Australian Government
Department of Defence
Defence Science and
Technology Organisation

On the 'Boyd- Kuramoto Model': Emergence in a Mathematical Model for Adversarial C2 Systems

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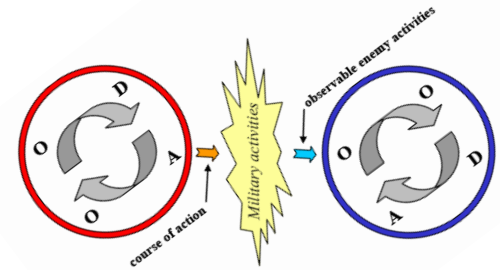
C2 Processes: many are cycles!

- Boyd's *O*bserve-*O*rient-*D*ecide-*A*ct Loop:



Interacting OODA:
Moon, Kruzins, Calbert 2002

- Snowden's Cynefin Framework:
Different loops depending on *context*

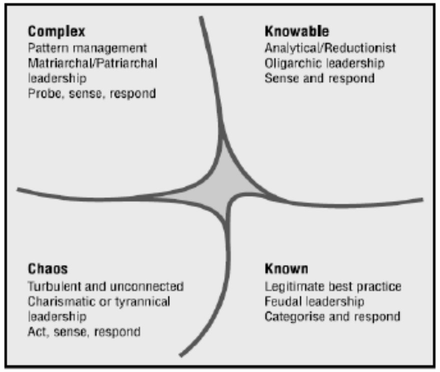
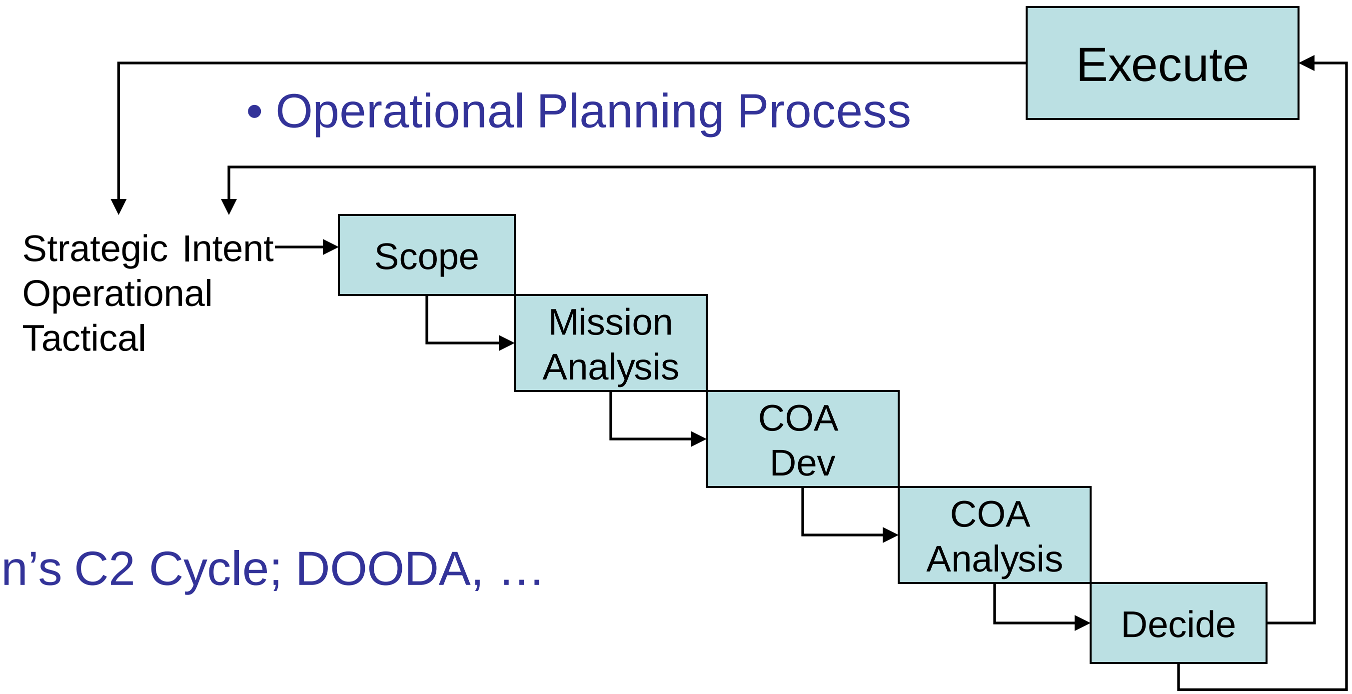


