16th ICCRTS

International Command and Control Research and Technology Symposium

A TOPOLOGICAL MODEL OF C^2 ORGANIZATIONS

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Let us recall our first Math classes...

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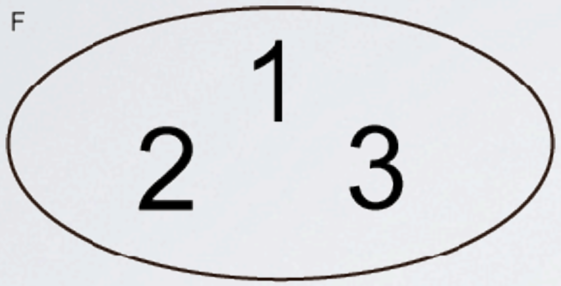
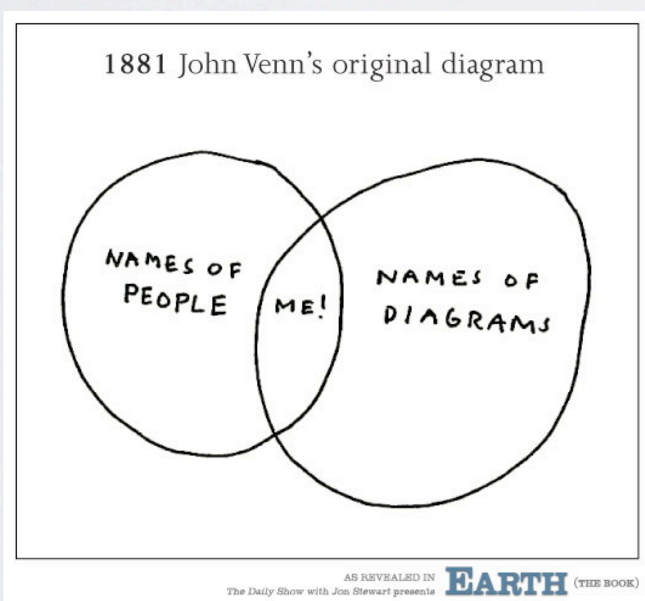
First, we learn about ***sets****....*

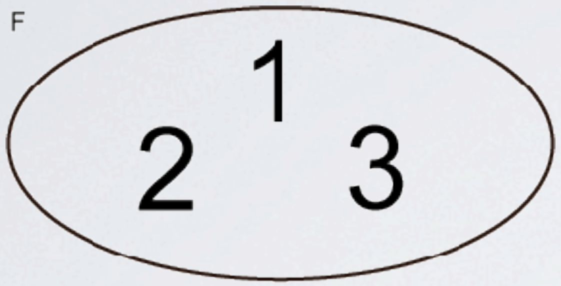


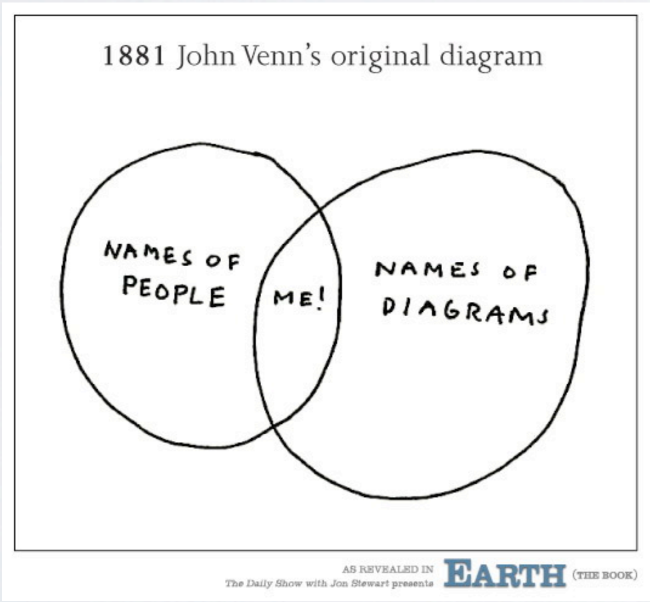


...to be able to understand ***numbers****!!!*

Let us recall our first Math classes...

*But once we learn **numbers**...*





+	0	1	2	3	4	5	6	7	8	9
0	0	1	2	3	4	5	6	7	8	9
1	1	2	3	4	5	6	7	8	9	10
2	2	3	4	5	6	7	8	9	10	11
3	3	4	5	6	7	8	9	10	11	12
4	4	5	6	7	8	9	10	11	12	13
5	5	6	7	8	9	10	11	12	13	14
6	6	7	8	9	10	11	12	13	14	15
7	7	8	9	10	11	12	13	14	15	16
8	8	9	10	11	12	13	14	15	16	17
9	9	10	11	12	13	14	15	16	17	18

Let us recall our first Math classes...

*But once we learn **numbers**...*

+	0	1	2	3	4	5	6	7	8	9
0	0	1	2	3	4	5	6	7	8	9
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4	4	5	6	7	8	9	10	11	12	13
5	5	6	7	8	9	10	11	12	13	14
6	6	7	8	9	10	11	12	13	14	15
7	7	8	9	10	11	12	13	14	15	16
8	8	9	10	11	12	13	14	15	16	17
9	9	10	11	12	13	14	15	16	17	18

*...we forget **sets** for the rest of our lives!!!*

OBJECTIVES

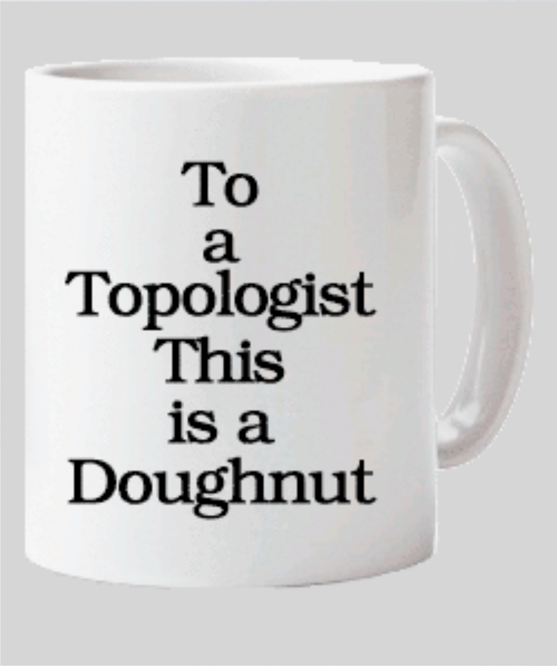
- *To motivate the C^2 research community for the application of Topology to C^2 theory building*
- *To present a topological model of C^2 organizations*

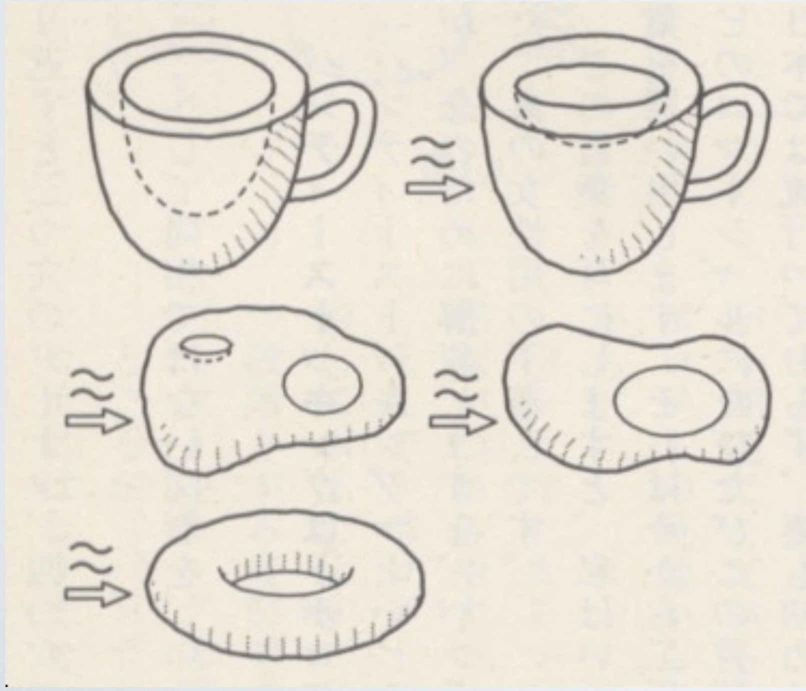
SUMMARY

- Topology in the military
- Topology in C^2
- Definitions
- The *Model*

TOPOLOGY

- From the Greek **τόπος**, "place", and **λόγος**, "study"
- Concerned with properties of objects preserved under continuous deformation
- '*What's the difference between a mug and a donut?*'

