Relating Large and Small in C2 and Operations

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Abstract

We address the following two questions on a global scale. First, how is it possible for a Commander in Chief to conceive of, let alone command, the complex engagement processes involved in theater level warfare? Second, is it possible that this means of command and control would yield reasonably tractable mathematical regularities? Although our answers are simplified to the cartoon level, they preserve a common picture across all echelons in the command hierarchy and provide a plausible path to a more complete representation. We employ scaling relationships to compare echelons and discover many unexpected findings. For example, we found embedded connections between deliberate planning at different echelons and the quick reactions with weapons of all types, between the distribution of casualties and engaged units, and between several characteristics and the durations of engagements. We also found a scaling relationship representing the fastest rates of advance a relationship that captures conditions found in both historical and modern warfare.

1. **Introduction**

For clarification, this work is my own and does not represent IDA or its sponsors in any official manner. However, this detachment also offers a freedom to present ideas that might not otherwise be exchanged widely and, possibly, to stimulate interest in several fields.

Military conflict is extraordinarily complex virtually anything imaginable to man and more can have an impact on the course of battle and its outcomes. This paper addresses the following two questions. First, how is it possible for a Commander in Chief to conceive of, let alone command, the complex engagement processes involved in theater level warfare? Second, is it possible that this means of command and control would yield reasonably tractable mathematical regularities?

The first question, for example, applies with almost equal relevance to Corps, Division, and subordinate commanders. In other words how can a “large” Corps” level military engagement be effectively controlled by a “small” Corps headquarters team directed by a Corps Commander? An analogy to weather forecasting may apply here. Short-term forecasting reaches a chaotic limit in about seven days the physical equations of turbulent fluid motion become too complex and sensitive to initial conditions. Nevertheless, researchers can model or assess the possibilities of climate change over thousands of years. One can also characterize the general patterns of climate change how change takes place, what are the likely extremes, what is the distribution of wind or rain intensities over a period of time. Although these are incomplete descriptions of the weather and not predictions of outcome they are adequate to guide in building design and river basin projects, and are useful in disaster warning and forecasting. It follows that, a commander and his staff may be able to anticipate the range of possibilities well enough that his subordinates can manage the unpredicted particulars.

Weather satisfies several scaling relationships. For example, the pattern created by a small cloud is indistinguishable in shape and convolution from those of large clouds. This self-similarity has been demonstrated on all geographic scales up to the entire globe. Thus, the mathematics describing weather scales at all size ranges. For military organizations, the echelon hierarchy establishes the scaling pattern. For example, there are analogous command post cells for intelligence, logistics, and operations at all echelons from Battalion on up. These cells become more specialized and larger, but are analogous to one another. This illustrates scaling in a qualitative manner.

2. **Quantifying the Battlefield Scaling Relationships.**

The Unit Size (*Size*) at each echelon of a force, the depth of the commander's area of interest *(D-AOI*), and the duration of the Deliberate Planning Cycle (*DPC*) characterize the force, space, and time of the battlefield. Doctrine and established military practice dictate the quantitative values for each of these characteristics, especially from Company up through Army, as shown in Table 1. Table 1 also shows the values of the scaling relationship fitted to the values for each characteristic. A number *(E)* represents the echelon level for each of these equations as follows:

(1) $$Size(E) = 10.65 \bullet 4.288^{E}$$

(2) $$DPC(E) = 25.0 \bullet 2.288^{E}$$

(3) $$D - AOI(E) = 0.953 \bullet 2.587^{E}$$

In general, the scaling relationship with respect to echelon takes the form:

$$Characteristic = Factor \bullet Ratio^{E}$$

Note that in each case, we obtained the factors and ratios by regression on the values within the boxes in Table 1. This range of echelons Company through Army appears to be the most universally accepted by doctrine. Although the military situation may call for an adjustment of these values, they are stable for most situations. We also show four digits to limit roundoff errors for future calculations; the actual uncertainties range up to 30 percent on the *factor* but range only from 2 to 6 percent on the *ratio*. Thus, the scaling ratio of *DPC(E)* is close to that for *D-AOI(E)* but is unlikely to be equal by chance. It is possible that future work could show that the distribution of values for these characteristics would lead to equal estimates for both ratios.

