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**“Collective C2 in Multinational Civil-Military Operations”**

## **Analyzing C2 Structures and Self-Synchronization with Simple Computational Models**

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***Dr Anthony H. Dekker***  
*DSTO Joint Operations Division*  
*Defence Establishment Fairbairn*  
*Department of Defence, Canberra*  
*Australia*  
[dekker@acm.org](mailto:dekker@acm.org)

## Analyzing C2 Structures and Self-Synchronization with Simple Computational Models

*Dr. Anthony H. Dekker (Defence Science and Technology Organisation, Australia)*

### Abstract

Although Command and Control (C2) is a complex activity, useful lessons about C2 can be learnt from simple computational models. In this paper, we describe experiments with two such models. The Kuramoto Model, though with some serious limitations, provides a representation of information flow and self-synchronization in an organization. A second (agent-based) model, based on factorization, provides a representation of planning that is slightly more realistic. These models suggest that the time for an organization to reach a decision is related to the average distance in the organizational network, although our two experiments disagree on the nature of this relationship. Comparing the simulation results to empirical real-world studies confirms the relationship between time and the average network distance. Although the empirical studies suggest that this relationship is linear, the Kuramoto Model might be more realistic in its suggestion of a nonlinear relationship, since it captures the idea of information being “attenuated” during transmission by misunderstandings. The Kuramoto Model therefore reveals a need for further empirical studies in this area.

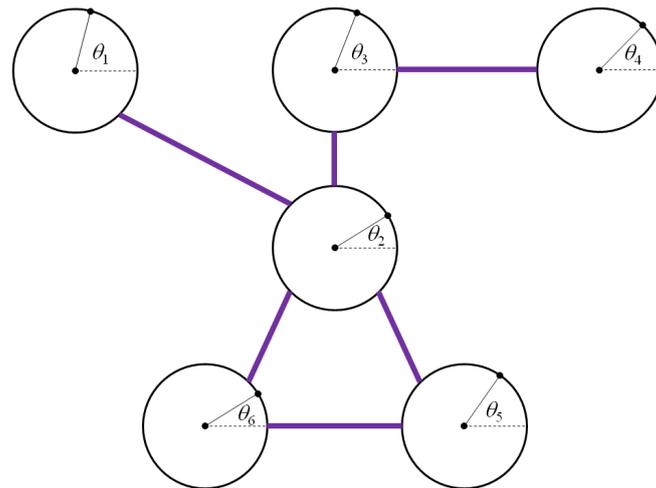
### Introduction

Command and Control (C2) of military operations is a complex activity (Alberts and Hayes 2006; Alberts and Hayes 2007). Nevertheless, useful principles for C2 can be derived from simple computational models, and lessons have been learned from simple agent-based models such as ISAAC (Brandstein et al. 2000).

We have been exploring the Kuramoto Model (Strogatz 2000; Dorogovtsev *et al.* 2008) as a simple model which can shed light on some aspects of C2, specifically on organizational topology (Dekker 2007a; Kalloniatis 2008a; Kalloniatis 2008b; Dekker 2010a). The generalized Kuramoto Model, which is drawn from the physical sciences, is a simple system of  $n$  networked oscillators  $O_1 \dots O_n$ , each with a natural frequency  $f_i$  (assumed to come from a unimodal and symmetric distribution) and a phase angle  $\theta_i$  (Figure 1 provides an example). Phase angles change so as to become closer to those of neighboring oscillators, according to the differential equation:

$$\theta'_i = f_i + k \sum_j^{i \leftrightarrow j} \sin(\theta_j - \theta_i)$$

where the sum is taken over all oscillators  $O_j$  connected to  $O_i$  in the network topology. The number  $k$  is the “coupling strength” along the links of the network, which may be taken as a representation of the quality of communication within an organization, or the extent to which information is “attenuated” as it moves across the network.



*Figure 1. The Kuramoto Model involves a network of coupled oscillators, and can be used as a very simple model of self-synchronization and information flow within a military organization. The  $\theta_i$  are oscillator phase angles, and these angles become similar as the network synchronizes. In this network, there is an average distance of 1.8 “hops” between nodes.*

Can such a simple model shed light on C2? The connection may not be immediately apparent, but the Kuramoto Model can be used to represent two important aspects of C2. First, the diffusion of information across a military organization (including sensor information, plans, and command intent); and second, the process of self-synchronization (Alberts and Hayes 2003).

