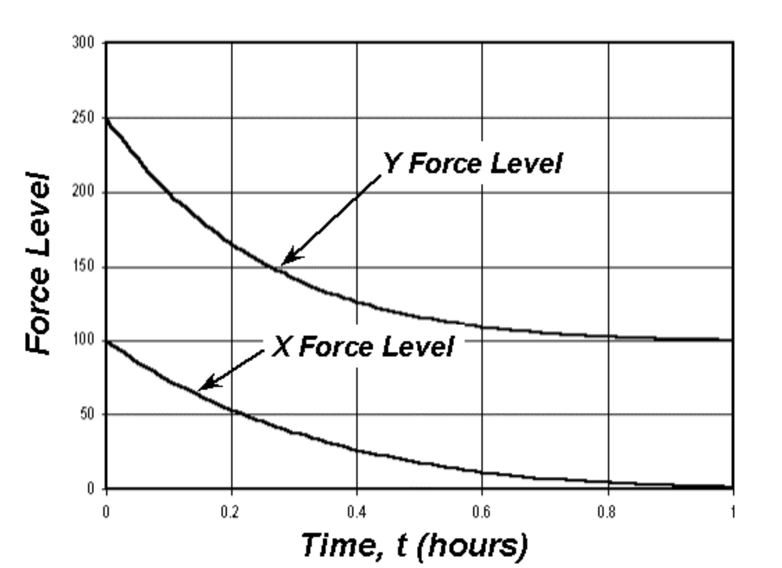

Fig. 9. Force-level decays for base-case battle (both sides use serial acquisition).

Lanchester-Type Combat: Intermittent LOS and Serial Acquisition

	X(Blue)				Y(Red)	
Initial Numbers	100				250	
Time Step			0.01	hr		
Single-Target Acquisition Rate	0.1	/hr			0.07	/hr
Conditional Kill Rate	8	/hr			1.5	/hr
Probability of LOS			0.9			
Rate of Losing LOS			0.01	/hr		
Attack Termination Rate	8.01				1.51	
Probability Target Killed before LOS	0.998752				0.993377	
Net Single-Target Acquisition Rate	0.09				0.063	

Time (hours)	X (Blue) Attrition-Rate Coefficient	*Y (Red) Attrition-Rate Coefficient*	*X (Blue) Loss in Period*	*Y (Red) Loss in Period*	*X (Blue) Force Level*	*Y (Red) Force Level*
0					100	250
0.01	5.899705	1.2099872	3.024968	5.899705	96.97503	244.1003
0.02	5.8625056	1.2027336	2.935876	5.685167	94.03916	238.4151
0.03	5.8253906	1.1953379	2.849866	5.478148	91.18929	232.937
0.04	5.788387	1.1877983	2.766821	5.278389	88.42247	227.6586
0.05	5.7515216	1.1801126	2.686628	5.085637	85.73584	222.573
0.06	5.7148209	1.1722786	2.609175	4.89965	83.12667	217.6733
0.07	5.6783111	1.1642942	2.534358	4.720191	80.59231	212.9531
0.08	5.642018	1.1561573	2.462073	4.547033	78.13023	208.4061

Fig. 10. Excel spreadsheet for calculating force levels as a function of time by Euler's method.

In the second battle, the X force now uses parallel acquisition, but with all other model parameters being exactly the same as for the first battle. Thus, in this case the numerical value for the Lanchester attrition-rate coefficient for the X force is computed from a formula analogous to (58). The battle dynamics are consequently now given by

$$\begin{cases} \dfrac{dx}{dt} = -\dfrac{\big(\alpha/(\alpha+\mu)\big)\,y}{\left\{\dfrac{1}{P_{LOS}\,\lambda_{XY}\,x} + \dfrac{1}{(\alpha+\mu)}\right\}} \\[2ex] \dfrac{dy}{dt} = -\dfrac{\big(\beta/(\beta+\mu)\big)\,x}{\left\{\dfrac{\left[1-B(t)\right]^{y}}{P_{LOS}\,\lambda_{YX}\,y} + \dfrac{1}{(\beta+\mu)}\right\}} \end{cases} . \tag{64}$$

As above, if we assume that the LOS process has reached steady state and that there are no enemy targets initially acquired, then Y target availability to an X firer is given by (46), which we will rewrite here as

$$B(t) = P_{LOS}\left(\frac{\lambda_{YX}}{\lambda_{YX}+\mu}\right)\left\{1 - e^{-(\lambda_{YX}+\mu)\,t}\right\}. \tag{65}$$

Numerical results for this second battle are shown in Fig. 11.