Note that the *factor* is the value for $E = 0$, which corresponds to a Squad. If one chose a different echelon for the baseline, the *factor* would change, but the *ratio* would not. The *ratios* are intrinsic characteristics of the structure of the Army, its command structure, and its dynamics. These ratios mean, for example, that commanders at each echelon of the hierarchy have approximately the same number of subordinate units to command and have proportionate areas of interest and deliberate planning cycle times relative to superior and subordinate echelon commanders. Although the absolute situations are very different for Battalion versus Corps commanders, their relative situations have comparable complexities measured against immediately higher and lower units.

Table 1: *Size, DPC,* and *D-AOI* from Theater Level Down to Buddies

Echelon Level	Echelon	Unit Size		Deliberate Planning Cycle		Depth of AOI (km)	
		Data	Fit	Data	Fit	Data	Fit
8	Theater	1,296,000	1,086,417	2 Weeks	13 Days	3,000	1,914
7	Army	250,000	256,987	5 Days	6 Days	1,000	740
6	Corps	65,000	60,789	3 Days	2.5 Days	200	286
5	Division	15,000	14,379	1 Day	1 Day	100	110
4	Brigade	3,000	3,401	12 Hrs	11 Hrs	50	43
3	Battalion	800	805	4 Hrs	5 Hrs	15	17
2	Company	200	190	2 Hrs	2 Hrs	7	6
1	Platoon	40	45	1 Hr	1 Hr	3	2
0	Squad	9	11	3 Min	25 Mins	1	1
-1	Buddies	2	2.5		11 Mins		0.4
	Factor		10.649		24.985		0.953
	Ratio		4.228		2.288		2.587

One observation that seems remarkable is that the force *Size* scales down to a pair of buddies rather than to an individual. The scaling relationships reveal an underlying buddy system at its foundation. Another observation is that, with typically three forward maneuver units at each lower echelon, 3/4.288 = 70 percent of a commander's force is forward deployed and 30 percent is in his rear areas, more on this later.

3. **Relating the Spatial and Dynamic Ranges of Battle**

Before plotting the deliberate planning process characteristics, we now consider the opposite extreme, the "weapon reaction cycle." This cycle consists of a sensor that detects a target or threat, the shortest possible command and control cycle to direct the weapon against the target, and the final engagement of the target. For example, the "sensor" may be a soldier's visual acquisition of a hostile soldier, the weapon is his (or her) automatic rifle, and the engagement is quick reaction firing upon the hostile soldier. This cycle typically would apply to distances of about 100 meters in obscuring terrain, and the reaction time is on the order of human reactions for swing and fire about one second. Each weapon has such a reaction cycle; for example, an artillery battery may be dedicated to support a unit so that the forward observer calls in fires directly to the battery. This cycle may operate over 20 Kilometers of visual range and take about 2 minutes to execute. Table 2 gives nine examples of weapon systems and their associated quick reaction cycle. At the highest level, Inter-Continental Ballistic Missiles (ICBMs) retargeting from intelligence sources took roughly a week during the late 1980's.

Table 2: The Reaction Cycle – from Sensor to Engagement

Integrated Cycle			Target range from weapon at initiation (approximate)	Engagement time (approximate)
Sensor	**Weapon**	**Target**		
Visual acquisition	Automatic handheld	Soldier	100 meters	1.0 seconds
Visual with aids	Armor main gun	Armor	500 to 1000 meters	10 seconds
FLIR, optics, radar	Helicopter-launched missile	Armor	3 km	30 second pop up
FLIR, optics radar	Ground-launched missile	Attack helicopter	5 km	1 minute exposure
Forward observer	Artillery battery on direct line	Near FEBA	20 km	2 minutes
Observer of commander	CAP in holding	Near FEBA	50 km	5 minutes
Sensors in forward area	CAS (at base)	Interdiction depth	300 km	30 minutes
AEGIS or airborne radar	CAP interceptor missiles	Incoming aircraft or missiles	400 km	40 minutes
Strategic sensor (e.g., satellite)	ICBM	High-value asset	20,000 km	One week