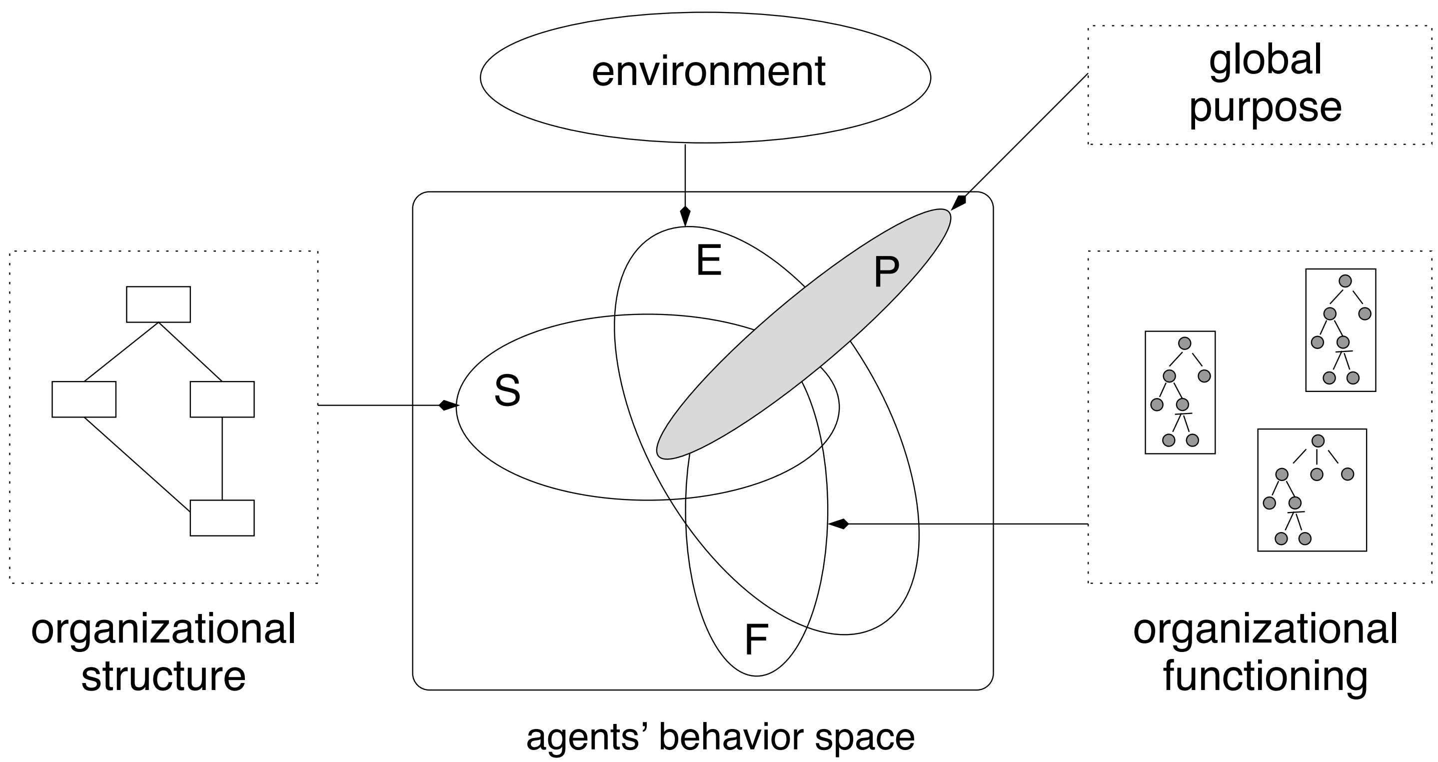




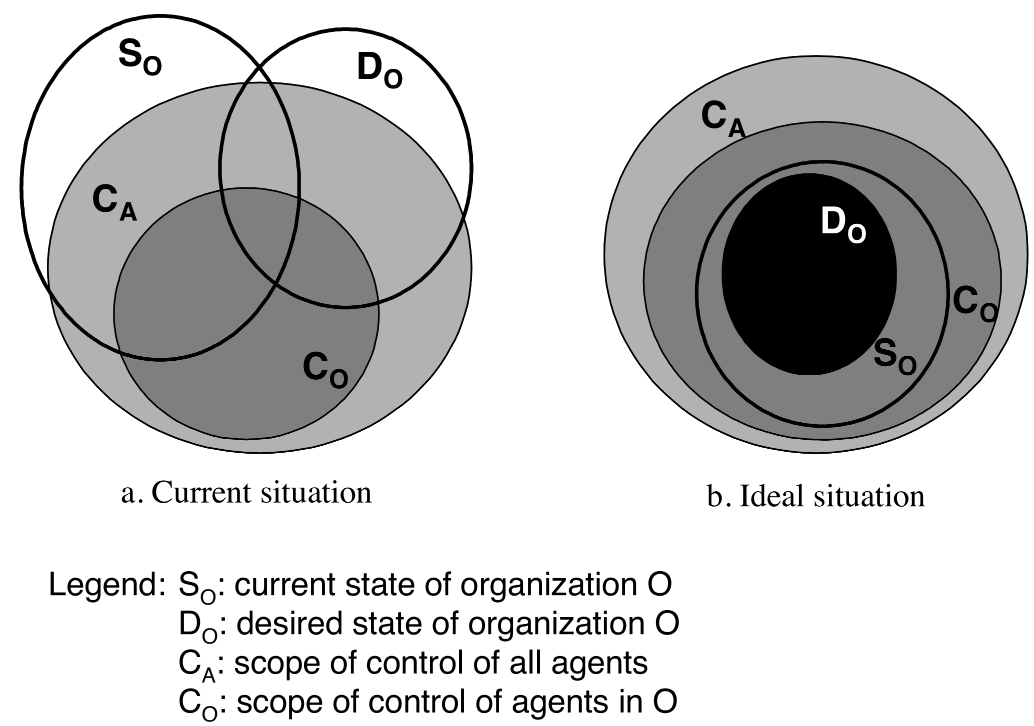
- For the point of interest to Command and Control:
  - Structures of complex objects and their combinatorial relationships
  - Combinatorial Topology (older name for Algebraic Topology)

TOPOLOGY IN AI

- From research on organizations of multiagent systems:
- ***Organizational Congruence*** is intuitively expressed as topological constructs
- 2 examples below:



(a) Behavior Space - Example 1 (HÜBNER; SICHMAN; BOISSIER, 2004)



(b) Behavior Space - Example 2 (DIGNUM; DIGNUM, 2007)

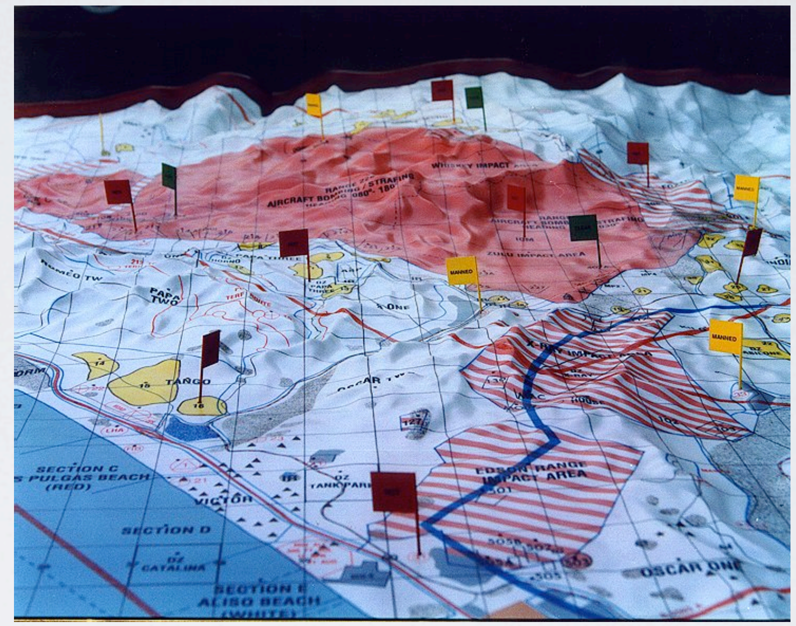
TOPOLOGY IN THE MILITARY

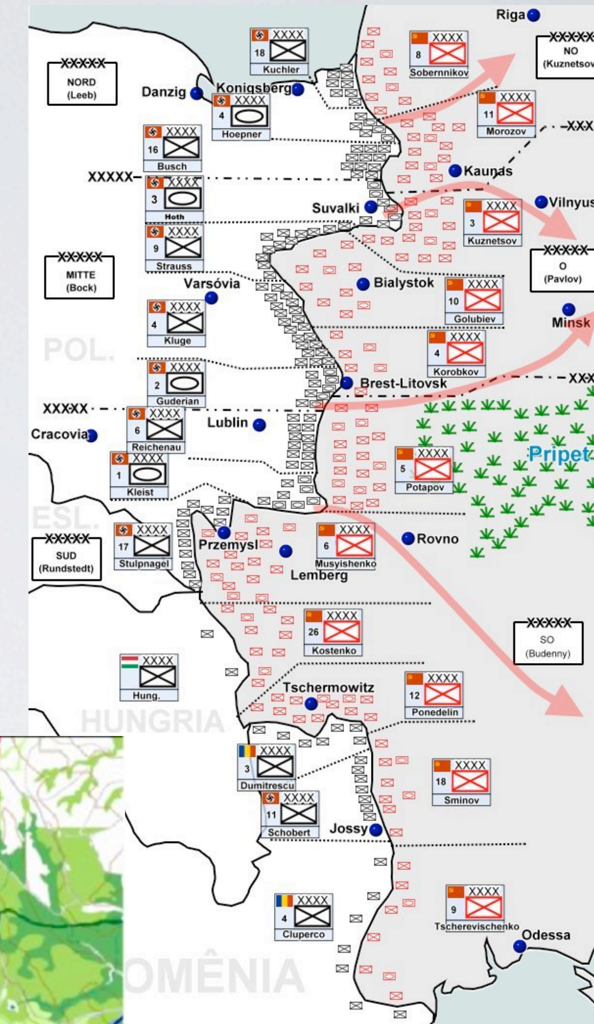
- (DoD definition) **Battlespace**: 'the environment, factors, and conditions that must be understood to successfully apply combat power, protect the force, or complete the mission'. This includes the air, land, sea, space, and the included enemy and friendly forces; facilities; weather; terrain; the electromagnetic spectrum; and the information environment within the operational areas and areas of interest.'

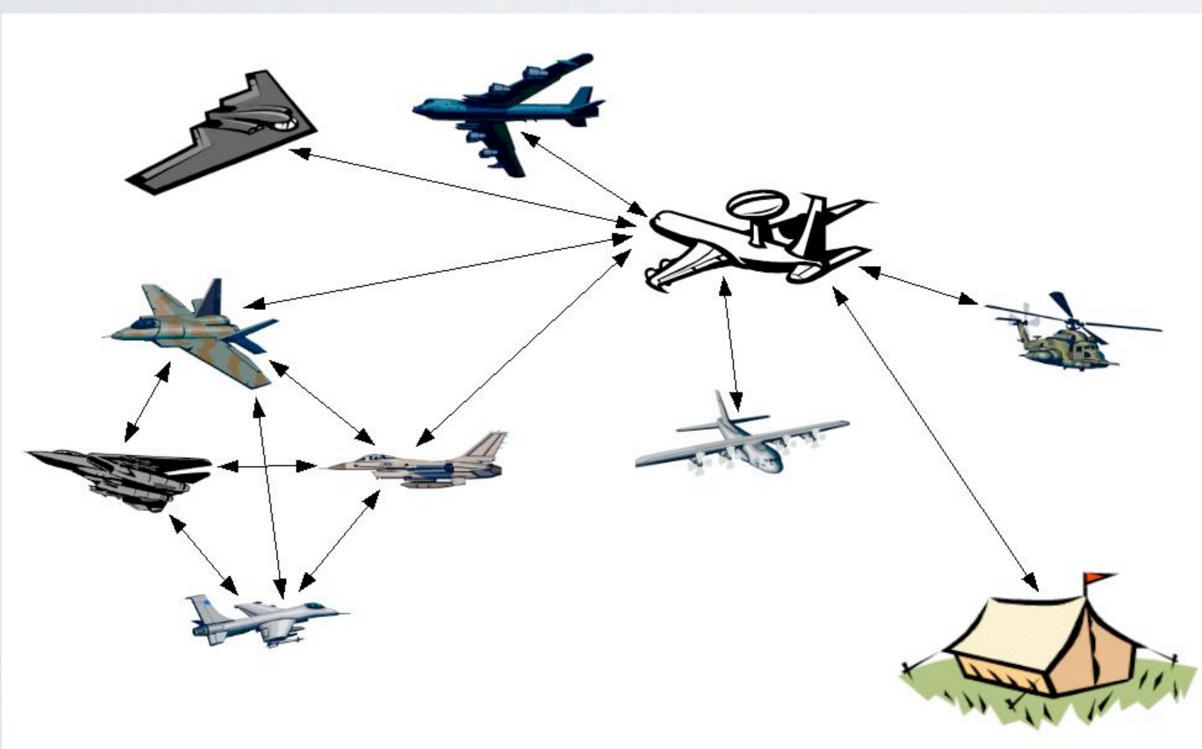
- ***'To shape the battlespace'***: to *conform* or to *give form* to all these elements to a configuration better fit to the mission