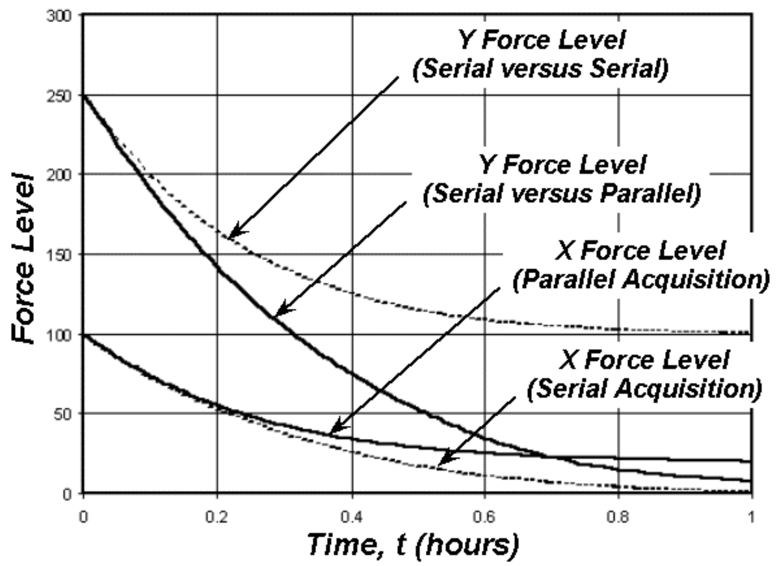

Fig. 11. Effect of changing from serial to parallel acquisition. Force-level decays for base-case battle appear as dotted lines.

The differences in outcomes for these two slightly different battles are really dramatic. By just changing from acquiring targets in the serial mode to the parallel mode for any X firer, the X force inflicts 62% more casualties and suffers 19% less attrition. Moreover, it must be emphasized here that the input parameters for each of the two opposing forces are exactly the same (see Table I) for both of these two battles, with the one exception that X changes from serial acquisition to parallel acquisition of targets. Furthermore, the X force is able to turn defeat into victory (see Fig. 10).

12. Insights into Effectiveness of Systems with Parallel Acquisition of Targets

Fig. 11 shows that the ability to acquire new targets while a (previously acquired) target is being engaged (i.e. attacked) can greatly increase the fire effectiveness (i.e. casualties produced) of a force and also reduce its own losses. The case depicted in this figure, however, was not typical of the numerous cases (i.e. different sets of input data [cf. Table I]) that were investigated by us in various spreadsheet calculations, but represents a fairly optimistic one. We suspect that the reason for the relative rareness of such an occurrence is that already superior forces will not reduce their own casualties so dramatically as depicted here (and we were looking for such a case), although they do inflict casualties faster with parallel acquisition. Thus, further investigation of the conditions under which parallel acquisition of targets is markedly advantageous is definitely required. However, this work here definitely does show that when a system (like the modern tank) does effect parallel acquisition of targets in practice, this feature must be reflected in combat models (especially those for aggregated forces).

Thus, much further experimental computing should be done in order to investigate the functional dependence of combat outcomes on model inputs for parallel acquisition of targets (and compared to results for serial acquisition). One aspect that peaked our curiosity was the effect of a limit on the number of targets that can be tracked on the attrition process. In the computations presented above, there was no limit on the number of previously acquired targets that could be tracked while one was being engaged (i.e. attacked). Investigation of this point so far has not led to any modification of the mathematical model for a Lanchester attrition-rate coefficient, but did lead to consideration of the number of "active targets."

13. The Number of Active Targets for a Firer

Lanchester-type differential equations such as (1) do not explicitly consider spatial distribution of the fighting capabilities of the two opposing forces[19]. All fighters on a particular side are considered to have identical characteristics, both as firers and also as targets. In order to develop the Lanchester attrition-rate coefficient for one of the two opposing forces, one considers a single typical firer and computes (for the case of intermittent LOS modeled as a Markov process[20]) his instantaneous rate of killing enemy targets by, for example, equation (6). Typically, one then assumes that all enemy targets are within sensor (i.e. acquisition) and weapons range of the firer and act independently. The result is that large numbers of targets for a force yield short acquisition times (cf. equation (63)). Before the convenient use of computers (and spreadsheets), one essentially had no other choice.