Figure 4: Cynefin Framework

- Operational Planning Process



- *Elaborations:*
  Lawson's C2 Cycle; DOODA, …

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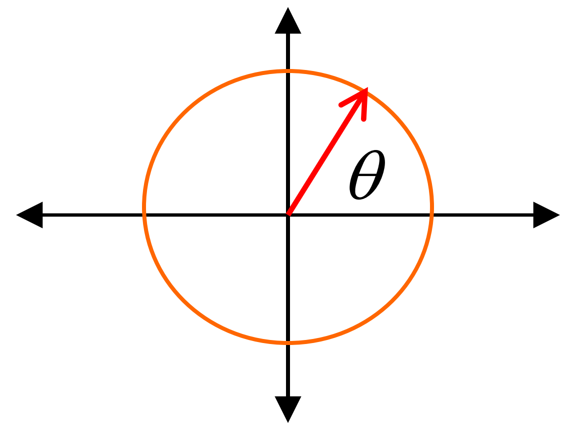
The Kuramoto* Model

Kuramoto Model -1-dim Oscillator:

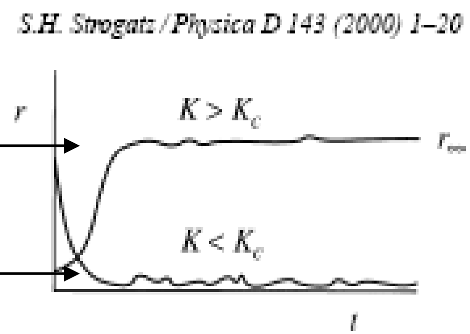
$$\dot{\theta}_i = \omega_i + \frac{K}{N}\sum_j A_{ij}\sin(\theta_j - \theta_i)$$

Natural Frequency Coupling Network adjacency matrix



Phase Synchronisation at High K

Incoherence at Low K



* Kuramoto, Chemical Oscillations, Waves and Turbulence, Springer, Berlin, 1984;
† Kalloniatis, Phys. Rev. E 82, 066202, 2010
Also contributions by R. Taylor and T. Dekker

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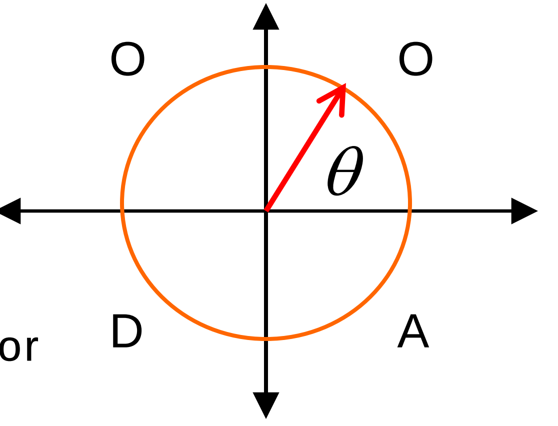
Mapping Kuramoto to Boyd



$\theta =$ Point of progress in decision cycle.

K = Coupling = degree of tightness of control.

ω = *Natural* frequency of each node = inverse time period for processing appropriate information according to "environment" in order to advance through cycle.

A = intra-C2 Network = not just communications connectivity, but also authority, collaborative, social, and visual networks.

- *Who are my points of reference for my decision cycle?*
- *With whom must I mutually adjust to progress decisions?*

Periodicity of *sine* response function: irrelevance of "stale" information or past decisions: the *current decision cycle is all that matters*.