The idea of self-synchronization implies that there are activities planned or executed within each node (person or unit) of a military organization, and that these activities are potentially in conflict with each other. These conflicts can be reduced to a minimum by communicating information across the organization, causing the various actions to be better aligned. Such a communication-driven negative-feedback process is exactly what is being modeled by the Kuramoto equation.

The phase angles  $\theta_i$  in the Kuramoto Model do not themselves necessarily represent anything meaningful, but the differences  $\theta_i - \theta_j$  represent the degree of conflict between the activities in nodes  $i$  and  $j$  of the network. Both in real military organizations and in the Kuramoto Model, communication-driven negative feedback acts to minimize these conflicts and to better coordinate the activities of the nodes. The Kuramoto Model therefore acts as an abstract representation of the process by which an organization synchronizes – whether synchronizing hierarchically in an organizational tree structure, or self-synchronizing in a non-hierarchical “edge organization” (Alberts and Hayes 2003). In the case of a simple organizational task where people choose colors from a palette, ensuring that people linked in the network choose different colors (Kearns *et al.* 2006), the link to the Kuramoto model has been demonstrated directly (Lee and Lister 2008).

## **Kuramoto Experiments**

In previous work (Dekker 2010a), we reported some experiments studying the time required for the Kuramoto Model to successfully synchronize, for a variety of network topologies with different attributes. Figure 2 summarizes the results. In these simulation experiments, we discretized the Kuramoto Equation, and counted the number of time-steps required to achieve synchronization, using a varied sample of 79 networks, all with  $n = 60$  nodes:

- 40 Random (Erdos-Rényi) networks (Bollobás 2001), with average degrees ranging from 3 to 10;
- 20 Scale-Free (preferential-attachment) networks (Barabási 2002), with average degrees ranging from 2 to 5;
- 10 Small-World networks generated by the Watts rewiring process (Watts 2003), with an initial antiprism structure, and a rewiring probability of 0.1;
- 1 social network resulting from a survey of informal communication within an organization; and
- the 8 networks in Figure 3, including 1 tree, 1 torus, and 6 spherical networks.

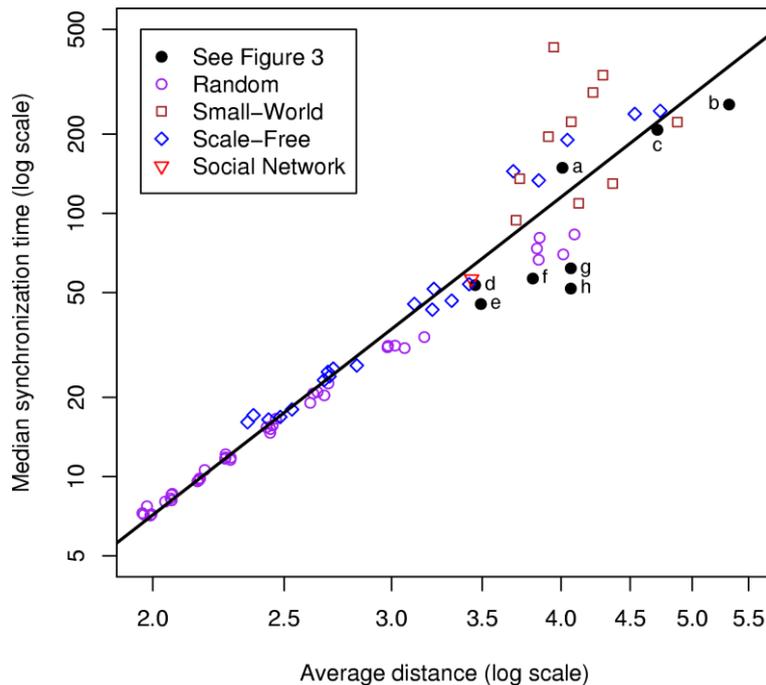


Figure 2. The median time for the Kuramoto Model to synchronize grows rapidly with the average distance of the network topology, fitting the power law  $T = 0.44 D^{4.0}$ . This figure is redrawn from Dekker (2010a).

Each of the 79 datapoints in Figure 2 represents the median of 101 simulation runs. Power-law regression (linear regression on the logarithms) indicated a statistically significant ( $p < 10^{-15}$ ) effect due to the average distance  $D$ , with a best-fit power law of  $T = 0.44 D^{4.0}$ , and with average distance explaining 93% of the variance in the results. In contrast, other network measures, such as the algebraic connectivity or the clustering coefficient, had less predictive value.