Now, however, one can approximate the underlying differential equations for force-on-force mutual attrition by difference equations and generate combat outcomes via step-by-step numerical integration of these difference equations on a spreadsheet (again, see Fig. 10). One can therefore consider explicit representation of the number of targets potentially available to a single typical firer on a particular side, and not be limited to only the case in which this number is equal to the size of the entire opposing enemy force. Dupuy [1983, Fig. 7] has presented some interesting data for the percent of a military unit directly exposed to enemy fire as a function of the unit's size. If one assumes something like that reserve force move forward and assume the positions of their fallen comrades, then one can hypothesize the following model for the number of active targets (i.e. targets potentially available for acquisition) to a single typical firer, denoted as $\boldsymbol{y_{act}}$,

$$\boldsymbol{y_{act} = minimum(n, y)}, \qquad (66)$$

where $\boldsymbol{n}$ denotes a quantity related (but not necessarily equal) to Dupuy's number directly exposed for a unit to enemy fire (as a function of the size of the opposing enemy military unit to the firer) and $\boldsymbol{y}$ denotes the total (average) number of live enemy targets. We have done some limited experimental computations using the model (64) with the number of active targets only considered for the X force, i.e. with as $\boldsymbol{y_{act}}$ given by (66) above only used in the calculation of target no availability for an X firer[21]. The results of this work will be discussed in the next section.

However, the main reason for discussing the concept of the number of active targets (even if the theoretical basis for the initial calculations presented here could be improved) is that we believe

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[19] See Taylor [1983b], however, for the development of a Lanchester-type model (taking the form of a coupled pair of integro-partial differential equations) that explicitly represents such spatial distributions of fighting capabilities. Furthermore, in practice, such spatial distribution is played by breaking a large force (i.e. military unit) into component subunits that engage in combat between opposing subunits according for equations similar to (2).

[20] See Section 6 above.

[21] In other words, target nonavailability to an X firer was computed in (64) as (cf. equation (47) above)

$$\boldsymbol{P_X^0 = \left[1 - B(t)\right]^{y_{act}}}.$$

However, we should have also used $\boldsymbol{y_{act}}$ to compute the time to acquire the first Y target by an X firer.

that it can be used as a means for representing some of the effects of network-centric warfare in a very convenient fashion. Although we have not worked out all the mathematical modeling details, we will discuss this point further in Section 15 below.

14. Some Further Computational Experiments

In this section we present some very preliminary results for which the target-direct-contact parameter for a firer (denoted as $\boldsymbol{n}$) in the model (66) for the number of active targets for a typical X firer (denoted as $\boldsymbol{y_{act}}$) is varied over a range of values for the same two battles as considered above in Section 11. As before, in one of these battles the X force uses serial acquisition, while in the other it uses parallel acquisition. Accordingly, we have done computations for the second battle with the following Lanchester-type equations

$$\begin{cases} \dfrac{dx}{dt} = -\dfrac{\left(\alpha/(\alpha+\mu)\right)y}{\left\{\dfrac{1}{P_{LOS}\,\lambda_{XY}\,x} + \dfrac{1}{(\alpha+\mu)}\right\}} \\[3ex] \dfrac{dy}{dt} = -\dfrac{\left(\beta/(\beta+\mu)\right)x}{\left\{\dfrac{\left[1-B(t)\right]^{y_{act}}}{P_{LOS}\,\lambda_{YX}\,y} + \dfrac{1}{(\beta+\mu)}\right\}} \end{cases}, \qquad (67)$$

with Y target availability to an X firer (denoted as $\boldsymbol{B(t)}$) given by (65) and the number of active targets (denoted as $\boldsymbol{y_{act}}$) for computing target nonavailability to an X firer (denoted as P_X^0) given by (66). Thus, Y target nonavailability to an X firer is given by

$$P_X^0 = \left[1 - B(t)\right]^{y_{act}}. \qquad (68)$$

Results for the target-direct-contact parameter for an X firer (denoted as $\boldsymbol{n}$) in the parallel case equal to 10 are shown in Fig. 12, which should be compared with Fig. 11 above. In Fig. 12 (similar to Fig. 11) results are shown for two different battles: a battle in which the X force uses parallel acquisition (modeled with equations (67) and with force-level decays drawn as solid lines) and a battle in which the X force uses serial acquisition (modeled with equations (63) and with force-level decays drawn as dotted lines). In both battles the Y force uses serial acquisition. Since results for the X force level are so similar is these two cases, the X force level when it uses serial acquisition has not been identified as such in Fig. 12. The common model parameters for both battles are shown in Table I.