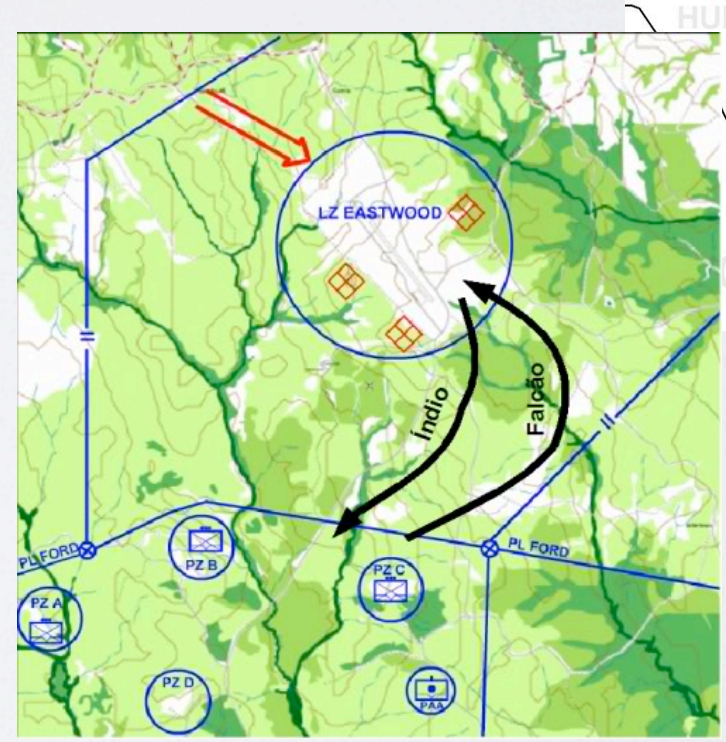
TOPOLOGY IN THE MILITARY

- Geographic space
- Situation maps
- Communication networks









TOPOLOGY IN C^2

'The most interesting and challenging [C^2] endeavors are those that involve a *collection* of military and civilian sovereign entities with *overlapping* interests that can best be met by sharing information and collaboration that *cuts across the boundaries* of the individual entities.'

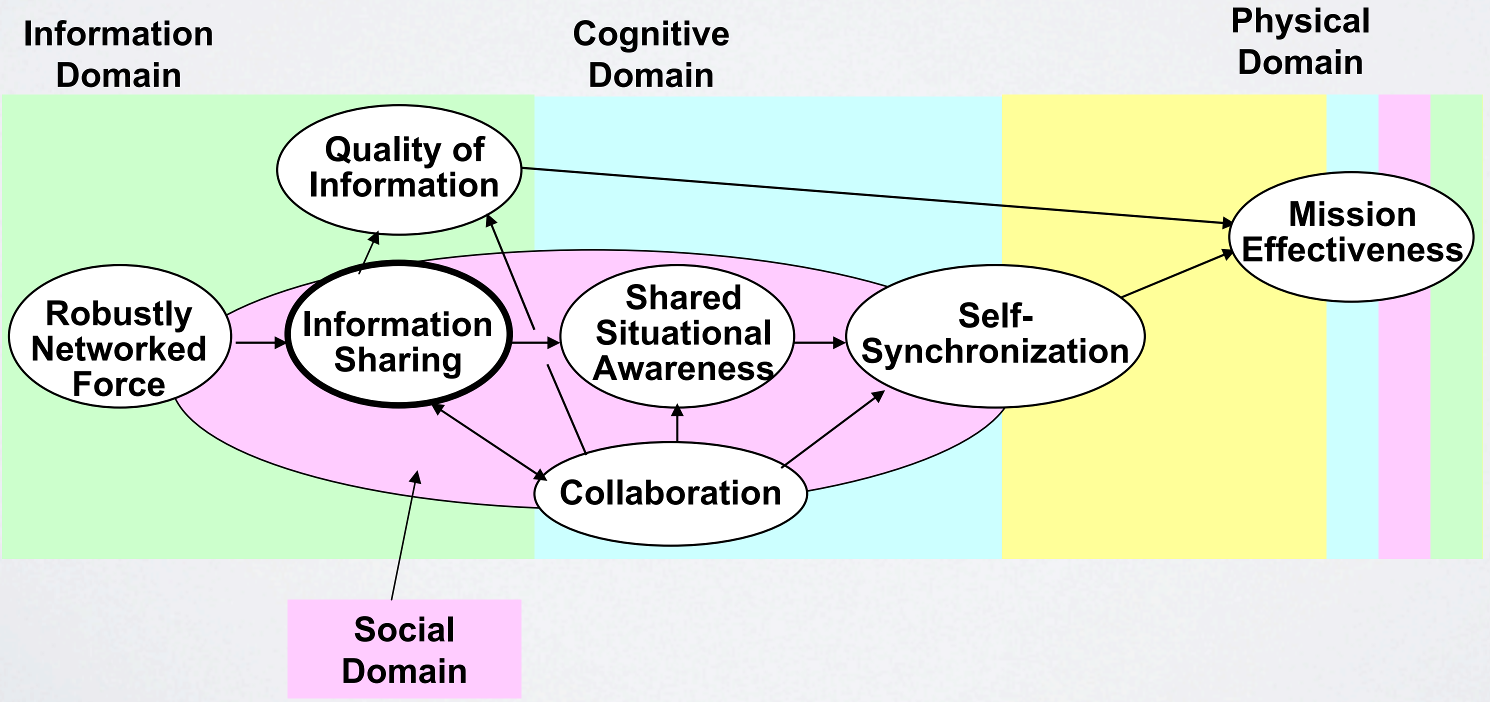
Understanding Command and Control, Alberts & Hayes, 2006.

TOPOLOGY IN C^2

- The Twin Curses of a C^2 Theory:

Dimensionality and *Complexity* (Levis&Athans,1987)

- Dimensionality: **4 key domains** (Alberts,2009)
- Complexity: **their interrelationships**:
  - Conceptual Model: ≈Hundreds of variables ≈**Thousands of relationships** !!!



TOPOLOGY IN C^2

- ‘Limitations present in C^2 Analysis:
  - **there are no proper investigation tools;**
  - **there is no *theory*; and**
  - **the treatment of *structure* has been neglected.’**

Mathematics of Command and Control, Dockery, 1984

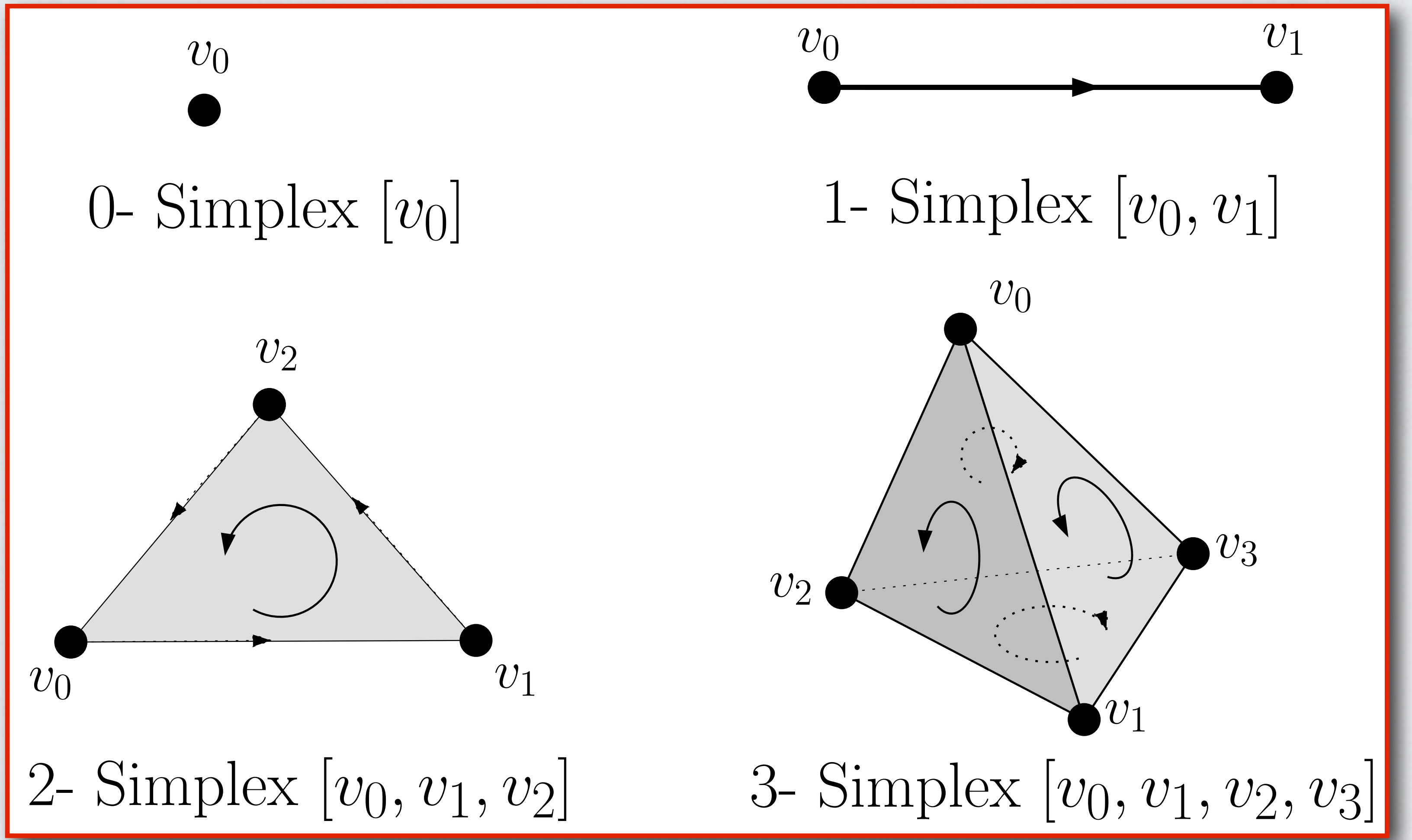
DEFINITIONS

- **Definition**: A ***Topological Space*** is a set ***X*** together with a collection ***O*** of subsets of ***X***, called ***open sets***, such that
    - the union of any collection of set in ***O*** is in ***O***;
    - the intersection of any finite collection of sets in ***O*** is in ***O***;
    - Both ∅ and ***X*** are in ***O***.