This experiment underscores the fact that the ability of networks to synchronize improves as the average distance shrinks. Simple combat simulations produce a similar conclusion (Dekker 2005). However, as a model of the real world, the Kuramoto Model has some serious limitations. In particular, the Kuramoto equation has the property that large ring structures may fail to synchronize. As a result, the Small-World networks in Figure 2 have a longer-than-predicted median synchronization time. From the point of view of modeling C2, this appears to be a modeling artifact, reflecting properties specific to systems of coupled oscillators – although it is interesting to note that in the classic experiments of Leavitt (1951), a ring-like team structure also displayed a surprisingly large decision time.

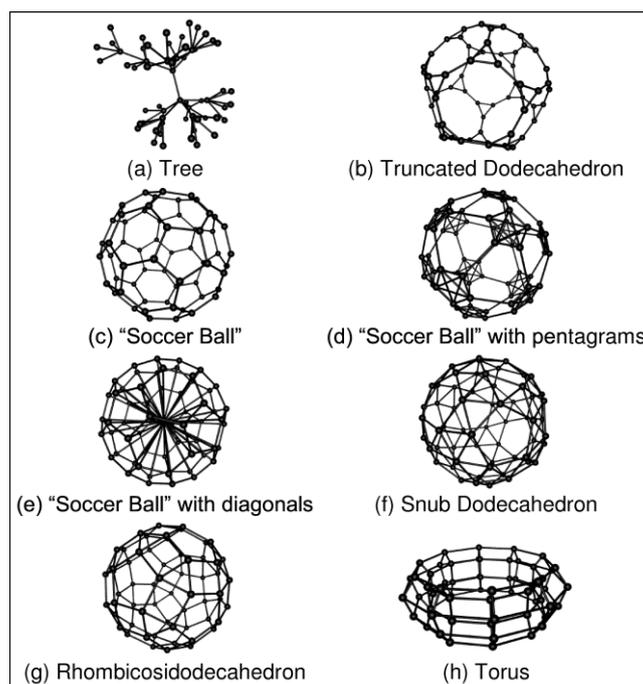


Figure 3. Eight special networks used in the Kuramoto simulation experiment. This figure is redrawn from Dekker (2010a).

## A Simple Planning Model

To further explore the effects noted in the experiments above, we explored another very simple, but slightly more realistic, agent-based simulation model. In this model, a collection of networked agents collaborate on a simple task, which provides an abstract representation of the collaborative planning process.

Specifically, the agents collaborate on factorizing a large number, such as  $6,598,886,315,082,427 = 571 \times 1,019 \times 1,303 \times 2,371 \times 3,671$ , with prime factors in the range 2 to 5987. Just as members of planning teams build up a plan by identifying and sharing key items of information, agents in this model use division to test potential factors, and build up the complete factorization by communicating the results of those tests. At each time step, each agent (that has not yet found the complete factorization) with equal probability either tries one new potential factor (adding a new fact about divisibility to its knowledge base), or forgets one fact in its knowledge base. The agent then communicates some fraction  $q$  of its knowledge base to its neighbors in the network.

This agent definition ensures that each agent is a finite Markov process, with successful factorization being an absorbing state (Norris 1997). Agents must therefore eventually reach their goal of finding the complete factorization, although the time taken for an agent on its own is very large – just as a single person could theoretically produce an entire military plan, but only if given unlimited time. As in the real world, communication within the model significantly speeds up the process, with the time until all agents complete the factorization goal being approximately inversely proportional to the square root of  $q$ . To ensure reasonable completion times, in the experiment reported in Figure 4, we took  $q = 0.1$ . This experiment used a similar set of 79 networks to the first experiment, and we again calculated median times over 101 simulation runs for each network.