In Fig. 12 the number of active Y targets for a typical X firer (using parallel acquisition) is essentially equal to 10 for the entire battle, and parallel acquisition is of little benefit to such an X firer, since target nonavailability is generally fairly large due to the relatively small exponent in (68). However, when the target-direct-contact parameter for an X firer in the parallel case is equal to 50, the number of active Y targets for a typical X firer is equal to 50 for most of the battle so that a sizeable list of previously acquired targets can build up for an X firer (see Fig. 13). Thus, parallel acquisition is more of an advantage for an X firer for the battles shown in Fig. 13 than it is for the battles shown in Fig. 12, but still not as much as it is for the battles shown in Fig. 11, which essentially applies in the parallel acquisition case when all Y targets are active for a typical X firer.

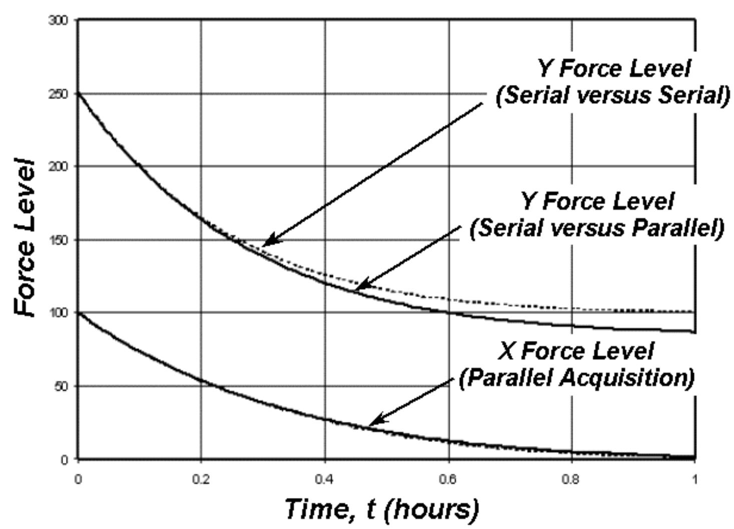

Fig. 12. Effect of changing from serial to parallel acquisition with the target-direct-contact parameter (denoted as ***n***) in parallel case equal to 10. Force-level decays for base-case battle appear as dotted lines.

Finally, in Fig. 14 the target-direct-contact parameter for an X firer (denoted as ***n***) in the parallel case is equal to 125. In this case the number of active Y targets for a typical X firer is equal to 125 for only about the first third of the battle (until the Y force level has decayed to 125), but this has been long enough for parallel acquisition to really pay off for the X force: reduced target acquisition times for X firers have caused a significantly higher attrition rate against the Y force when X uses parallel acquisition than in the battle in which they both use serial acquisition (depicted with the dotted lines for force-level decays). One should also observe that the serial versus parallel battle in Fig. 14 is fairly close to that depicted in Fig. 11 (in contrast to those shown in Fig. 12 and 13).

Moreover, one sees that the larger the number of active Y targets to a typical X firer becomes, the closer results become to the case in which all Y targets are active (i.e. those depicted in Fig. 11). If one inquires as to what could be the cause of all targets in an enemy force being active, one possible answer is that it may be because targeting information is being shared by X firers, i.e. they are collaborating in network-centric warfare. Thus, one can possibly represent the effects of network-centric warfare not so much by directly modeling it, but by degrading the effectiveness of firers without equipment and means to share targeting information. Consideration of parallel acquisition by modern tanks has led to this remarkable conclusion. Furthermore, when the effects of limiting the number of active targets are played also for the case of serial acquisition (which was not done for these preliminary results reported here), attrition will be further slowed down in such Lanchester-type models. This is very important, since such models are notorious for running "too hot" as far as attrition is concerned. Thus, the concept of the number of active targets for a typical firer in the modeling of a Lanchester attrition-rate coefficient may become a means of representing network-centric warfare. The major difficulty would appear to be how to measure or estimate such a quantity for representation in a Lanchester attrition-rate coefficient.