The collection ***O*** is called a ***Topology***.

DEFINITIONS

- **Definition**: An ***Abstract Simplicial Complex*** Δ on a finite set ***X*** is a family of subsets closed under deletion of elements. We refer to the singleton sets ***x*** in Δ as ***0-simplices*** or ***vertices***. It is not required that ***x*** $\in \Delta$ for all ***x*** $\in$ ***X***.



Extracted from: GHRIST, R.; MUHAMMAD, A. ***Coverage and hole-detection in sensor networks via homology***. In: IEEE PRESS. Proceedings of the 4th Intl.Symp. on Information Processing in Sensor Networks, 2005. p. 34.

C^2 ORGANIZATION CONCEPTUAL MODEL

- **Basic elements**:
  - Roles (R)
  - Relationships (Rel)
  - Tasks (T)

$$O = (O_{struct},\ O_{func},\ O_{assign})$$

$$O_{struct} = (R,\ Rel)$$

$$O_{func} = (T, P(T))$$

$$O_{assign} = \{(r_i, \{t_i\})\} \mid r_i \in R, t_i \in T\}$$

$$O_{behaviorstate} = (r_{i,k},\ \{t_{i,k}\})$$

$$O_{behaviorstate,k} \longrightarrow O_{behaviorstate,k+1}$$

C^2 ORGANIZATION
TOPOLOGICAL MODEL

- A tiny example - Air Defense:

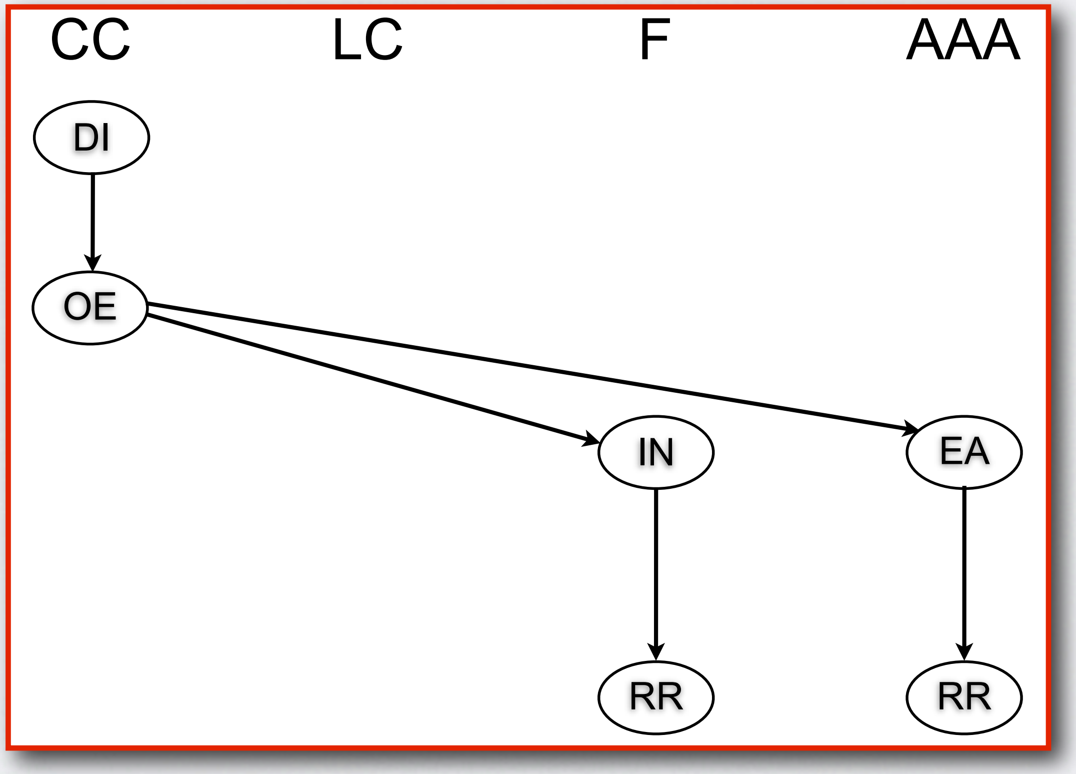
- 4 Roles
  - **CC** - Central Command Post
  - **LC** - Local Command Post
  - **F** - Fighter Aircraft
  - **AAA** - Anti-Aircraft Artillery

- 5 Tasks
  - **DI** - Detect and Identify
  - **OE** - Order to Engage
  - **IN** - Intercept
  - **EA** - Engage Artillery (AAA)
  - **RR** - Report Results

C^2 ORGANIZATION
TOPOLOGICAL MODEL

- Two steps process:
  - Step 1: <u>To build an *interaction poset* (partially ordered set) *P* from the task dependency graph</u>

$$O_{struct} : (\{CC,LC,F,AAA\},\{CC < LC,LC < F,CC < AAA\})$$
$$O_{func} : (\{DI,OE,IN,EA,RR\},\{DI < OE,OE < IN,OE < EA,IN < RR,EA < RR\})$$
$$O_{assign} : \{(CC, \{DI, OE\}), (LC, \emptyset), (F, \{IN, RR\}), (AAA, \{EA, RR\})\}$$

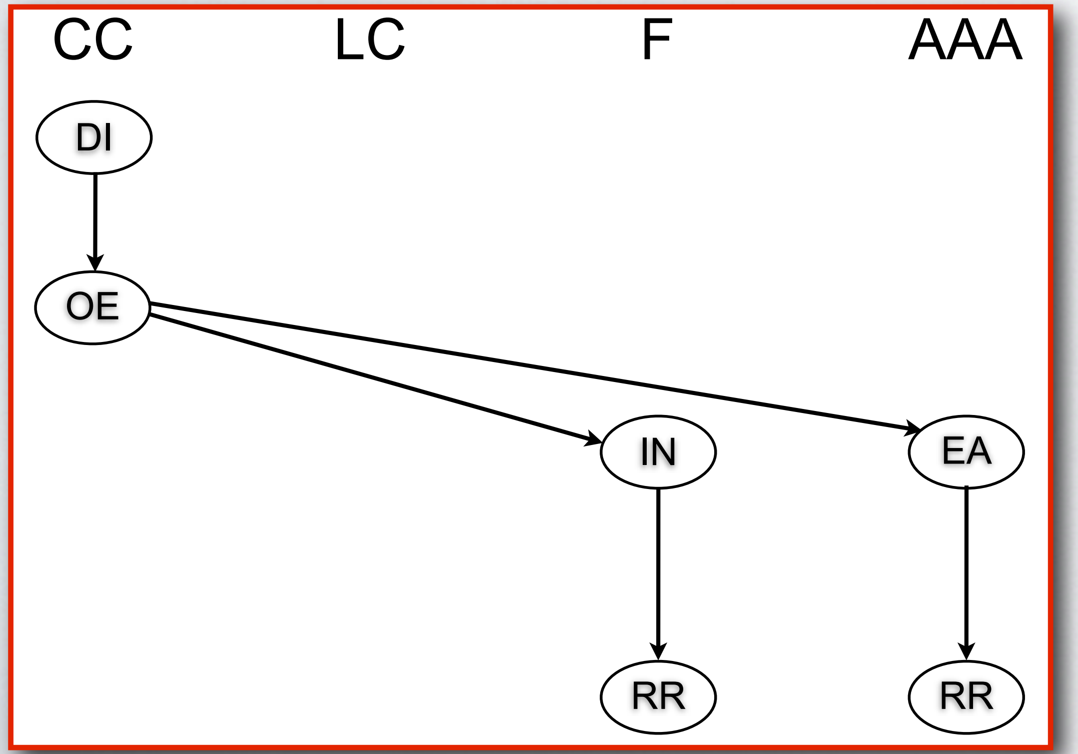


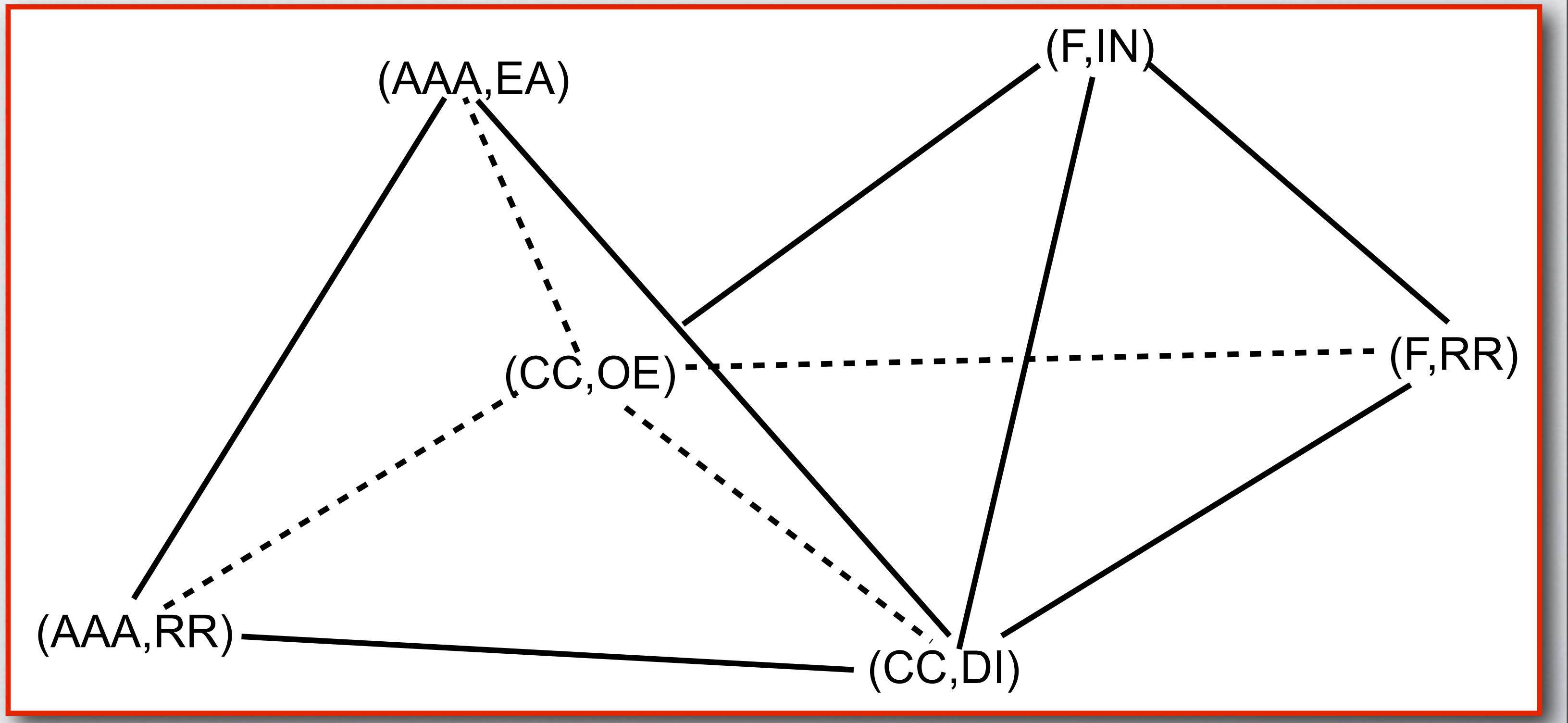
C² ORGANIZATION
TOPOLOGICAL MODEL

- **Definition**: An ***Order Complex*** ***$\Delta_0(P)$*** is a simplicial complex whose vertex set contains all elements of the interaction poset ***P***. A subset of ***P*** is a simplex of ***$\Delta_0(P)$*** if and only if its elements form a chain in ***P*** ,i.e., they can be arranged to form a totally ordered subset of ***P***.