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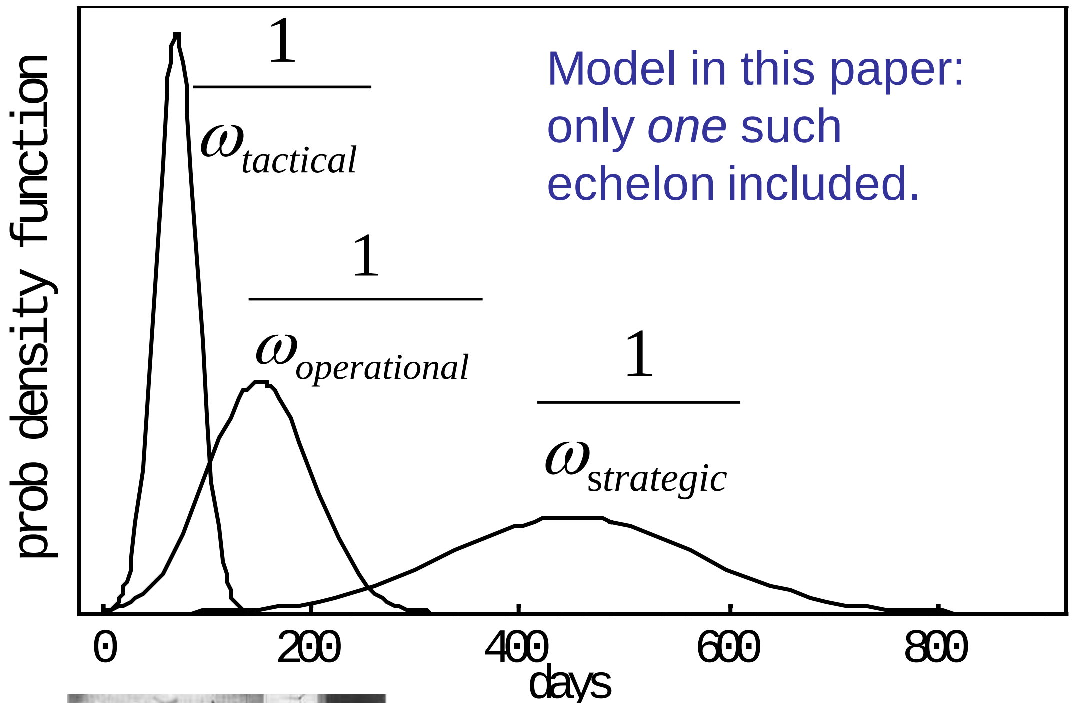
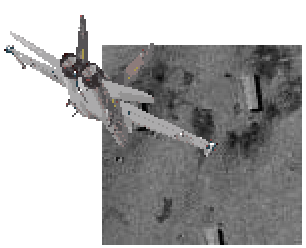
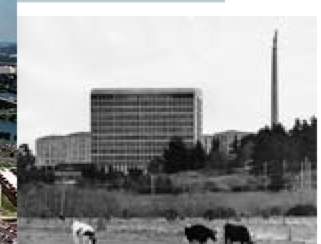
C2- Time Period Spectrum



Modern military operations involve diverse time scales and networking of processes.





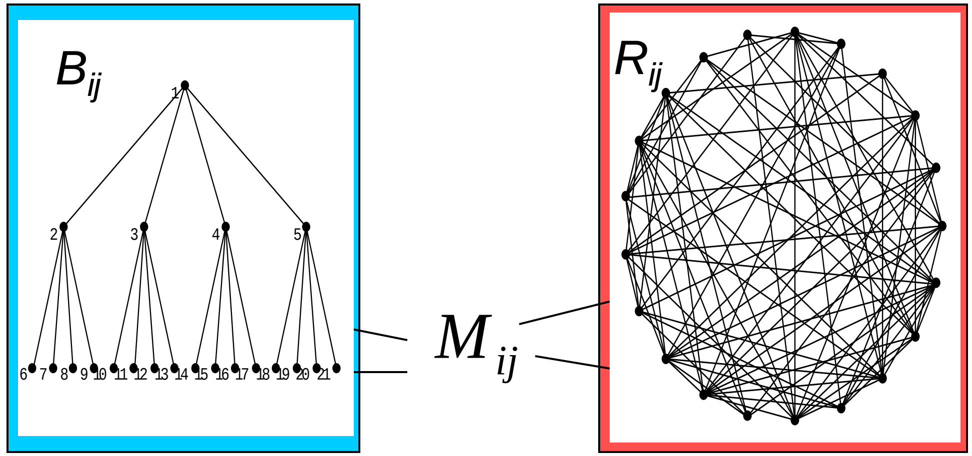
The 'Boyd- Kuramoto' equations

cf Lanchester attrition and Hughes salvo equations

Phase angles

Intrinsic frequencies

Adjacency matrices



N.B. This is a *Caricature*:
Informal Networks in Traditional Military (Ali 2011); Hierarchy in Insurgent Networks (Memon et al 2008)

$$\dot{\beta}_i = \omega_i + \sigma_B \sum_{j=1}^{N_B} B_{ij} \sin(\beta_j - \beta_i) + \varsigma_{BR} \sum_{j=1}^{N_{BR}} M_{ij} F(\rho_j - \beta_i)$$

$$\dot{\rho}_i = \nu_i + \sigma_R \sum_{j=1}^{N_B} R_{ij} \sin(\rho_j - \rho_i) + \varsigma_{RB} \sum_{j=1}^{N_{BR}} M_{ij} G(\beta_j - \rho_i).$$