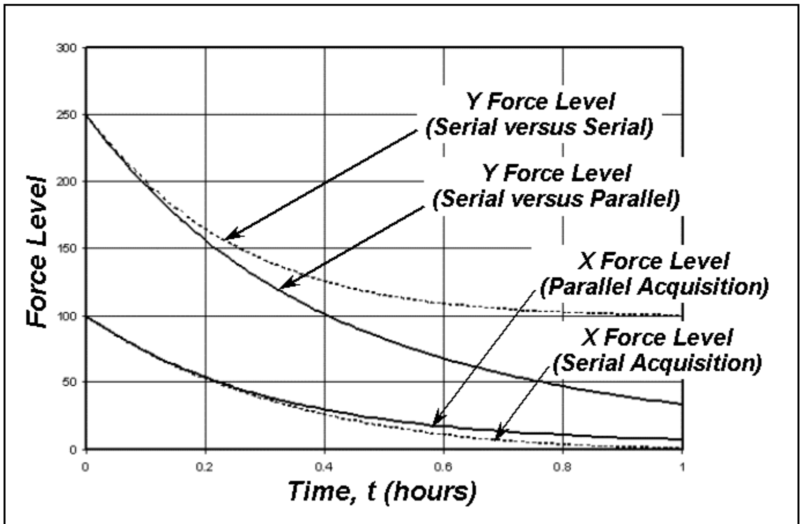

Fig. 13. Effect of changing from serial to parallel acquisition with the target-direct-contact parameter (denoted as $\boldsymbol{n}$) in parallel case equal to 50.

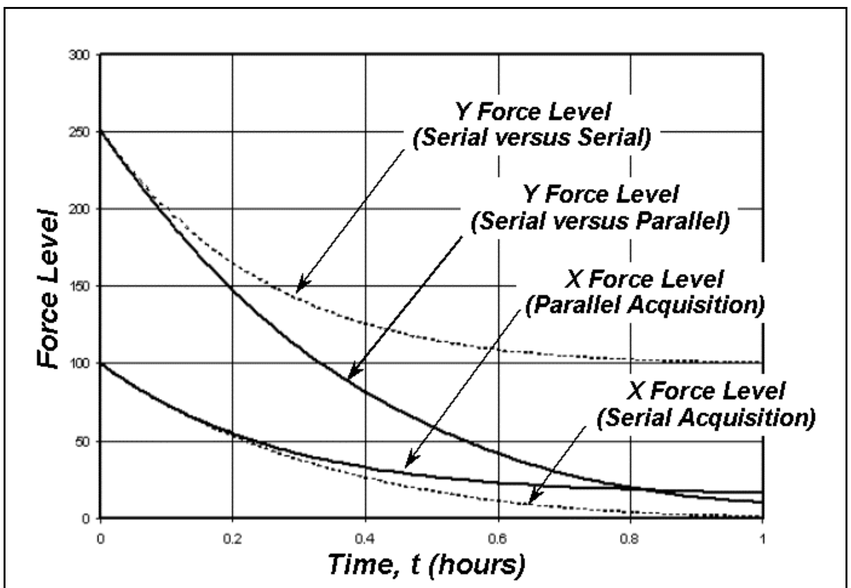

Fig. 14. Effect of changing from serial to parallel acquisition with the target-direct-contact parameter (denoted as $\boldsymbol{n}$) in parallel case equal to 125.

15. Relevance of Model for Network-Centric Warfare

The above models (i.e. parallel acquisition by itself and parallel acquisition combined with the modeling of the number of active targets) provide a very convenient mechanism for quantitatively transforming improvement in the processing of targeting information into improvement in combat outcomes. The model of parallel acquisition by itself is a very important new model that very conveniently converts the ability for a firer of being able to acquire new targets while a previously acquired target is being engaged (i.e. attacked) into production of more enemy casualties and reduction of one's own casualties. Parallel acquisition appears to be of particular value when target acquisition is the limiting factor in inflicting casualties on an enemy. Moreover, the combination of the concept of the number of active targets for a typical firer (as modeled by (66)) and parallel acquisition would appear to be a particularly valuable idea. Finally, experimental computing allows one to investigate UNDER WHAT CONDITIONS such hypothesized interactions might (or might not) really pay off as far as more combat effectiveness. Thus, we feel that the concepts and mathematical models developed in this paper will be very useful for further investigating in a quantitative fashion the possible benefits from improving weapon system and sensor capabilities (as well as the architecture of the processing of targeting information). The key characteristics, moreover, of such models are their convenience and accessibility.