Figure 1 shows both the scaling relationships between *DPC* and *D-AOI* and the corresponding relationships between time and range for the quick reaction cycle. Only the automatic rifle through the Aegis Missile contributed to the regression fit to the reaction cycle; therefore, it is surprising that the ICBM cycle was even close to the trend. Airborne operations require a day to plan, which puts them on the deliberate planning cycle trend as a "reaction" weapon. However, when employed with a pre-planned offensive, the airborne can move with the reaction envelope pace. This supports the concept that the gray band contains all types of operations – it covers the spatial and dynamic range of warfare.

3.1 ***A Different Mathematics of Scaling***

Some readers may want to skip to the significance of all this, although the mathematics requires only familiarity with manipulating exponents and logarithms. We ask, what *V* relates *D-AOI* to *DPC*:

$$D-AOI = V \bullet (DPC)^{a}$$

where enables the equality to hold for all echelons, *E*. Examination of equations (2) and (3) implies that

$$2.587 = 2.288^{a}$$

or $a = \log 2.587 / \log 2.288 = 1.148$. It also implies that, $V = 0.953/(25.0)^{a} = 0.0236$. Actually, we independently fit both the reaction cycle and the deliberate planning cycle to the same scaling exponent. Regression analysis produced an = 1.090, which equals 1.148 within error tolerances.

Regression showed that the reaction cycle is 196 times faster than the deliberate planning cycle. <u>Therefore, the fastest weapon reaction rate of response as about 200 times greater than the deliberate planning process</u>.