$\omega_i, \nu_i \in [0,1]$ uniform random distribution interactions only within one 'echelon'

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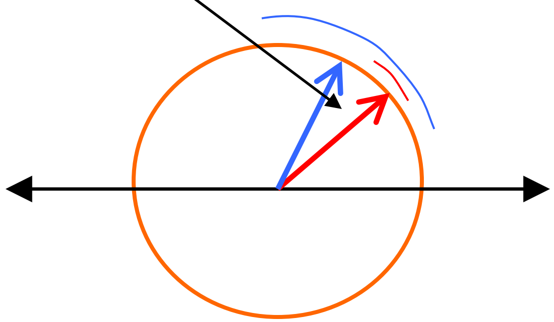
Intelligence- Surveillance- Reconnaissance & OODA

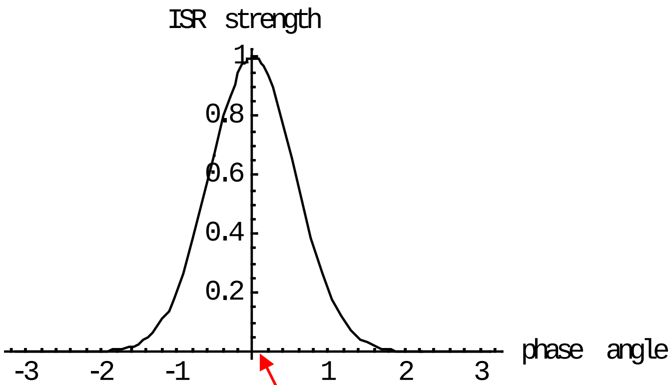
Blue has total ISR

$$F(\rho_j(t) - \beta_i(t)) = \sin(\rho_j(t) + \lambda - \beta_i(t)) \times 1$$

Blue seeks to 'get inside adversary OODA loop'

$\lambda = \pi/4 \approx 0.7854$





$$G(\beta_j(t) - \rho_i(t)) = \sin(\beta_j(t) - \rho_i(t)) \exp(-(\beta_j(t))^2 / 2s^2)$$

$s = \sqrt{\pi} \approx 1.772$

Red synchronises around Blue `Actions' with narrow ISR

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Measures of Performance

Measure of internal synchronisation
(B↔B, R↔R)

