

# Applications of Dynamic Systems Theory to Effects-Based Operations and Adversarial Modelling

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## Abstract

Effects-based operations (EBO) has become an increasingly important doctrinal concept used in the prosecution of war, most especially against terrorist organizations and the rogue states which support them. As a philosophy, EBO reaches beyond the realm of the propagation of simple physical effects. EBO encompasses the full spectrum of military activities, including psychological operations (PSYOPS). While a number of different accounts of EBO have been documented (Warden 1995, Barlow 1994), alarmingly little work has been conducted concerning the application of effects-based operations to organizations of human entities. Herein, we present a formal model of  $n^{th}$ -order cascading belief revision in the style of Warden's model. The considered approach is motivated by the theory of dynamic systems, and is able to be generalized through the manipulation of beliefs via information-theoretic (Shannon & Weaver 1949) metrics. We shall conclude with a simple example, and some future directions for research in this area.

# 1 Introduction

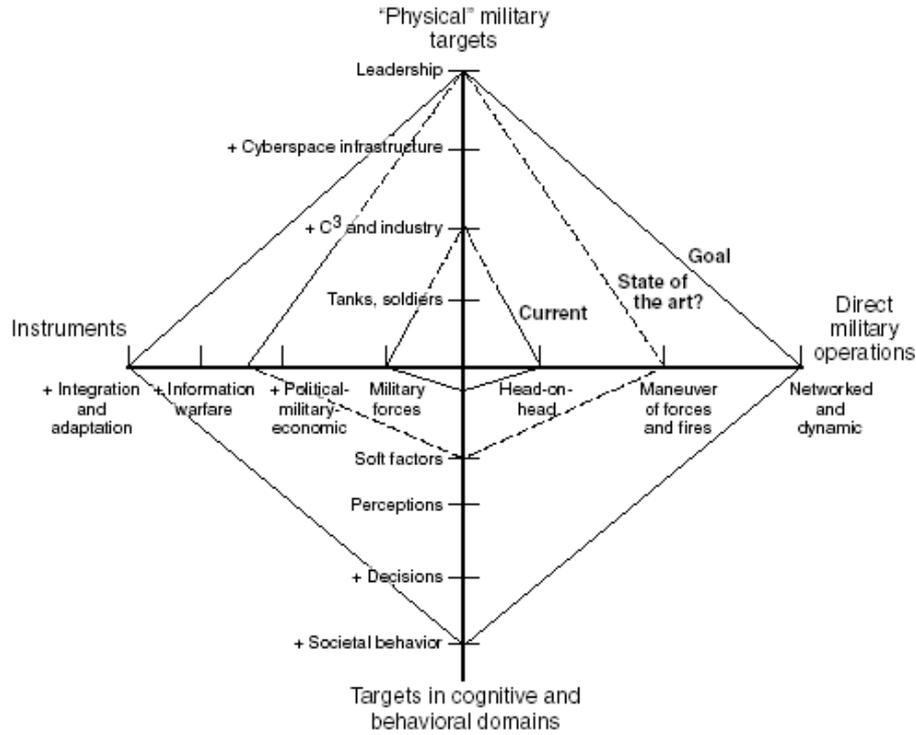
Understanding organizational behavior has traditionally occupied an important place in the conduct of warfare. In light of the tragedies of September 11th 2001, gaining knowledge pertaining to the infiltration of terrorist networks has become critical to maintaining our national security posture. Warden's model of the enemy as a system (Warden 1995) provides a general framework for the modelling of organizations. Other theories of organization, such as Simon's theory of administrative behavior (Simon 1997), can be applied on top of Warden's model to produce reasonable flexible models of goal-directed agent interaction via means-ends planning. While the goal-directed aspect of organizational design isn't central to our discussion, it can be seen as a logical next step in designing decision-aids for effects-based analysis of organizational behavior models. This paper is organized into three sections: a brief introduction to the effects-based modelling paradigm, our mathematical formulation of system-of-systems organizational modelling, and some promising new research directions in which to extend our methodology.

## 2 Effects-Based Operations and Military Rationality

Effects-based operations has been defined in a number of different ways, serving a number of different purposes. Rather than attempt to define a synthesis of all of these descriptions, we shall adopt the following:

Effects-based operations adopt a systemic view of both the environment and agents which act within it. As such, EBO focuses on the analysis of direct, indirect, and cascading ( $n^{th}$ -order) effects. The effects-based analysis framework incorporates the application of the full spectrum of politico-military resources to conflict situations. Effects-based planning is *outcome-centric*, focusing primarily on the achievement of a desired end-state through the application of these resources in a fashion consistent with the concepts of operations defined in (H. Shelton. General, Chairman - Joint Chiefs of Staff 2000).