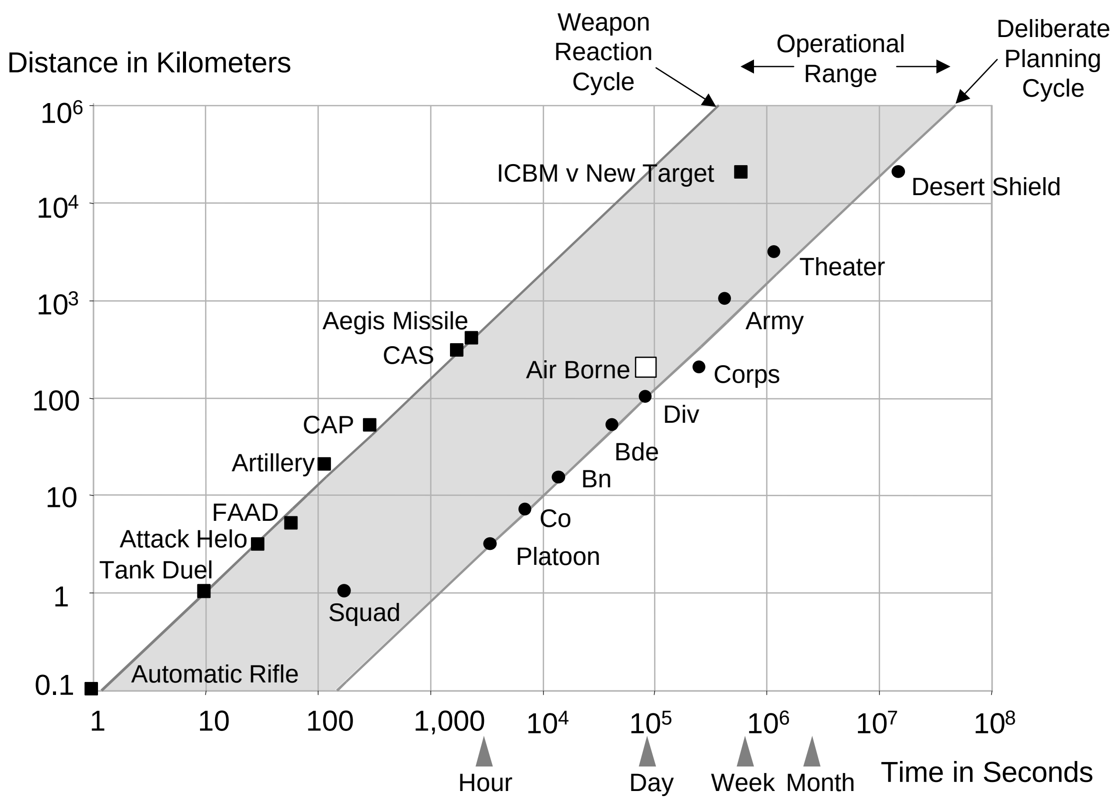

Figure 1: The Reaction Cycle and the Deliberate Planning as Scaling Relationships

3.2 *Interpreting the D-AOI*

The *D-AOI* appears to measure the region in which hostile actions, such as maneuver, reinforcement, resupply, and infrastructure changes can affect the commander's forces "in the near future." The precise value of the *D-AOI* arises from the characteristics of the weapons, technology, and mobility of the opposing force. However, the *D-AOI* is also proportional to the depth of friendly rear areas – representing the length of lines of communication and supply sources as well.

3.3 *Interpreting the DPC*

The duration of the *DPC*, in principle, could be any period the commander considered effective. However, in practice, this duration is constrained by staying within the opposing commander's planning cycle while providing one's own subordinates time to report and later to prepare. As a balance between opposing needs, the duration of the *DPC* measures the effective rate of change of the battle situation that the commander can influence.

Again, the entire dynamic range of the battlefield, from deliberate planning to reaction "planning" is, by fit, a factor of 196 – essentially 200:1.

3.4 *Lessons from Outliers*

The plotted value of *DPC* for a Squad, just as those for Platoon through Theater, represents the judgment of an experienced retired colonel. For the echelons with relatively defined command and control processes, the values fell along the fitted trend. Does a Squad plan for 25 minutes, as the trend indicates, or for only 3 minutes as experience estimated? Situational differences may create such a spread that *DPC* may not be defined for Squads. This shows that the durations were not artificially generated by some "rule of thumb" or other algorithm, which might produce apparent regularity.

For Desert Shield, the plotted point falls very close to the extrapolated trend line. Here the rear area lines of communication were global, of order 20,000 Kilometers, halfway around the world. Correspondingly, the Iraqies did not alter the tactical situation from the time they dug in. Coalition forces, on the other hand, took as much time as they deemed useful to improve the tactical situation to our advantage. From this perspective and the definitions of *DPC* and *D-AOI*, the point represents an extended deliberate planning cycle for global deployment of an Army-sized force of 258,000 soldiers. Thus, these scaling relationships may be valuable in estimating capabilities well beyond just the local battlefield.

4. **Quasi-Velocities**

Although the two trend lines for deliberate planning and reaction are scaling relationships, their ratio acts as a "quasi-velocity." It ranges from about 2 Kilometers per hour for Platoons to about 5 Kilometers per hour for an Army. However, these are not real velocities but, rather, measures of the rate of the changing battle situation. Let us now examine real maximum rates of advance for different types of operations and unit sizes.

Leonard Wainstein [Wainstein, 1984], who has subsequently retired from IDA, produced a historical catalog of major unit movements in an attempt to determine what paces rates of advance. Examples from WWII dominate the collection, but it also covers the Korean War, the Israeli 6-Day War, WWI, and some historical battles. We added the Persian Gulf "Left Hook" to these examples. All of this was to populate the outer envelope of fastest rates of advance in modern heavy warfare. Figure 2 shows the results.

Figure 2 plots the distance advanced in miles against the time duration of the advance in days. We plotted only the most rapid movements; otherwise, the lower portion of the plot would become black with symbols. Many of the campaigns and fronts in the plot contain some of the fastest attacks as a subset. Thus overall, forces do not sustain the fastest rates but converge, or "average out," to a progressively slower pace as the war continues.