$$r(t)e^{i\Psi(t)} = \frac{1}{N}\sum_i e^{i\theta_i(t)}$$

$$r_B(t) = \frac{1}{N_B}\left|\sum_i e^{i\beta_i(t)}\right|$$

$$r_R(t) = \frac{1}{N_R}\left|\sum_i e^{i\rho_i(t)}\right|$$

Measure of external synchronisation
B↔R

$$\Delta_{BR}(t) = \frac{1}{N_{BR}}\sum_i \left[\beta_i(t) - \rho_i(t)\right]$$

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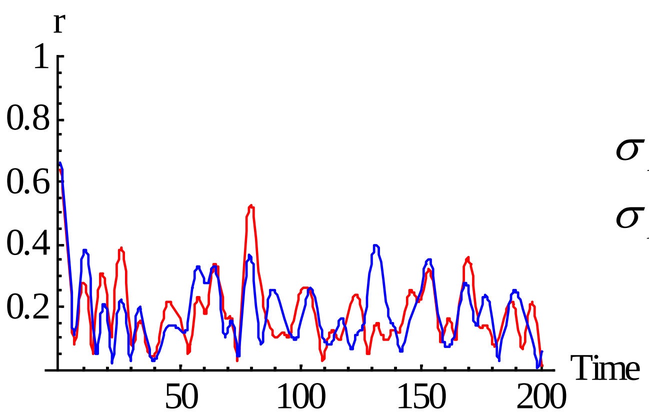
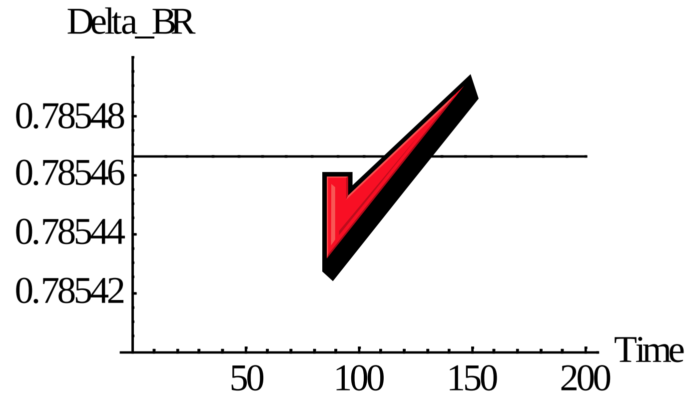
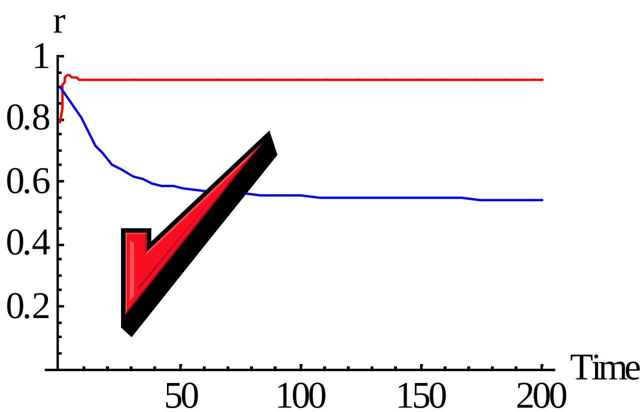
Basic (Extreme) Behaviours



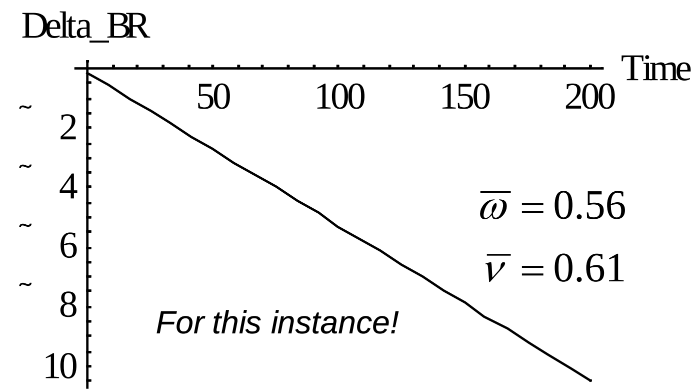
$$\sigma_B = 0 \quad \varsigma_{BR} = 30$$
$$\sigma_R = 0 \quad \varsigma_{RB} = 0$$



Blue focused exclusively on Red; neither internally coordinates.



$$\sigma_B = 0.8 \quad \varsigma_{BR} = 0$$
$$\sigma_R = 0.15 \quad \varsigma_{RB} = 0$$



Blue, Red focused exclusively on internal coordination but no regard for each other

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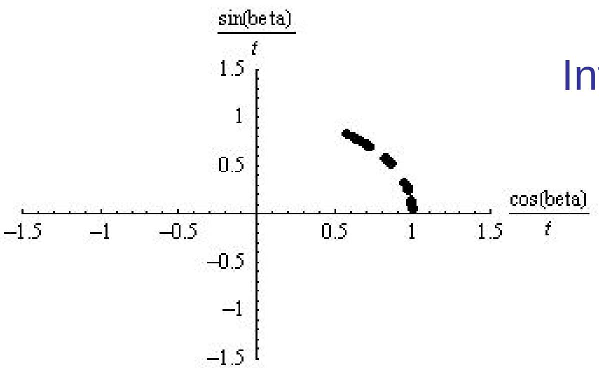
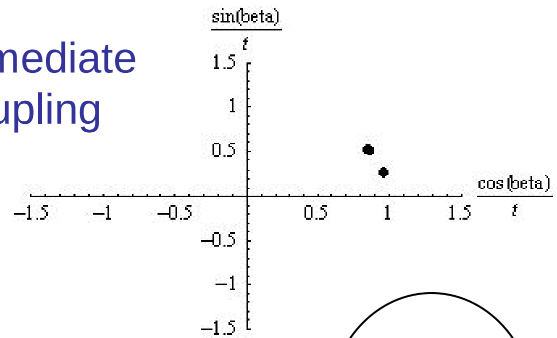
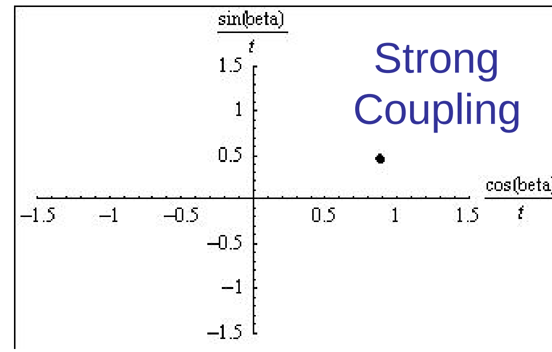
Emergence: the ‘surprise’