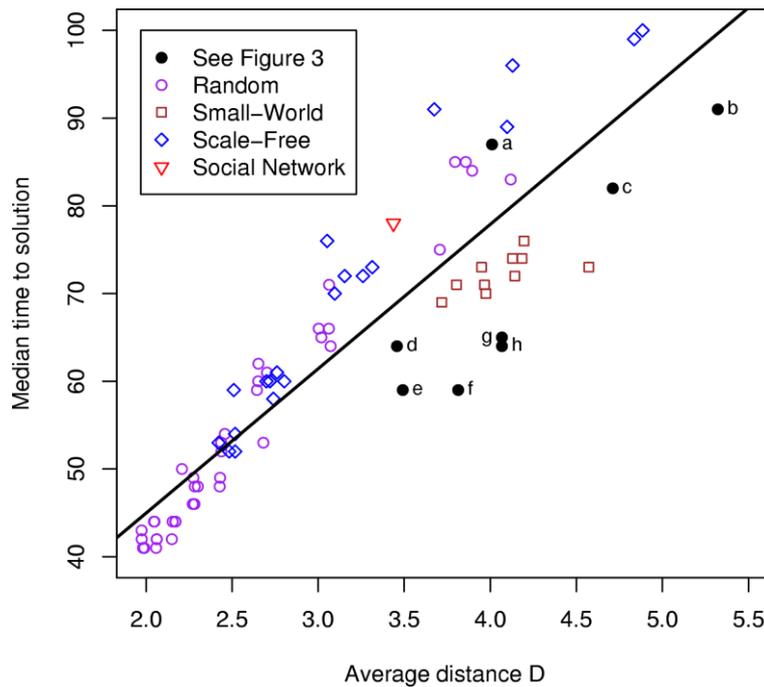


Figure 4. The median time for the Factoring Model to reach a solution grows linearly with the average distance of the network topology, with a best-fit line  $T = 12 + 16.5 D$ .

Two features of Figure 4 are particularly noticeable. First, the relationship to average distance is linear, not a power law as in Figure 2 (average distance predicts 81% of the variance, and the effect is statistically significant, with  $p < 10^{-15}$ ). Second, the modeling artifact of Figure 2 has disappeared. In fact, the Small-World networks, together with networks (b) to (h) of Figure 3, are particularly good network topologies for synchronization, falling below the best-fit line.

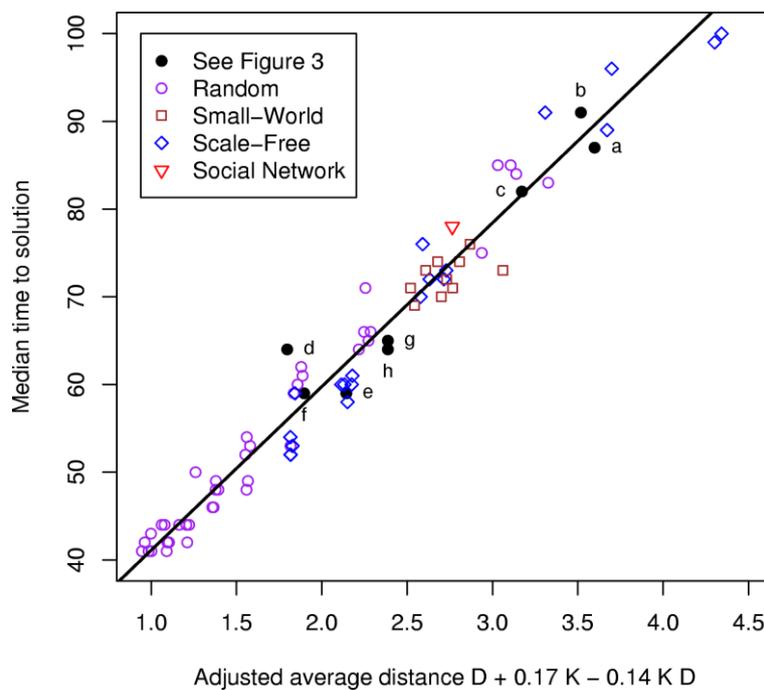


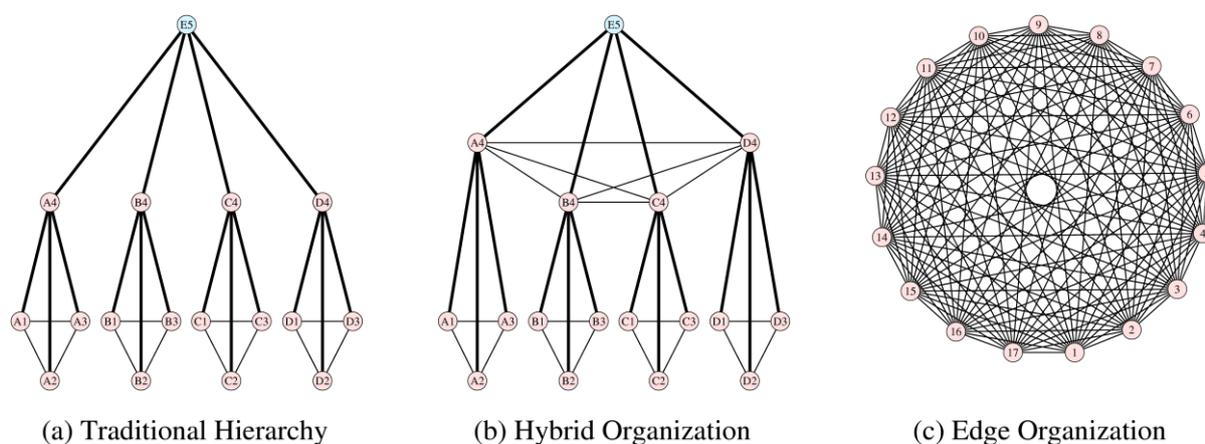
Figure 5. Incorporating the average connectivity  $K$  helps explain the good performance of Small-World and regular networks, and improves prediction of variance to 96%.