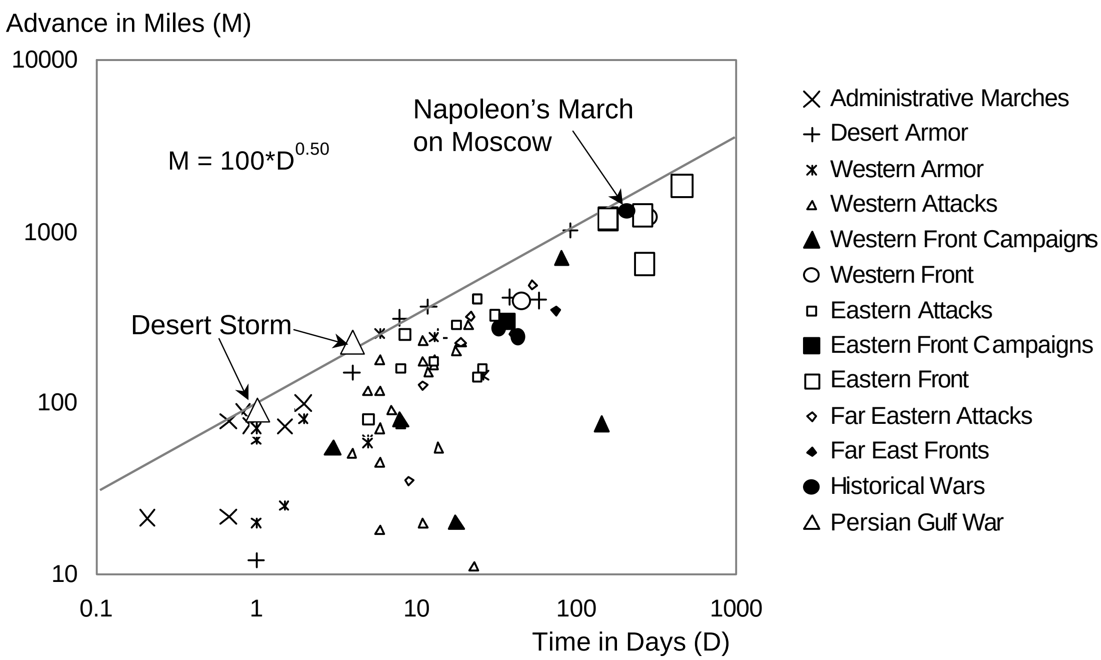

Figure 2: Scaling Relationship for Maximum Rate of Advance

4.1 *Not a True Velocity*

The interesting result is that the envelope curve representing the maximum rates of advance is a scaling relationship between time and distance. A true "velocity" would be a 45-degree upward trend line from lower left to upper right. The actual trend line is given by the equation on the graph, which states that the distance covered increases as the square root of the time of the advance for increasingly long advances.

The very different exponents in the scaling relationship of the maximum rate of advance from that for *D-AOI* and *DPC*, which more closely mimics a true velocity, proves that *D-AOI* and *DPC* cannot be directly associated with maneuver per se.

4.2 *Diffusion across the Battlespace*

Distance increasing as the square root of time is familiar to physicists; it represents a diffusion process such as a dye diffusing through a still liquid. The mathematics of diffusion is a random walk – successive steps taken in a random direction will spread from the origin at a rate proportional to the square root of time. Experienced commanders point out that advances push hostile forces from one defensible position to the next. This lurching progress conforms to the terrain barriers and the rate of supply of the pursing force.

4.3 *Persistence over Time*

At the high end of the plot, we see that Napoleon reached Moscow faster than Hitler's armies did – both with disastrous results for the aggressor. In modern times, a weakened position crumbled at a pace little different from the fastest WWII desert campaigns. Therefore, exceeding this well-established envelope for the fastest advances would truly represent a historical breakthrough in technology or tactics.