Laughlin: “system qualities or behaviours not reducible to the system components but arise from their interactions.”

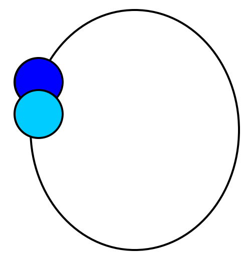


Intermediate Coupling

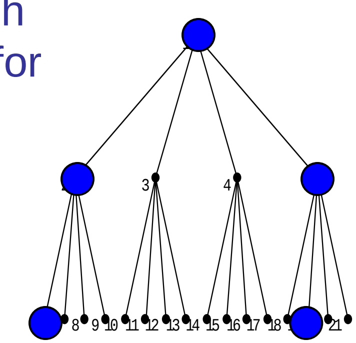




Strong Coupling



Eg. Random selection of frequencies makes nodes ● ‘close’ even though topologically `far’; creates affinity for dynamically forming a sub-cluster



This cannot be *designed for* given agent differentiation.
Sub-clusters ‘emerge’ at intermediate interaction strengths.
Each instance is different!

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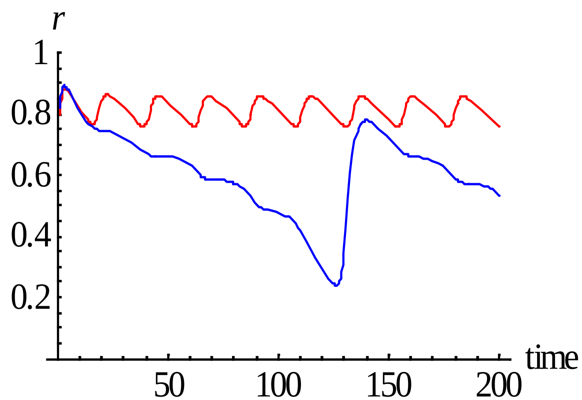
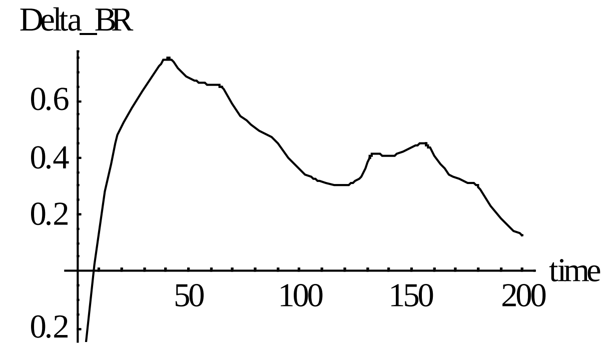
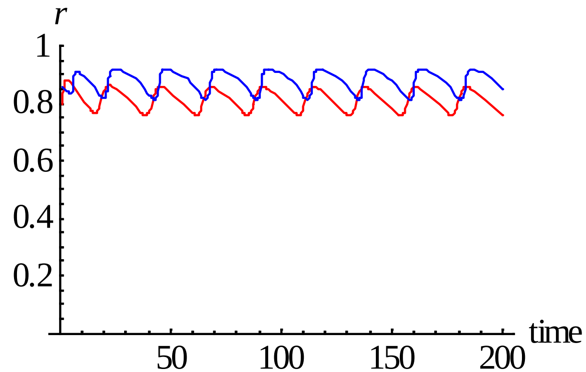
Blue v Red at the Edge of Chaos