The last point can be explained by incorporating the *average connectivity* (Dekker 2007a) of the various networks into the analysis, as shown in Figure 5. The average connectivity  $K$  measures the average number of independent paths between pairs of nodes (where “independent” means having no intervening nodes in common), and has a statistically significant impact on the results ( $p \leq 10^{-5}$  for both direct and interaction effects, improving the prediction of variance to 96%). Such redundant network pathways are beneficial because they help facts move through the network even when one particular node does not pass them on (note that the  $KD$  interaction effect, which acts oppositely to  $D$ , outweighs the direct  $K$  effect in Figure 5). The Small-World networks, together with networks (b) to (h) of Figure 3, all have average connectivities of at least 2.96, which helps explain their good performance. Experiments with simple combat simulations support the benefits of redundant network pathways (Dekker 2004).

## Relationship to Human Studies

The linear dependence of synchronization time on average distance is confirmed by a number of human studies. Kearns *et al.* (2006) for example, studied the time taken by a group of people to collectively solve the network coloring problem (Gibbons 1985). The networks which their experimental subjects were asked to color were exactly the networks by which the subjects were connected, and communication across these networks was essential, because participants only had a local view of the coloring. For classes of networks where subjects usually found a correct solution (bicolorable networks), the time taken in minutes was approximately 20 times the average distance of the network used (Kearns *et al.* 2006, Dekker 2010b).

Similarly, experiments with the ELICIT game by Thunholm *et al.* (2009) also showed a linear dependence on average distance. ELICIT, the Experimental Laboratory for Investigating Collaboration, Information-sharing and Trust (Ruddy 2007), is a tool for exploring organizational concepts, in which a team of 17 people plays the role of an intelligence analysis cell, discovering and communicating “factoids” concerning a fictional terrorist plot. The team goal is to assemble factoids and decide *who* will carry out an attack, *when*, *where*, and on *what*. The 17 participants in each team can be organized into a variety of team structures.



*Figure 6. Three network structures used by Thunholm et al. (2009). In the traditional hierarchy, four teams can communicate amongst themselves, while the hybrid organization adds communication among team leaders, and the edge organization allows completely symmetrical communication. The average distances for these networks were calculated to be 2.85, 2.15, and 1, respectively.*

Thunholm *et al.* (2009) explored the three different organizational network structures shown in Figure 6, and measured, among other things, the time taken for each 17-person organization to reach a decision.

Figure 7 shows the decision times (in minutes) for these three organizational structures. The average distance explains 61% of the variance in the results, and the effect is moderately statistically significant ( $p < 0.04$ ). The best-fit line  $T = 26.3 + 5.7D$  perfectly fits the three group averages.