5. **Scaling of Casualties and Engagement Durations**

Data from more than 200 battles yield estimates of the intensity and duration of modern warfare as experienced at different echelons [Dupuy, *et al*., 1986]. The measure of engagement intensity is the Average Daily Casualty Rate (*ADCR*). In this measure, the Company level *ADCR* is 21 percent while that for an Army is 0.3 percent. However, the engagement at Company level engagement of that intensity lasts only 30 minutes while the Army is engaged 150 days. Table 3 shows the *ACDR* and Duration of Engagements (*DENG)* for all echelons from Company to Corps. It also shows the scaling parameters and fits for each of these levels.

Table 3: ADCR and DENG by Echelon

Echelon Level	Echelon	Average Daily Casualty Rate (ADCR)		Engagement Duration (DENG)	
		Data	Fit	Data	Fit
7	Army	0.3%	0.2%	150 Days	146 Days
6	Corps	0.5%	0.5%	24 Days	25 Days
5	Divison	1.0%	1.3%	4 Days	4.3 Days
4	Brigade	2.6%	3.2%	18 Hrs	18 Hrs
3	Battalion	9.5%	7.8%	3 Hrs	3 Hrs
2	Company	21.0%	19.0%	30 Min	31 Mins
1	Platoon		46.0%		5 Mins
0	Squad		111.7%		53 Secs
-1	Buddies		271.0%		9 Secs
	Factor		1.117		0.895
	Ratio		0.412		5.856

5.1 ***Connections between Scaling Relationships***

The first connection provides some insight into why the *ADCR* should scale as it does. Note that *ADCR* scales approximately as the inverse of the *D-AOI* because the inverse of the ratio for *D-AOI* is 1/ 2.587 = 0.3865 ~ 0.412, which is the ratio for *ADCR*. This says engagement intensity falls off inversely with distant from the Forward Line of Own Troops (FLOT).

An even more surprising connection is that between *ADCR* and *DENG*. The duration of an engagement appears to be directly proportional to the inverse square of the engagement intensity because the ratio for *DENG* = 5.856 ~ 5.891 = $(0.412)^{-2}$. Here the difference represented by similarity, ~, is equality within the range of uncertainty on *ADCR* and *DENG*. We have no explanation for mathematical precision of this relationship, only that it seems plausible qualitatively.

Because the *DPC* scales very closely to the *D-AOI*, the inverse of the *ADCR* also approximately scales as the inverse of *DPC*. Similarly, the *DENG* scales approximately as the cross product of any combination of *DPC, D-AOI*, or the inverse of *ADCR*. The product of the scaling ratios of *DPC* and *D-AOI* are within the uncertainty limits of the fits, $2.288 \bullet 2.587 = 5.919 \sim 5.856$. Thus, the duration of engagements varies approximately in proportion with the contested area measured by the square of *D-AOI*, the square of *DPC*, and so forth. These relationships again seem plausible, but they beg for a deeper explanation.

5.2 ***Influencing the Outcome of an Engagement***

The *DENG* determines a cutoff time for assistance from more powerful weapons because more powerful weapons cannot react quickly enough to enter the engagement. Although the Dupuy data do not reach down to the Platoon or Squad levels, the DENG can be extrapolated to these smaller values. At the Platoon level, Close Air Support (CAP) can barely arrive in time, *DENG* = 5 minutes as is the time delay for CAP. Since these time values are single estimators of an entire distribution of times, there will be longer engagements or more opportune locations of CAP for which they could shape the engagement outcome.

At the Squad level, we would estimate that the typical engagement is less than a minute – too quick for even artillery support. Armored units, however, might slug it out in this time frame.

The key point here is that if the reaction cycle scaling holds for ALL weapon systems and if the *DENG* scaling held within a fairly tight range of times, then one could estimate the likely degree of support a unit could expect from more powerful weapons. It also points out the increasing military advantage of less sophisticated forces as they are able to close the engagement distance to our forces. Put another way, detecting, identifying, and engaging hostile forces at a greater distance brings an additional advantage of buying time for the arrival of heavier weapon systems. Scaling relationships quantify this gain from this strategy.