$\sigma_B = 0.6$ $\quad \varsigma_{BR} = 0.1$

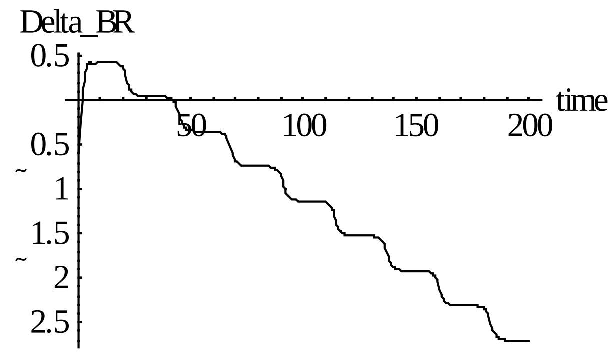
$\sigma_R = 0.075$ $\quad \varsigma_{RB} = 0$

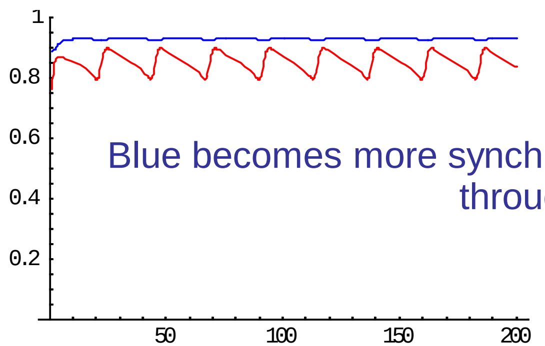




$\sigma_B = 0.6$ $\quad \varsigma_{BR} = 0.8$

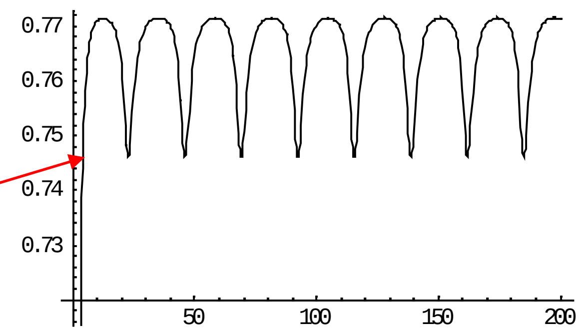
$\sigma_R = 0.075$ $\quad \varsigma_{RB} = 0$





$\sigma_B = 0.6$ $\quad \varsigma_{BR} = 2$

$\sigma_R = 0.075$ $\quad \varsigma_{RB} = 0$



Blue becomes more synchronised
through stronger coupling *to Red*

at cost of oscillations in
staying inside Red's OODA

DID YOU EXPECT THAT?

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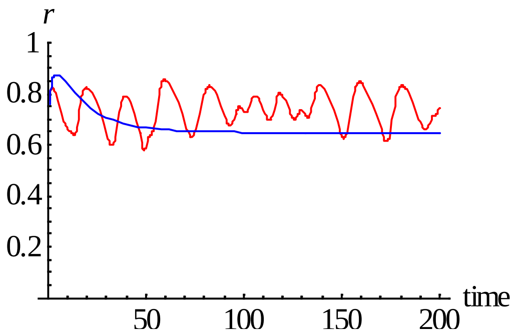
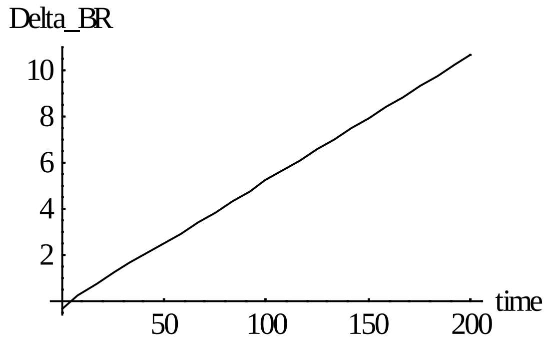
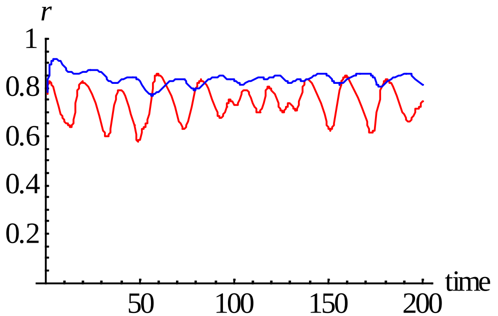
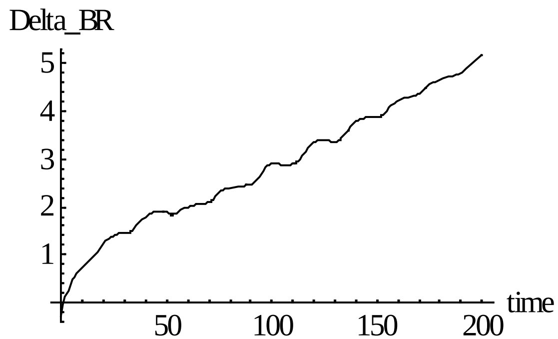
Another example: Only Red ‘at the Edge’