16. Future Enhancements

Bonder and Farrell [1970] developed expressions for a Lanchester attrition-rate coefficient only for the very restrictive case of exponential interfiring times, which is at variance with all the data played in U.S. Army high-resolution Monte-Carlo simulations[22]. However, Taylor has developed very general results that allow one to play any distribution for interfiring times and develop an analytical expression for the corresponding Lanchester attrition-rate coefficient that is not appreciably more complicated than those given above. Moreover, experimental computing with such coefficients has revealed that playing such more realistic interfiring times can lead to appreciably different outcomes than those obtained form just playing exponential interfiring times. Thus, one extremely important future enhancement would be the playing of more realistic interfiring times (including having the first round be different form subsequent rounds). Other important future enhancements would include playing the number of

(1) active targets for a firer,
(2) active firers against an enemy force.

17. Final Comments

Taking Taylor's [to appear] new methodology for developing an analytical expression for a Lanchester attrition-rate coefficient from detailed mathematical modeling of the target-engagement cycle, this paper has developed new methodology and paradigms (e.g. parallel acquisition of targets, the number of active targets) to reflect the effects of information superiority and network-centric warfare on fire effectiveness in aggregated-force combat models. Moreover, such mathematical models have apparently not been previously considered in the combat modeling literature. Furthermore, once such an analytical structure for Lanchester attrition-rate coefficients has been developed, parameter values could be estimated by a hierarchy-of-models approach that uses high-resolution Monte-Carlo combat simulation (e.g. see Taylor et al. [2000]). Thus, this work could in the future build on the latest developments in entity-level combat simulations that consider the playing of information, information superiority, and network-centric warfare. However, other mathematical modeling approaches may be possible (e.g. expansion of the state space in such an aggregated-force model).

The results given in the paper at hand were all developed for intermittent line of sight (LOS) that is mathematically modeled as a stochastic process (see Section 6 above). This LOS model was first developed by Bonder and Farrell (e.g. see Miller et al. [1978], CCTC [1979], TRAC-FLVN

[22] See Footnote 6 above.

[1992], or Bonder [2002a, p. 29]). However, we have not been able to verify that they ever developed any theoretically correct analytic expression for a Lanchester attrition-rate coefficient under these conditions of playing stochastic LOS. However, such results are given here for the first time, both for serial and also for parallel acquisition of targets (see also Taylor [to appear]). Although the results given here are for exponential interfiring times (which is not reflected in actual AMSAA data) and the assumption that all rounds are mathematically identical, they are readily extended to cases of an arbitrary interfiring-time distribution and also to cases in which a finite number of rounds can have different characteristics. Moreover, work has already been completed on these extensions by us (with all the mathematical details worked out) and just needs to be written up.

The work here has focused on the detailed modeling of the target-acquisition process to reflect the effects of information superiority and network-centric warfare. The continual acquisition of targets throughout the target-engagement cycle for parallel acquisition was represented through target availability (i.e. the probability that an observer/firer has a particular target of a given type available for immediate engagement at the beginning of a new target-engagement cycle) from which one can compute the probability that one or more previously-acquired targets are available for immediate engagement at the beginning of a new target-engagement cycle. In turn, a three-state Markov- chain model is used to determine target availability, which then plays a critical role in computing a Lanchester attrition-rate coefficient for parallel acquisition. Obtaining an explicit analytic expression for target availability played a key role in these developments.

Moreover, this model that considers parallel acquisition of targets at the platform level can be extended from such a platform-centric force-on-force model to a network-centric one. Such extensions were briefly considered and initial insights into the greater combat effectiveness to be gained from being able to share targeting information and operating in the network-centric mode were developed (including consideration of different architectures for the U.S. Army's future combat system (FCS)). We have barely scratched the surface here and much more important work remains to be done.

Acknowledgement

The work by Taylor and Neta was partially supported by the Defense Threat Reduction Agency (DTRA).

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