  - Step 2: **To build the *order complex* $\Delta_0(P)$ from the *interaction poset P***

$$\Delta_o(P) = \{\{(CC,DI),(CC,OE),(F,IN),(F,RR)\}, \{(CC, DI), (CC, OE), (AAA, EA), (AAA, RR)\}\}$$



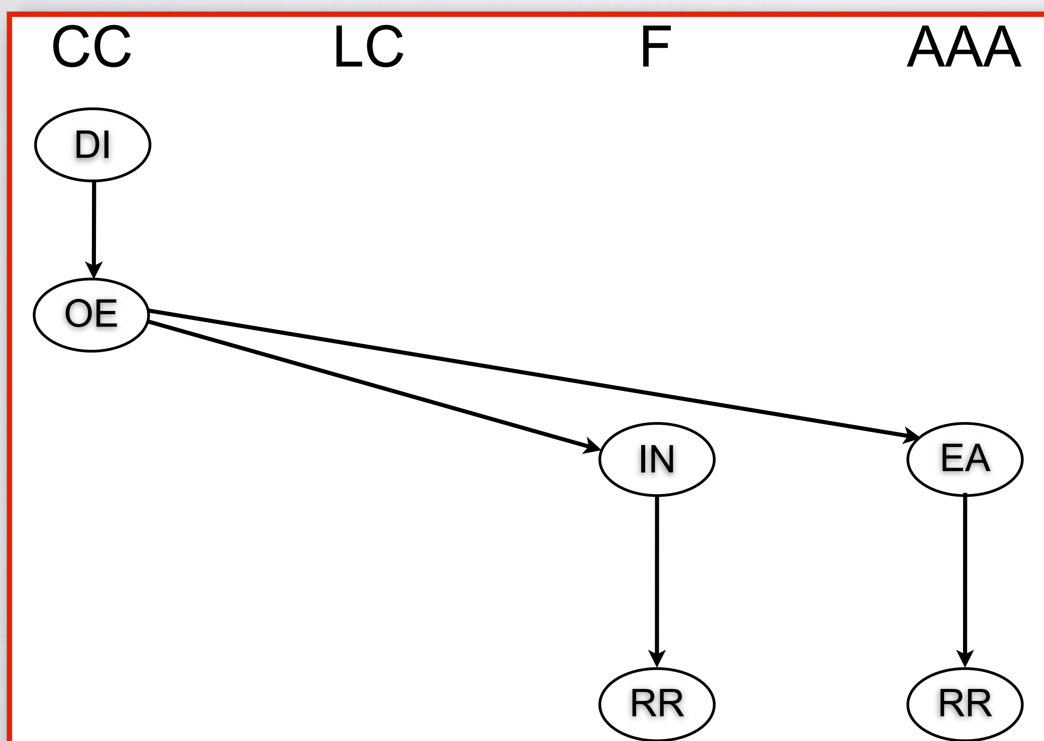


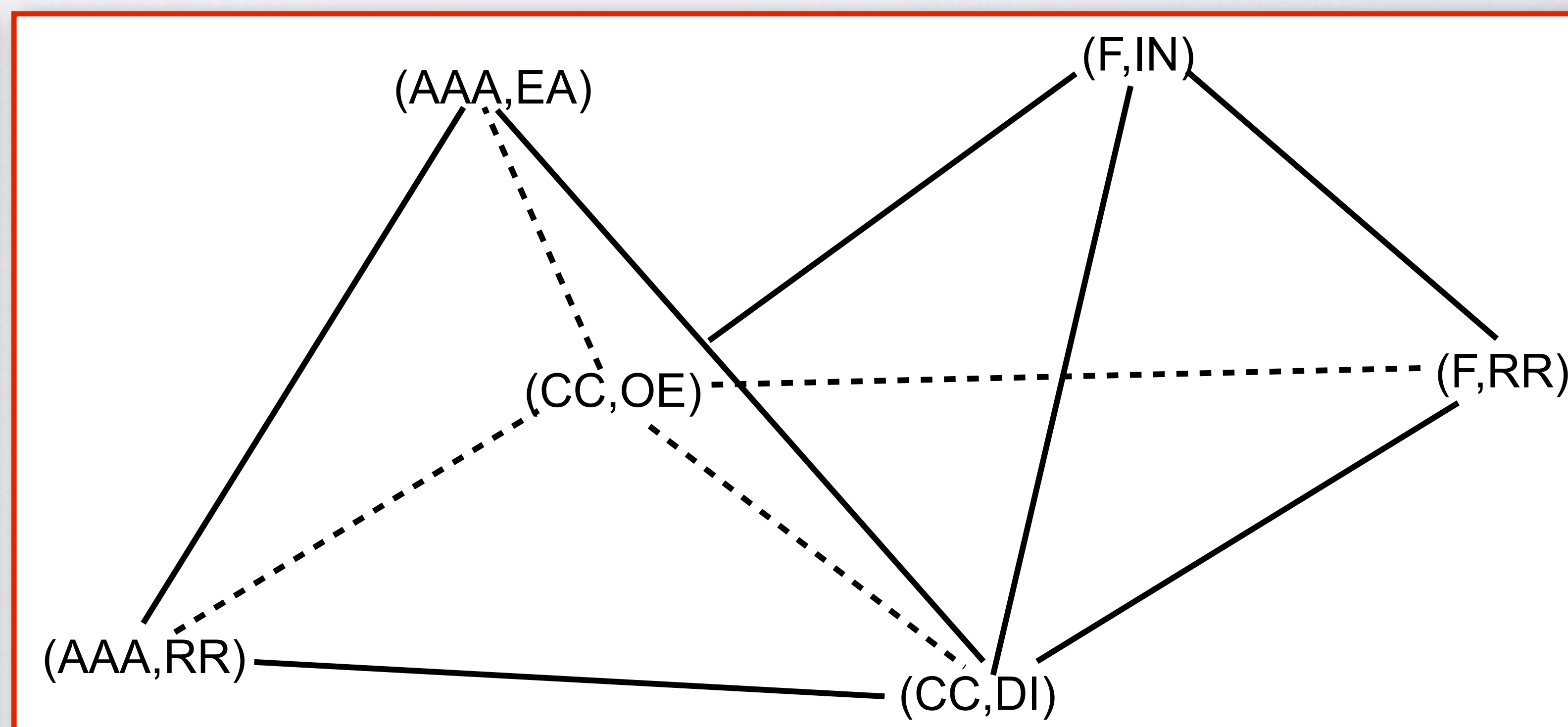
C^2 ORGANIZATION
TOPOLOGICAL MODEL

- Baseline organization:

$$O_{struct} : (\{CC, LC, F, AAA\}, \{CC < LC, LC < F, CC < AAA\})$$
$$O_{func} : (\{DI, OE, IN, EA, RR\}, \{DI < OE, OE < IN, OE < EA, IN < RR, EA < RR\})$$
$$O_{assign} : \{(CC, \{DI, OE\}), (LC, \varnothing), (F, \{IN, RR\}), (AAA, \{EA, RR\})\}$$