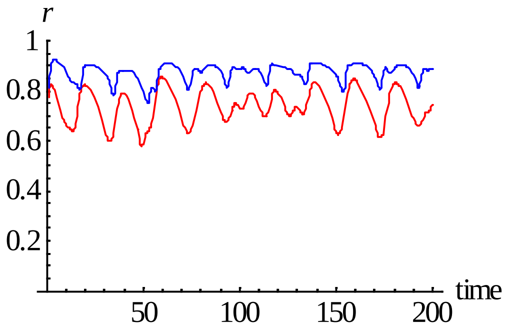


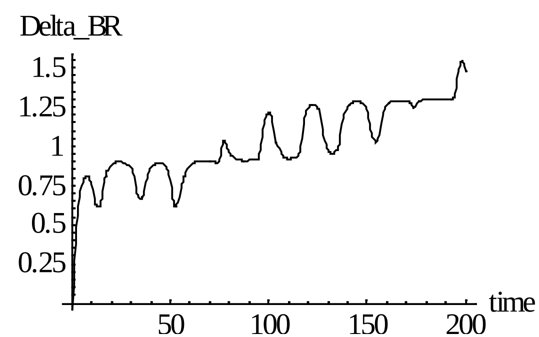
$$\sigma_B = 0.6 \quad \varsigma_{BR} = 0$$
$$\sigma_R = 0.08 \quad \varsigma_{RB} = 0$$



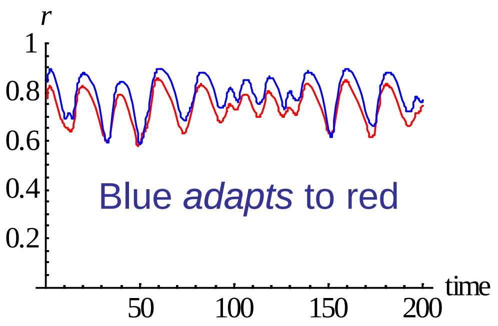


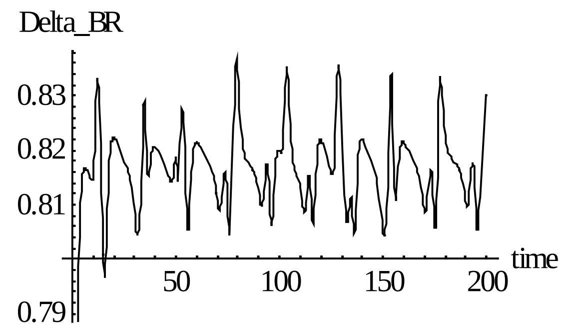
$$\sigma_B = 0.6 \quad \varsigma_{BR} = 0.2$$
$$\sigma_R = 0.08 \quad \varsigma_{RB} = 0$$





$$\sigma_B = 0.6 \quad \varsigma_{BR} = 0.6$$
$$\sigma_R = 0.08 \quad \varsigma_{RB} = 0$$





$$\sigma_B = 0.6 \quad \varsigma_{BR} = 2$$
$$\sigma_R = 0.08 \quad \varsigma_{RB} = 0{,}1$$

Blue *adapts* to red

at cost of zig-zag in staying inside Red’s OODA

Conclusions

There are more variables by which traditional C2 structures can achieve Agility; they are subject to mathematical modelling.

Model enables finding the balance point for given C2 structures and time scales between internal coordination and responsiveness to adversary.

Emergence is nothing mystical: mathematical models can capture such surprises in representations of C2.

Applications: realistic network data, human factors data, limited/interrupted ISR functions also for Blue.

Multi-echelon, multi-time spectrum: needs modification of equations.

'Boyd-Kuramoto' can intermediate/cross-validate between high/low fidelity models of C2 systems.

Thanks to Tony Dekker, Richard Taylor, Brian Hanlon, Sharon Boswell and Paul Whitbread for discussions and encouragement over the years.

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