Conversely, preplanned operations enable the coordinated employment of the entire spectrum of weapons on up the hierarchy of options. This is one great advantage to surprise or initiative versus a reactive or defensive posture. Unfortunately, the United States appears to be moving more toward “peace keeping” reactive postures, a much more difficult challenge.

6. **Number of Units Engaged Simultaneously**

Since the total average daily losses of an entire Army is a fixed quantity, no matter what echelon level is used to evaluate it, we can calculate the number of units engaged on average. After presenting the simple version of this calculation, we discuss its ramifications or modifications.

6.1 ***Estimating the Average Number of Units Engaged***

The following expression can be evaluated for the number of units engaged (*NENG*) at any echelon level:

$$Total\ Army\ Casualties = ADCR \bullet Size \bullet\ NENG$$

$$640 = (1.117 \bullet 0.4121^E) \bullet (10.65 \bullet 4.288^E) \bullet (R_{NENG}^{7-E})$$

where the 640 results from evaluating the above expression at E = 7, and

$$640 = 1.117 \bullet 10.65 \bullet R_{NENG}^7 \bullet \left(\frac{0.412 \bullet 4.288}{R_{NENG}}\right)^E .$$

Now the expression in parentheses raised to the E power must equal 1.0 if the result on the left is to be independent of echelon – this determines R_{NENG}.

$$R_{NENG} = \frac{1}{0.4121 \bullet 4.288} = 1.767$$

Because this is a scaling ratio, it means that there are 1.77 subordinate units engaged on average out of the total of the 4.288 subordinate units to each echelon. Thus, 1.77/4.288 = 0.413, or 41.3 percent, of each engaged unit's subordinate units are themselves engaged.

Note that this calculation is approximate because of roundoff errors when raising the ratios to the 7th power.

6.2 ***Standing on Your Thumbs***

Table 4 shows a more precise spreadsheet calculation of the Army-wide implications of the scaling of the number of units engaged. Table 4 shows clearly that raising ratios to powers of 7 or more causes significant drift due to round off errors, as exhibited in the above more approximate example calculation.

Table 4: Fractions of an Army Engaged and Unit Loss Rates per Engagement

Echelon Level	**Echelon**	**Total "Units"**	**Units Engaged At Random Moment**	**Fraction of Army Engaged**	**Average Engagement Casualty Rate (AECR)**
		Fit	**Fit**	**Fit**	**Fit**
7	Army	1	1.0	100.00%	44.71%
6	Corps	4	1.7	41.21%	12.15%
5	Divison	18	3.0	16.98%	4.32%
4	Brigade	76	5.3	7.00%	1.91%
3	Battalion	319	9.2	2.88%	1.19%
2	Company	1,350	16.0	1.19%	0.45%
1	Platoon	5,708	27.9	0.49%	0.16%
0	Squad	24,132	48.7	0.20%	0.07%
-1	Buddies	102,020	84.8	0.08%	0.03%
	Ratio	4.228	1.742	0.412	2.413

If we counted each of the "unit equivalents" implied by the ratio 4.288, an entire Army would be made up of 1,350 companies. Although artificial, this illustrates the scaling. Similarly, of the 1,350, only 16 would be engaged on average. Since their engagement times are short, a different group of Companies would be engaged about each half hour. This is like a giant standing on his thumbs.

Since the overall level of engagement intensity is not constant over time even for an Army, more subordinate units would be engaged one moment and fewer in the next. It is accurate, however, to say that, at the Company level, only 1.19 percent of the force is engaged "on average."

The average percentage of the unit lost over each engagement also scales but in just the opposite manner. Since the Army as a whole is engaged continuously for a very long time relative to the Company, its losses are 44.7 percent for its long engagement, while a Company typically loses only 0.5 percent of its force during one of its relatively short engagements.