$$\Delta_O(P) = \{\{(CC,DI),(CC,OE),(F,IN),(F,RR)\}, \{(CC, DI), (CC, OE), (AAA, EA), (AAA, RR)\}\}$$



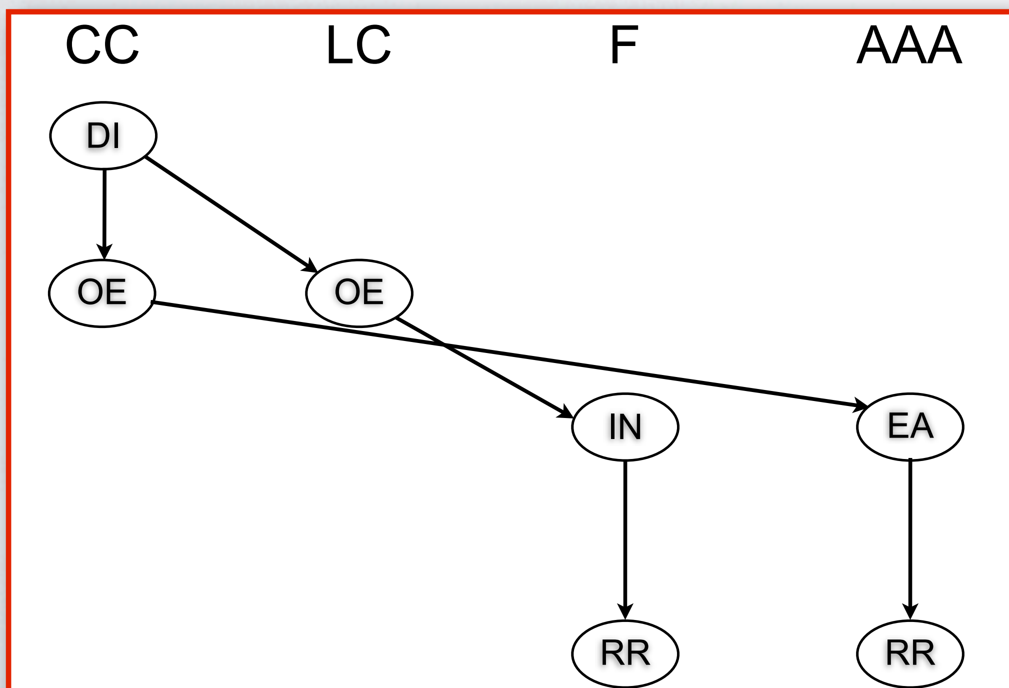


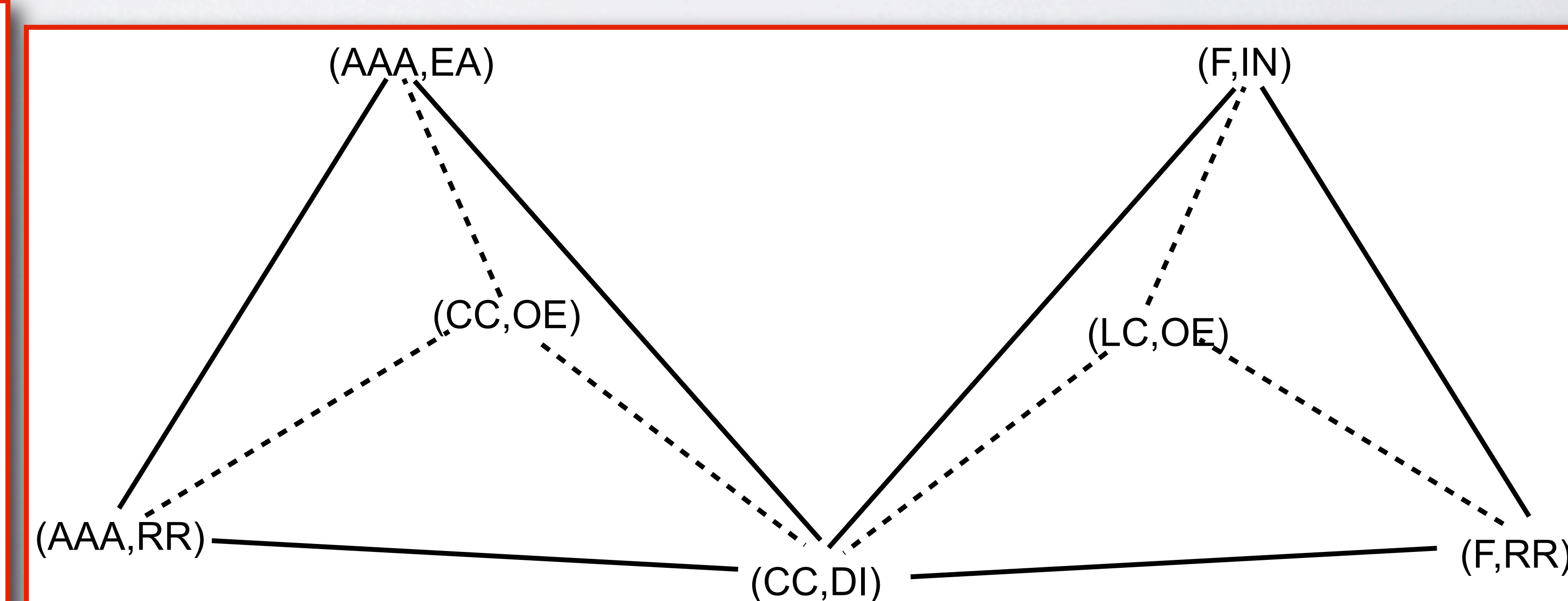
C^2 ORGANIZATION
TOPOLOGICAL MODEL

- Change 1: **CC** delegates to **LC** responsibility to order fighters.

$$O_{struct}: (\{CC, LC, F, AAA\}, \{CC < LC, LC < F, CC < AAA\})$$
$$O_{func}: (\{DI, OE, IN, EA, RR\}, \{DI < OE, OE < IN, OE < EA, IN < RR, EA < RR\})$$
$$O_{assign}: \{(CC, \{DI, OE\}), (LC, \{OE\}), (F, \{IN, RR\}), (AAA, \{EA, RR\})\}$$

$$\Delta_o(P) = \{\{(CC,DI),(LC,OE),(F,IN),(F,RR)\},$$
$$\{(CC, DI), (CC, OE), (AAA, EA), (AAA, RR)\}\}$$



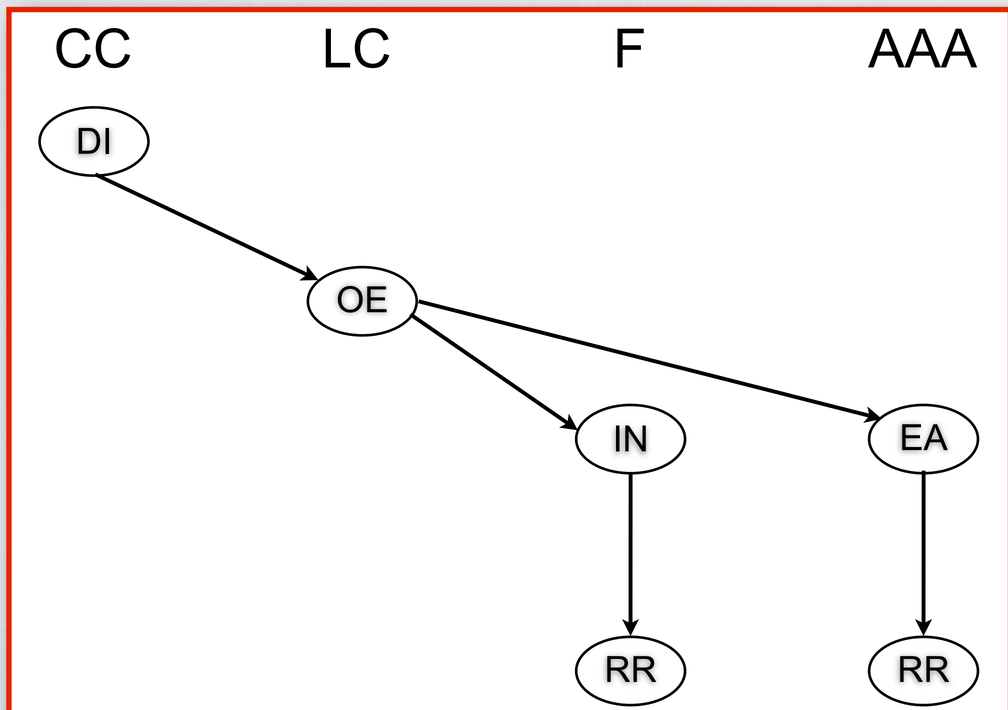


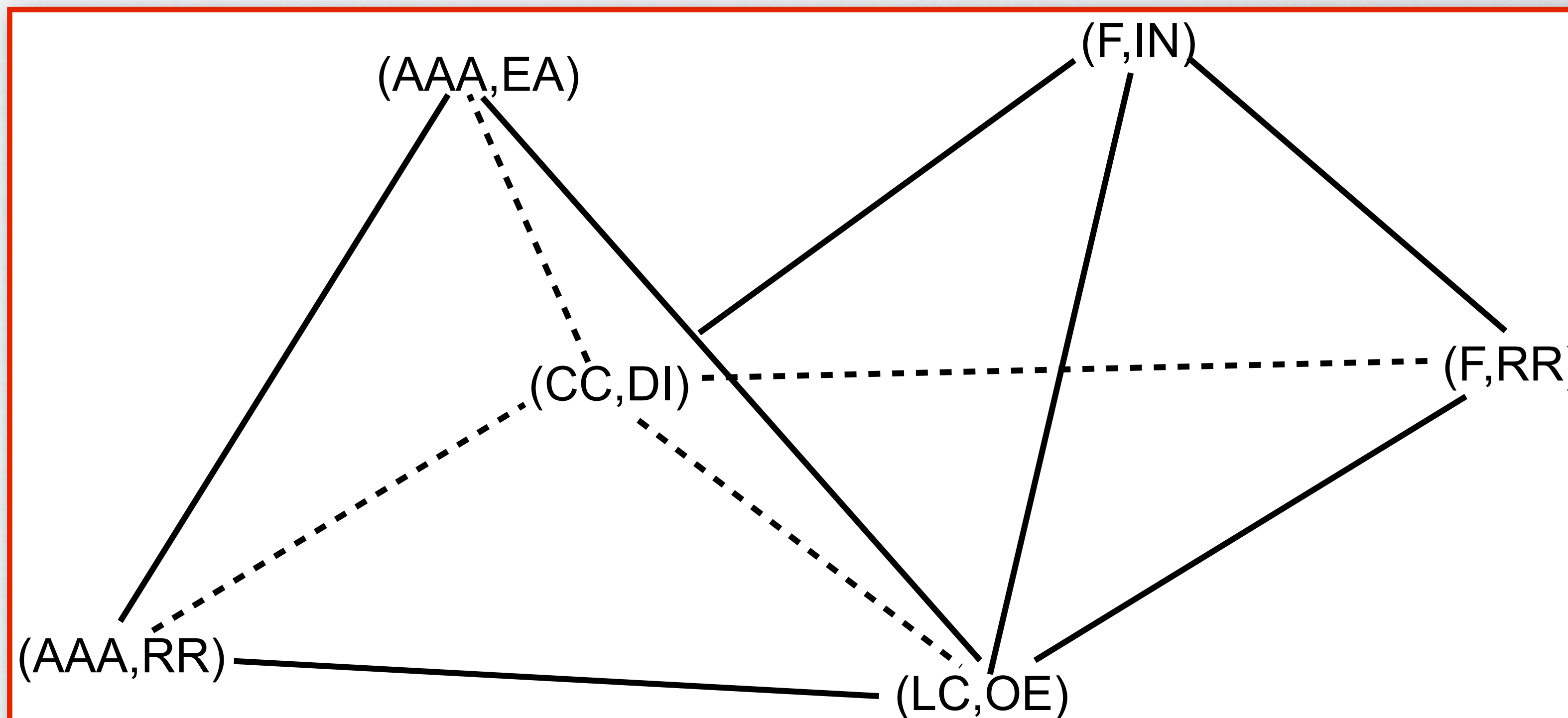
C^2 ORGANIZATION
TOPOLOGICAL MODEL

- Change 2: **LC** now also commands **AAA**

$$O_{struct} : (\{CC, LC, F, AAA\}, \{CC < LC, LC < F, CC < AAA\})$$
$$O_{func} : (\{DI, OE, IN, EA, RR\}, \{DI < OE, OE < IN, OE < EA, IN < RR, EA < RR\})$$
$$O_{assign} : \{(CC, \{DI\}), (LC, \{OE\}), (F, \{IN, RR\}), (AAA, \{EA, RR\})\}$$