An excellent discussion of effects-based operations can be found in (Davis 2002), which provides a taxonomic description of potential effects-based evaluation methodologies. One of the major sticking points in most of the documents concerning current conceptions of effects-based operations is the relative lack of success in defining how the EBO methodology can be construed formally, and most especially, how these formal models operate at the sociocultural level. As the reader will notice in figure 1, the negative y-axis denotes the "cognitive dimension" in terms of affecting targets. Unfortunately, the cognitive aspects of effects-based operations are at best, ill-defined. Attacking all of the problems associated with cognition (perception, affect, memory, learning, reasoning, decision-making, and so on) will be the grand challenge for a new generation of artificial intelligence researchers who are gradually looking at human intelligence (through the seminal empirical research) to answer questions about the limitations of rationality and cognitive capacity. It is our feeling that we can leverage some prior work done at the level of organizations in order to better understand



NOTE: The axis of physical targets relates to both direct physical and systemic effects; dashed lines indicate where current capabilities are poor in capturing indirect effects.

Figure 1: Characterization of Baselines, State of the Art, and Goal (Courtesy of P.K. Davis)

some of the salient issues in modelling the cognitive dimensions of effects-based operations. One of the authors (PB) has developed a conception of the individual decision-maker based on a concept inspired by Warden’s five-rings model which is called cognitive effects-based operations (CEBO). The CEBO paradigm was inspired by the following question: “*What happens when an entity belongs to multiple systems, with (potentially) conflicting value structures?*” Figure 2 provides a clearer conception of the issue at hand:

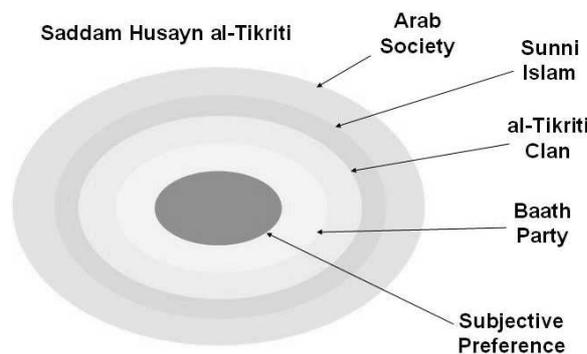


Figure 2: Context-Dependent Decision Making via CEBO

In this case, the decision-maker in question is Saddam Huseyn al-Tikriti, the former president of Iraq. While we make no claims about what Saddam was actually thinking when making the numerous decisions that he made during his tenure, it seems reasonable to suppose that he certainly had to take the many different organizational contexts that he was connected to into consideration while doing so. In concordance with other formal models of decision-making (Von Neumann & Morgenstern 1944, Savage 1964), we may loosely characterize the military decision-maker in the following way:

- $A$ : The set of foreseeable alternatives.
- $KB$ : Heterogeneous knowledge base (syntactic knowledge, semantic (model-theoretic) knowledge, explicit beliefs, explicit deontic knowledge, et cetera)
- $(c_i \in C) \subseteq KB$ : a set of “contexts”  $C$ .
- $\succsim_c$ : a preference ordering over the contexts.
- $f_\pi(A, c_i, \succsim_c) : Ac_i \longrightarrow A'_{c_i}$ : a context-sensitive *permissibility function*.
- $\mathring{A} = \bigcup_{c_i \in C} f_\pi(A, c_i, \succsim_c) = \{A'_{c_0}, A'_{c_1}, \dots, A'_{c_m}\}$  for each of the  $m$  contexts.
- $U_p(x)$ : A nonlinear prospective utility function which returns the perceived utility for a given alternative.

This conception of “military man” stands in stark contrast to the normative theory posed in (Von Neumann & Morgenstern 1944). While some have called Von Neumann’s creation “the economic man”, or “economic rationality”, we refer to this formal incarnation of the military decision-maker as being inspired by the “military rationality” framework under development by Bello, Yang, et al.