6.3 ***Non-Forward Area Losses***

In the picture we have presented, lower echelon units suffer all the losses. Our attempts to modify the scaling by assigning some losses to rear areas failed to satisfy the empirically demonstrated simple scaling relationships. Therefore, one could consider the rear area losses to be suffered by fractionally sized rear area units with an equivalent scaling down a hierarchy to lower echelons. Alternatively, it is possible that rear area losses are dwarfed by the forward area casualties and do not appear in such a simple scaling model of an Army. More sophisticated versions of this form of modeling could introduce the time variations and the rear area losses.

6.4 ***Scaling of Tooth to Tail***

In simplified terms, army doctrine has three maneuver units subordinate to each higher unit. In this scaling picture, 3/ 4.288 = 0.70, or 70 percent of each unit, consists of forward maneuver units – the other 30 percent are rear area support. As one goes down the echelons, the fraction of the Army designated to maneuver unit status shrinks as a scaling relationship. It goes down as 0.7 raised to the E – 7 power. At Company level, only $0.70^{E-7} = 0.168$, or 16.8 percent of the force is up front as maneuver units. Therefore, the tooth to tail "ratio" is actually the "tooth to tail" scaling relationship the "ratio" is an exponential function of echelon.

Doctrine also advises commanders to maintain two maneuver units engaged and one in reserve. This is very close to the scaling model result of 1.77 engaged versus the three maneuver units for each <u>engaged</u> unit. Those units that are not engaged, by the definition of the scaling, <u>have no engaged subordinate units</u>. Played out along a FLOT, this pattern of engagements leaves wide gaps in a staggered saw-toothed pattern, which can only be represented by "fractal" mathematics. Engaged units are more likely to have collateral engaged units. This causes a localized drain on logistic supplies, if taxes the superior unit's headquarters, and it generally sets up a competition for supporting fires from higher echelon weapons.

7. **Revisiting the Original Two Questions**

Although such a simple picture as that created by analyzing simple scaling relationships cannot hope to completely answer the original two very deep questions, scaling does seem to offer an avenue to mount a much more penetrating attack upon these two questions.

7.1 ***How is it possible for a Commander in Chief to conceive of, let alone command, the complex engagement processes involved in theater level warfare?***

Because many of the most important considerations to a commander appear to scale with echelon, the workload and demands upon command and control resources at any given echelon appear to be comparable to those at other echelons. Thus, the relative circumstances of commanders remain comparable even though their total span of control varies widely. Although higher level commanders lead many more soldiers, he or she commands the same number of immediately subordinate units as do other commanders at lower levels. Similarly, the relative level of detail represented by the ratio between the area of interest of a commander and the immediate subordinate is constant across all echelons. The “information contents” of these relative pictures are comparable. This relationship from one level of command to the next evolved over centuries of warfare and represents a fundamental quality of command that needs to be explicitly captured in the mathematics.

Future development of an understanding of the commander’s perspective might engage such questions as what difference it makes that the deliberate planning process at the Company level is shorter than the duration of a typical battle engagement, while at Brigade they are comparable and, at Corp, they are reversed. Also, better data on the variability of the characteristics covered in this paper would provide a much more realistic representation of the challenge to commanders.

7.2 ***Is it possible that the commander’s means of command and control would yield reasonably tractable mathematical regularities?***

The scaling regularities built into the echelon hierarchy provide a clue to the appropriate form of mathematics scaling relationships. Once we fit scaling relationships to the available data on casualty rates and engagement durations in addition to deliberate planning and areas of interest, we found a wealth of unexpected interrelationships. Scaling relationships revealed known facts such as; Armies are built upon the buddy system, on average two maneuver units are engaged and one is back, and battle intensity declines in proportion with the distance from the FLOT.

We also discovered that the maximum rate and distance of weapons to react to a newly identified target also followed a scaling relationship among the spectrum of modern weapons. This fastest reaction cycle was 200 times the rate of the deliberate planning process at comparable distances and times. In fact, all of warfare appears to be conducted between these two limits. We also saw that the maximum rate of unit advances led to a scaling relationship analogous to diffusion advance progresses as the square root of time.

Future development of scaling relationship models would soon require fractal mathematics to properly represent the spatial, temporal, and engagement characteristics of warfare.

8. **References**

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