$$\Delta_o(P) = \{\{(CC,DI),(LC,OE),(F,IN),(F,RR)\}, \{(CC, DI), (LC, OE), (AAA, EA), (AAA, RR)\}\}$$



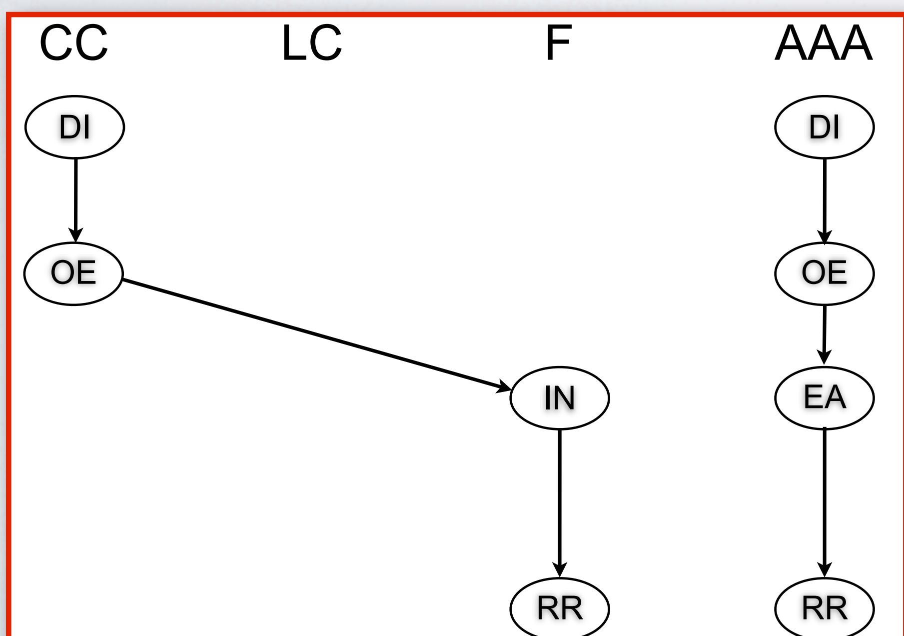


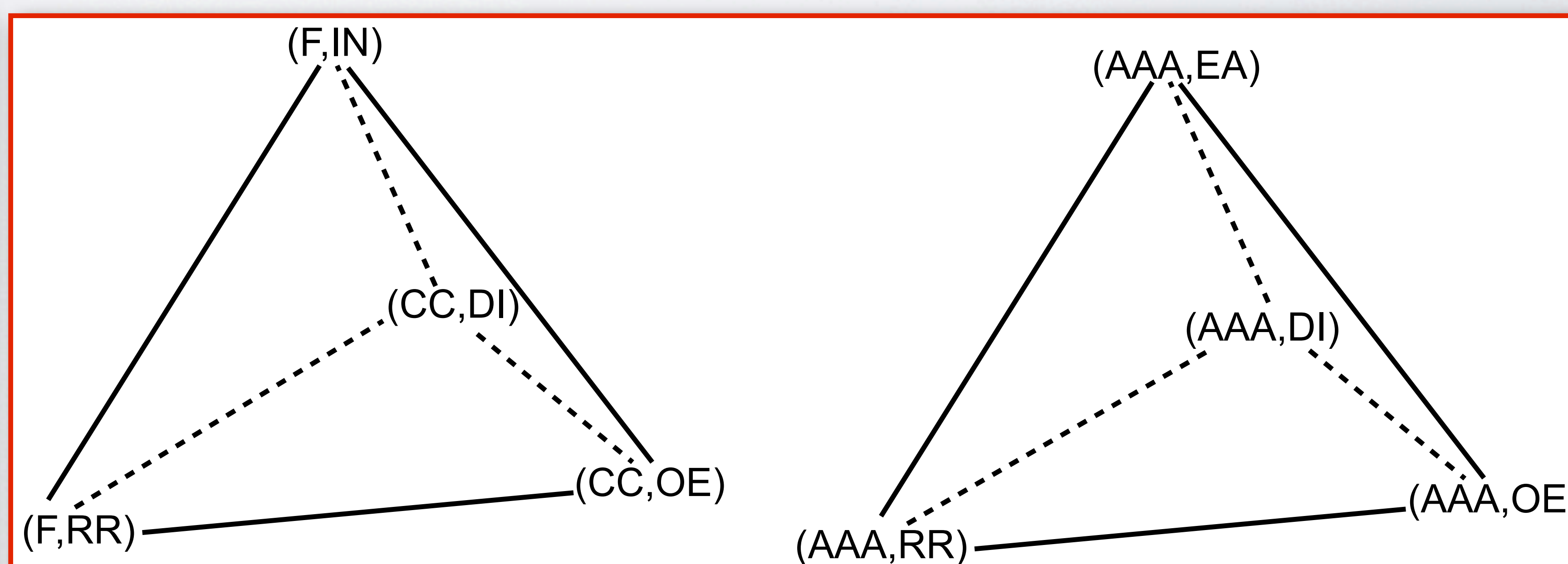
C^2 ORGANIZATION
TOPOLOGICAL MODEL

- Change 3: **AAA** goes autonomous...

$$Ostruct : (\{CC, LC, F, AAA\}, \{CC < LC, LC < F, CC < AAA\})$$
$$Ofunc : (\{DI, OE, IN, EA, RR\}, \{DI < OE, OE < IN, OE < EA, IN < RR, EA < RR\})$$
$$Oassign : \{(CC, \{DI\}), (LC, \{OE\}), (F, \{IN, RR\}), (AAA, \{DI, OE, EA, RR\})\}$$

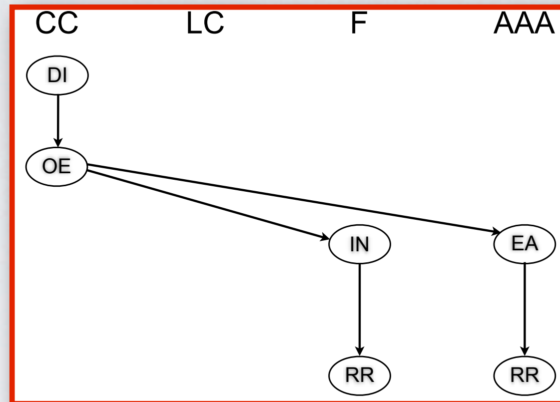
$$\Delta o(P) = \{\{(CC,DI),(LC,OE),(F,IN),(F,RR)\}, \{(AAA,DI), (AAA,OE), (AAA, EA), (AAA, RR)\}\}$$

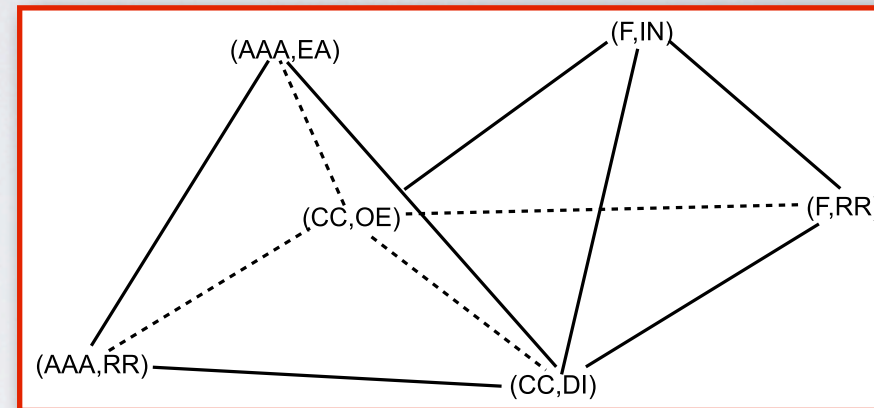




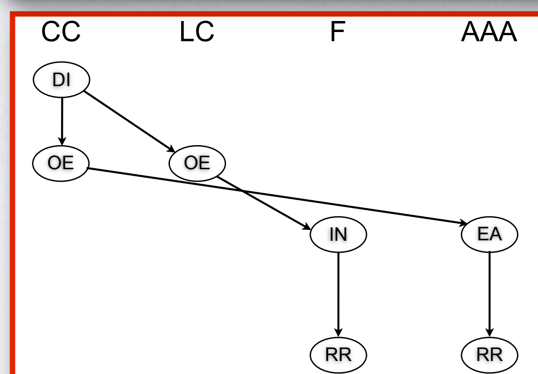
C^2 ORGANIZATION
TOPOLOGICAL MODEL

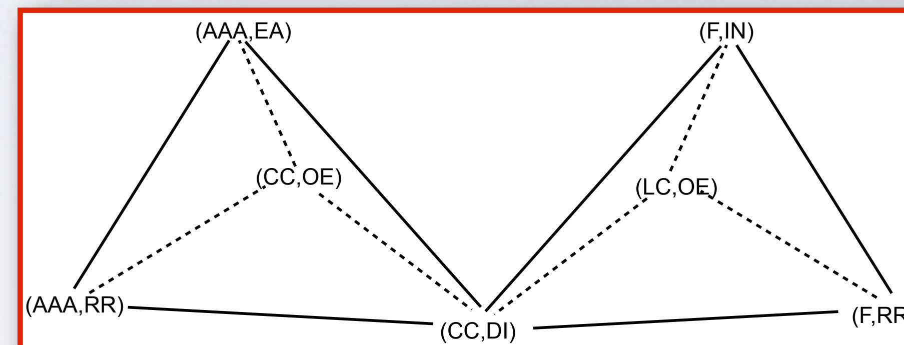
- Dynamics:
  - Baseline:



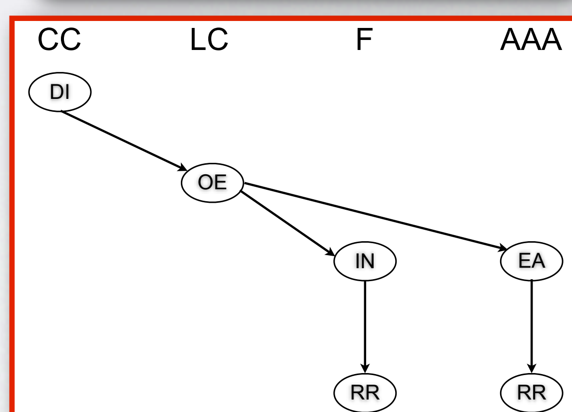


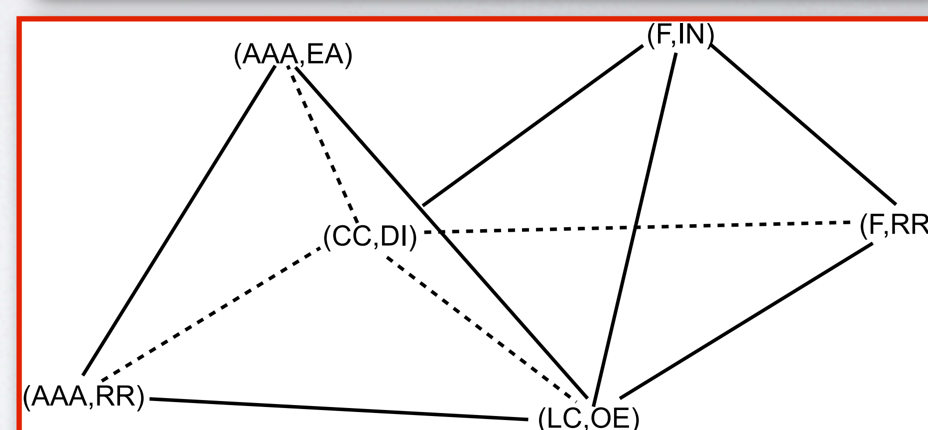
  - Change 1:



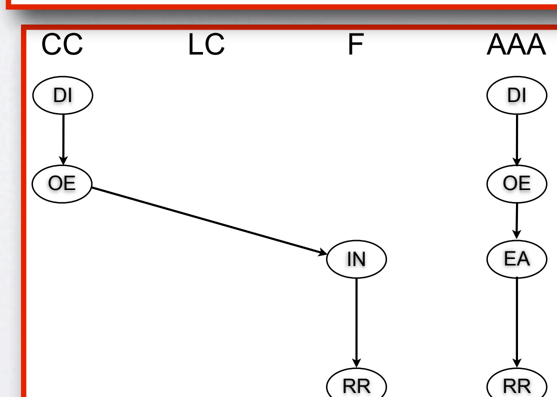


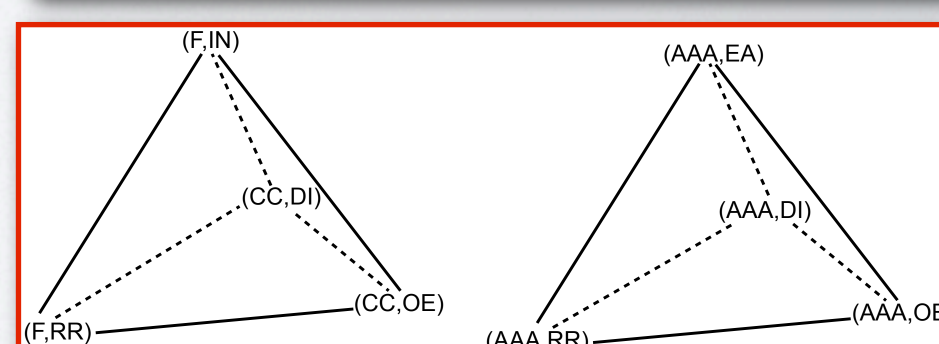
  - Change 2:





  - Change 3:





CONCLUSIONS

- A topological model of C2 organizations, based on simplicial complexes, that captures tasks' dependencies and distribution, was presented;

- Simplicial complexes have the main advantage to capture higher dimensional relationships, where graphs are only unidimensional;

- By changing task dependencies or task allocation the connectivity of the resulting simplicial complex changes;

- It means, the ***combinatorics*** related to the behavior of these organizations is changed!

CONCLUSIONS

- Next steps:
  - Information flow (*concurrency*)
  - Geospatial and temporal topological relationships
  - Power relationships

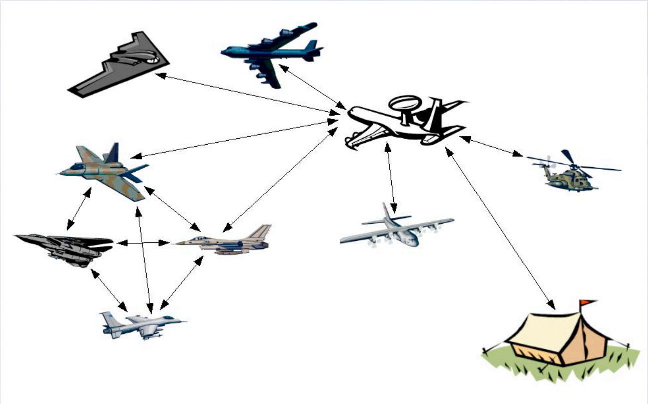
CONCLUSIONS

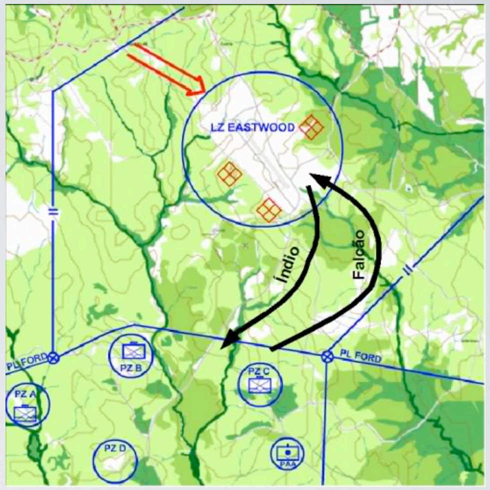
- ***Main tenets of this approach:***
  - Any C^2 organizational model will have to deal with the ***combinatorics*** of multidimensional parameters (structure, functions, environment, cognition, capabilities and resources);
  - Maybe the relationships between these parameters cannot be captured in a graph style, one dimension only;
  - ***Simplicial complexes*** are an adequate mathematical construct to capture these higher dimensional relationships.

‘We have therefore searched for a **theory**. More fundamentally, we have searched for a starting point for a theory. In the end we focused on **structural aspects** of C^2. In fact structure is but one of three aspects of the problem which we have identified. The other two are **data/information** and **transactions**. The complete characterization is therefore ***transactions within a structure involving the flow of data/information through that structure***.’

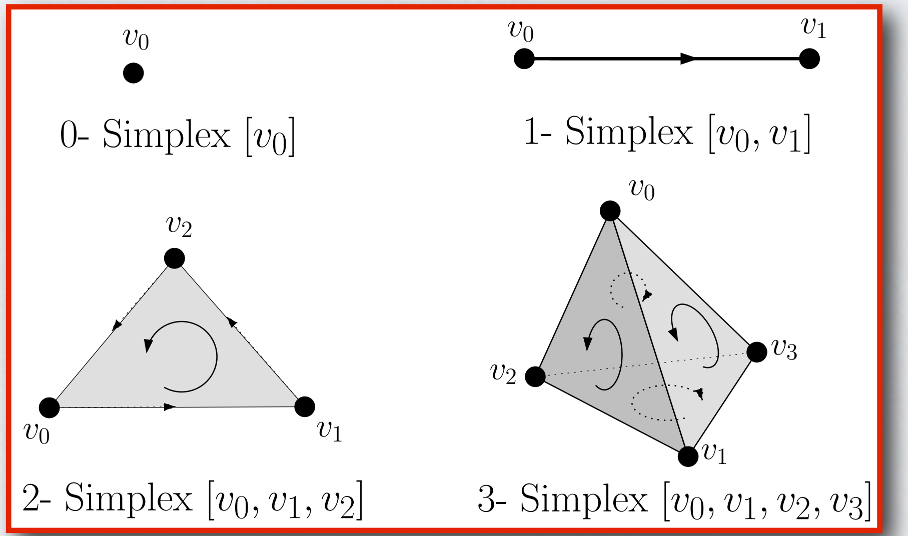
Mathematics of Command and Control, Dockery, 1984

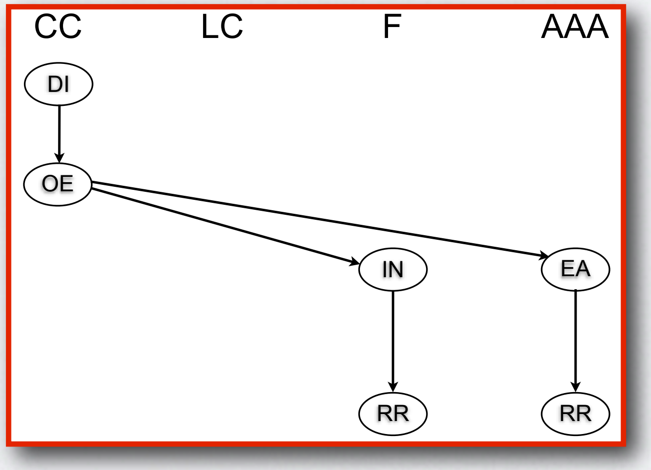
SUMMARY

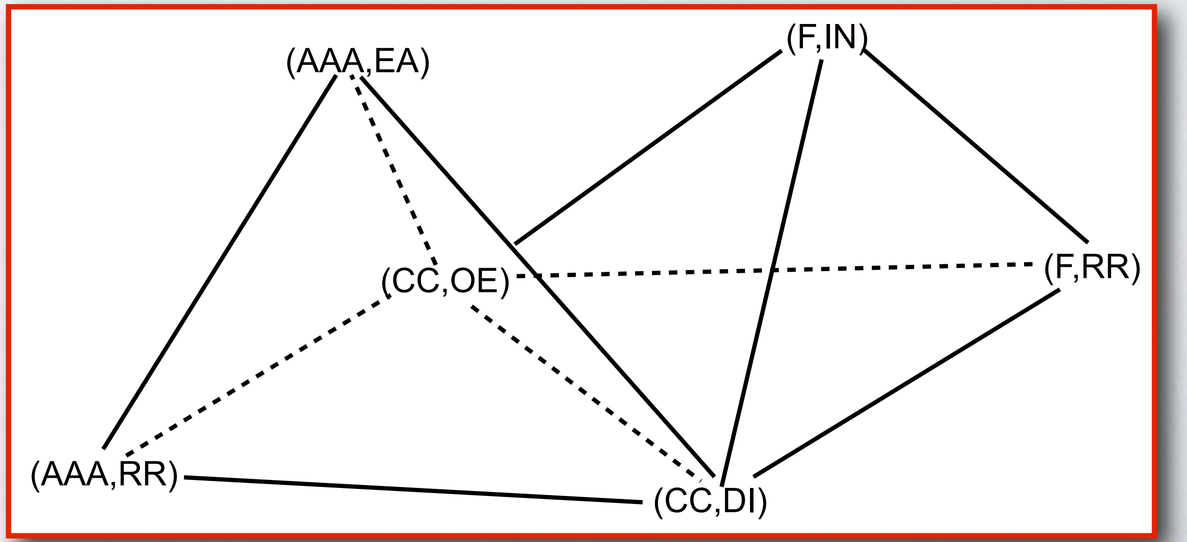




- Topology in the military
- Topology in C^2
- Definitions
- The *Model*







OBJECTIVES

- *To present a topological model of C^2 organizations*
- *To motivate the C^2 research community for the application of Topology to C^2 theory building*

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