A context, in our terminology, can be construed as a set of deontic principles (ethical rules, social norms, etc) which act as a meta-property of the organization it is associated with. The context defines permissibility of action in the presence of various conditions which compose the decision-maker’s mental representation of the situation at hand. For the sake of brevity, we shall not go into the formal mechanisms for representing and computing over information of this variety, but a full characterization shall appear in (Bello 2005). Military rationality is the confluence of several different intellectual traditions, ranging from Bayesian epistemology (on the nature of truth and probability in decision theory) (Jeffrey 1965) to research in the psychology of higher-order cognitive processes (Tversky & Kahneman 1990, Johnson-Laird 1983, Braine 1990). In the opinion of the authors, the defining distinction to be made here is the following: should a theory of decision/cognition be based on desirability of a particular outcome, or the probability that said outcome will occur. The evidence seems overwhelmingly in favor of the former. Kahneman’s Nobel-prize winning experimentation on this issue seems to be a remarkable vindication of the fact that behaviorally, humans display marked inability to deal with probability in a normative way while processing contextual cues. Taking this as a starting point, the development of a new theory of decision-making should, in turn, be more sensitive to conceptions of desirability, rather than the strictures of a purely probabilistic world-view. A full discussion of these issues is warranted, and will also appear in (Bello 2005).

### 3 Organizational Theory and Dynamic Systems

The interdisciplinary nature of multi-agent systems modelling theory (Woolridge 2002) naturally lends itself to effects-based analysis. The agent-based paradigm has proven to be a rich testbed for looking at so-called “emergent” properties of collections of agents. The question laying at the heart of this research is in regards to how the beliefs (and subsequent intentions) of organizations change when they are related through multi-resolution hierarchical groupings of agents. Are these beliefs related purely to the sharing of (possibly incomplete/uncertain) information, or do exogenous factors influence the intentions of an organization? For the sake of computational convenience, we shall represent beliefs and ethics as simple probability distributions displaying Gaussian characteristics. A more complex development of belief and deontic statements is currently under development as a quantified modal logic. We take the liberty of modelling organizations in an extraordinarily simplified way, focusing on the representations of belief and ethics in the generalized guise of “information”. Let us start with a few definitions:

**Def:** Let  $\mathbf{B}$  be the set of belief functions for a given organizational domain  $\mathbf{O}$ .

**Def:** Let  $\mathbf{E}$  be the set of ethical concern functions for  $\mathbf{O}$ .

**Def:** Let an *information parameter*  $\tilde{\alpha}_i$  be an indexed value, associated with the sets  $\{\mathbf{B}, \mathbf{E}\}$  which describes the relevant belief or ethical concern function.

**Def:** Let  $\phi \in \Phi$  be an *alphabet*, which is the set of  $\tilde{\alpha}_i$  which defines an interaction space completely, and let  $\Phi$  be the set of all information parameters relevant to  $\mathbf{O}$ .

**Def:** Let the  $I_n$ -space be an n-dimensional interaction space over which all agents of identical alphabets are defined.

**Def:** An agent  $\mathbf{a}$  is defined by a set of  $\{\alpha_b, \alpha_e\}$  which specifies the distributions defined for each parameter.

**Axiom:** All agents  $\mathbf{A}$  defined in  $I_n$  contain values for each  $\tilde{\alpha}_i$  in  $\phi$ .

**Theorem:** A system can be changed by either changing the value of an information parameter directly, or by allowing two agents to interact with one another.

**Def:** Let an interaction  $I_{ij}$  be a sharing of information between two agents  $\mathbf{a}_i$  and  $\mathbf{a}_j$  with the intention of altering a belief. Let this interaction formally be defined as the following:

$$I_{ij} = \sum_1^m \sum_1^n \frac{\alpha_{b_n}^i \alpha_{b_m}^j}{\lambda \alpha_{e_m}^j}, \quad (1)$$

where the parameter  $\lambda$  is defined as the *Kullback-Leibler Distance* (Kullback & Leibler 1951, Mackay 2003) between two belief functions  $\alpha_b^1$  and  $\alpha_b^2$  (shown in Figure 3 for three belief-type information parameters):

$$\lambda = \alpha_{b_n}^1 \log \left[ \frac{\alpha_{b_n}^1}{\alpha_{b_n}^2} \right], \quad (2)$$

and where an individual interaction relative to a specific belief is given by:

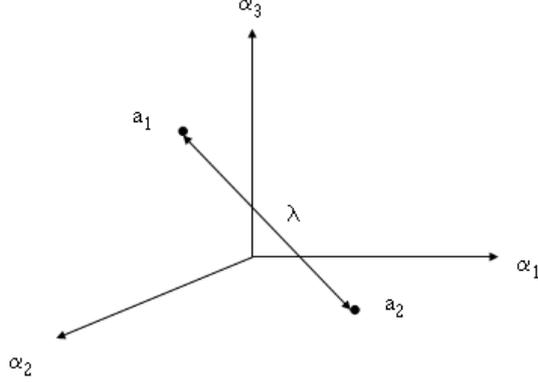


Figure 3: Interaction space

$$I_{ij}(\alpha_{b_n}^1) = \sum_1^m \frac{\alpha_{b_n}^i \alpha_{b_m}^j}{\lambda \alpha_{e_m}^j} \quad (3)$$

In interactions such as these, the ethical concern function  $\alpha_{e_n}$  is always constant, assuming organizations and their constituents rarely ever change their ethics. In the context of the interaction equation (1),  $\alpha_{e_n}$  serves as a weight representing the strength of the relevant belief, or the difficulty to change the belief of the relevant agent.

**Corollary:**  $I_{ij} \neq I_{ji}$  (Interactions are asymmetric).

The change in  $\alpha_{b_n}^1$  as two agents interact can be described by the differential equation:

$$V(\alpha_{b_n}^1) = \frac{\partial I_{ij}}{\partial t}(\alpha_{b_n}^1) \quad (4)$$

To analyze a system of agents in a single interaction space  $I_n$ , we must solve a system of differential equations:

$$\text{solution}(i) = \begin{cases} V(\alpha_{b_i}^i) = \frac{\partial I_{ij}}{\partial t}(\alpha_{b_p}^1) \\ V(\alpha_{b_i}^i) = \frac{\partial I_{ij+1}}{\partial t}(\alpha_{b_p}^1) \\ \vdots \\ V(\alpha_{b_p}^i) = \frac{\partial I_{im}}{\partial t}(\alpha_{b_i}^i) \end{cases}$$

where  $p = 1 \dots n$  is the length of the alphabet  $\phi$ , and  $m$  is the number of agents in the interaction space. This set of equations must be solved completely with the set of initial conditions being the values of  $\phi(\tilde{\alpha}_p)$  at the time of interaction for agent  $i$ . Interactions such as these generate a 3-dimensional tensor  $T$  with dimensions  $m \times m \times i$ , since similar systems such as the above must be solved for each agent beyond the first.

The complete solution for a set of such equations yields an equation for each  $\alpha_b$  in terms of  $\lambda$ , which may be optimized using standard optimization techniques for n-dimensional spaces or through non-linear analysis techniques.

### 3.1 Multi-Resolution Organizational Modelling

Possible mechanisms behind multi-space interaction: any space  $I_{\eta prime}$  can be thought of as an individual agent in the supervening space  $I_j$ :

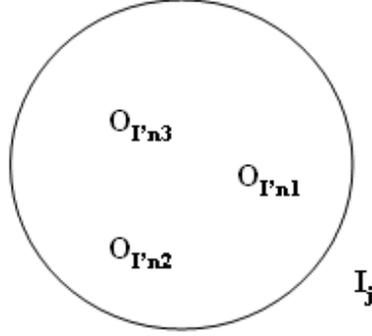


Figure 4: Multi-Resolution Space

an  $\alpha_b$  in  $\phi_{I_j}$  is proportional to the corresponding  $\alpha_b$  in the lower level spaces which compose it:

$$\alpha_{e_n}^{I_j} = \frac{1}{m} \sum_{i=1}^m \alpha_{e_i}^{I'_{n,m}} \quad (5)$$

and

$$\alpha_{b_n}^{I_j} = \frac{1}{m} \sum_{i=1}^m \alpha_{b_i}^{I'_{n,m}} \quad (6)$$

and  $\phi_{I_j} = \{\alpha_{e_n}^{I_j}, \alpha_{b_n}^{I_j}\}$

## 4 Example

As a very simple example (possibly the most simple example worth studying with this model), two agents interact with each other concerning two beliefs  $\alpha(b1)$  and  $\alpha(b2)$ . For simplicity, we will assume that only agent 1 will be affected by the interaction, emphasizing the asymmetry of the interaction as a whole. Therefore,  $\alpha_1$  and  $\alpha_2$  will be held constant for agent 2. This system produces two first order differential equations, the solutions of which produce functions for the changes in  $\alpha_1$  and  $\alpha_2$  in terms of time:

$$\Delta\alpha_1 = \left( \frac{b_1(t)}{\left( be_1 \cdot \ln \left( \frac{a_1(t)}{b_1(t)} \right) \right)} \right) + \left( \frac{b_2(t)a_1(t)}{\left( be_2 \cdot \ln \left( \frac{a_2(t)}{b_2(t)} \right) \cdot a_2(t) \right)} \right) \quad (7)$$

$$\Delta\alpha_2 = \left( \frac{b_1(t)a_2(t)}{\left( be_1 \cdot \ln \left( \frac{a_1(t)}{b_1(t)} \right) \cdot a_1(t) \right)} \right) + \left( \frac{b_2(t)}{\left( ae_2 \cdot \ln \left( \frac{a_2(t)}{b_2(t)} \right) \right)} \right) \quad (8)$$

It may seem that this model behaves very much like a physical system of point masses interacting via a gravitational force (or similarly, two charged particles interacting via a Coulomb force). The analogy is not complete, but still worth discussing. The factor of lambda represents a distance in interaction space between two agents. The space is defined over the alphabet, so the coordinate system for the space is represented by the alphabet. Unfortunately the relationship between lambda and alpha is logarithmic, so the “distance” analogy is not perfect. Solutions to the differential equations above are analogous to the “velocities” of the alphas in interaction space. They represent the rate at which alpha changes with time. The interaction function (1) itself, then, is analogous to a force acting on the agents in interaction space.

It is important to realize that “time” in the context of this model is not even completely analogous to “real” time. It acts as a system parameter which increments independently of the system. It can be used as an independent variable to parameterize the alphas by, but certainly does not correspond to any change in “seconds,” per se.

The solutions to the above system of differential equations are best represented in a phase (or solution) space. Figure 5 shows the solution space for two agents with very different ethical concern functions, and figure 6 shows the solution space for two agents with identical ethical concern functions.

## 5 Conclusions and Future Work

When analyzing the above phase portraits, it is important to realize that the trajectories represent changes in alphas rather than alphas themselves. All relevant information can still be extracted from such plots, but without the expense of solving systems of second order differential equations. Most importantly, unstable equilibria (example is identified by the rectangle in figure 5) represent regions that are most easily perturbed from their equilibrium. These regions correspond to specific values of the belief functions that are most easily, or most effectively, changed.

Even with an example as simple as this, it begins to become apparent how valuable such a model may be. Potentially, such systems could provide analysts or scientists with data concerning what beliefs or intentions of certain individuals or groups of people might easily be changed, allowing for advantage in insight against our opponents.

The authors would like to emphasize that this research is in its infancy. There are several research directions to pursue in the immediate future, mostly in relation to empirical data

2-D Interaction Space for alpha 1, alpha 2

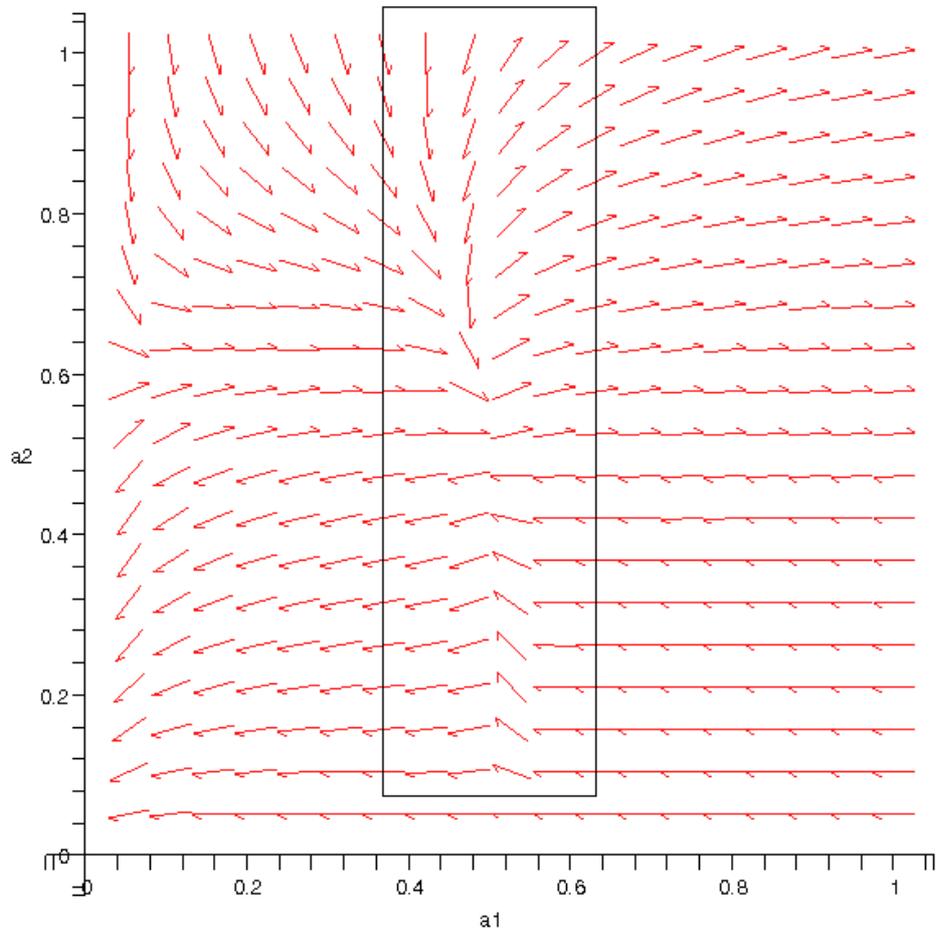


Figure 5: Solution Space: Different Ethics

on methods for coercion, a more suitable representation for belief, and new methods for the visualization and analysis of this sort of data.

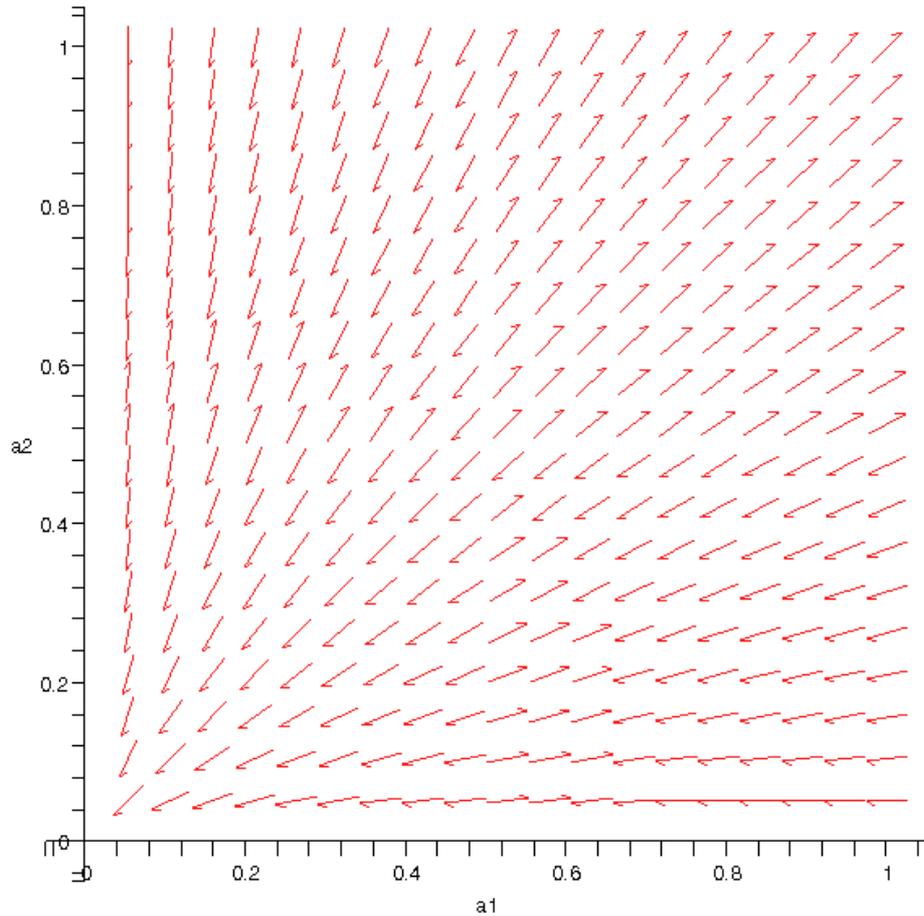


Figure 6: Solution Space: Identical Ethics

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