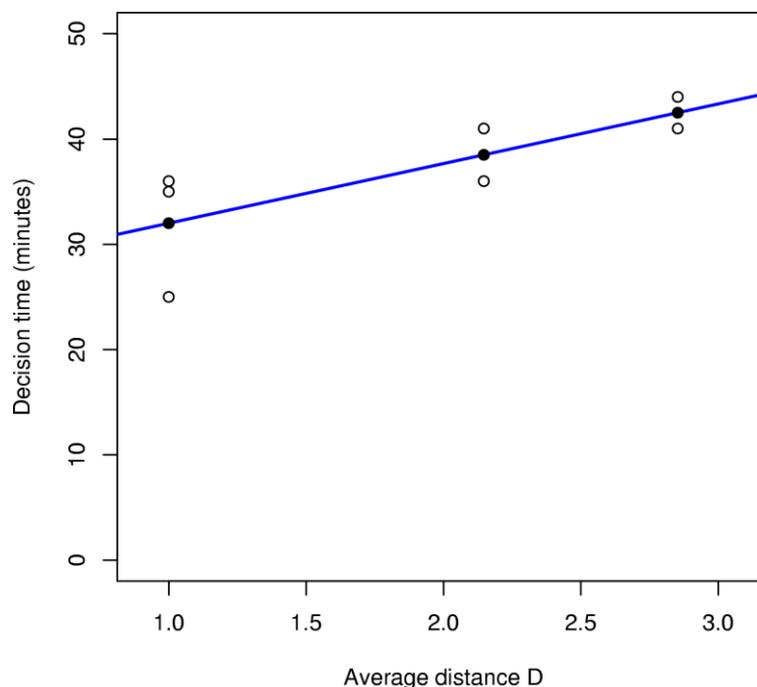


Figure 7. Decision times (open circles) for the experiment of Thunholm *et al.* (2009, Table 3). The best-fit line  $T = 26.3 + 5.7D$  perfectly fits the group averages (solid circles).

## Implications for C2

It is tempting to believe that this linear dependence of time on average network distance – demonstrated both in the agent-based factorization experiment, and in the human studies of Kearns *et al.* (2006) and Thunholm *et al.* (2009) – is a universal factor. However, both these human studies, as well as the factorization simulation, involve information that is entirely black-or-white. Individual facts are either communicated, or they are not, with no possibility of information being “attenuated,” as in the Kuramoto Model.

In real C2 systems, on the other hand, it is possible to “attenuate” information by misunderstanding it. Hard information about target coordinates may be black-or-white, and this information is relatively difficult to misunderstand – it is either transmitted or it is not. However, subtle information (e.g. about command intent, relative priorities of tasking, or civilian attitudes) is much less cut-and-dried, and can be “attenuated” by being only partially understood. For example, at the Battle of Gettysburg, General Robert E. Lee sent word to Richard S. Ewell “to seize that hill south of town [Cemetery Hill] if practicable” (Stackpole 1982, 173). Ewell received this directive, but apparently misunderstood its urgency, and did not attempt to seize the hill, severely disadvantaging the Confederate forces.

Where such a message is transferred across a chain of people (i.e. a path of length 2 or more), repeated transmission and re-transmission can progressively degrade meaning (Pratt and

Bennett 1989, 88; Hone *et al.* 2007; Kashima and Yeung 2010). As average distances increase, so does the chance of information losing its value (Baber *et al.* 2004).

To avoid such problems, General Fred Franks, the highly successful US VII Corps commander in the Gulf War, emphasized the importance of direct communication with his subordinates (i.e. short distances) as well as the higher quality of information transfer in face-to-face, rather than electronic, communication:

“The main thing was that I wanted to get my subordinate commanders’ sense of what was happening, and then give them my own sense and tell them what I wanted them to do in the next twelve to twenty-four hours. When I was there with them, I could look them in the eye and see if they understood what I wanted. That way, there could be no ambiguity in orders. ... Commanders shouldn’t be staying in their command post. They should be out and around the soldiers, where they can be feeling the pain and the pride, and where they can understand the whole human dimension of the battle.” (Franks and Clancy 1999, 103).

By modeling the progressive “attenuation” of information, the Kuramoto Model might therefore represent an element of real-world C2 which is missing from the factorization experiment, and also from the human studies of Kearns *et al.* (2006) and Thunholm *et al.* (2009). If the Kuramoto Model’s power-law average-distance effect holds for real C2 structures, then these human studies may be underestimating the problems associated with traditional hierarchical structures, and hence underestimating the benefits of structures with low average distances – structures like “edge organizations” (Alberts 2003; Dekker 2007b). For large organizations, there will be a substantial difference between decision times proportional to the average distance  $D$ , and times proportional to  $D^4$ . Consequently, the Kuramoto Model reveals that there remains a need for further human experiments, in the style of Thunholm *et al.* (2009), which explore the “attenuation” of more subtle kinds of information – such as attitudes, priorities, and intent – in different organizational structures.

As a first step to addressing this need for experimentation, we have planned a series of experiments in which distributed teams attempt to solve a verbal form of the assignment problem (Christofides 1975; Dekker 2006). Using a chat tool which enforces one of a number of network topologies, members of each team will discuss a series of verbally-expressed constraints on the assignment of a set of hypothetical platforms to a set of hypothetical areas of operation. The wording of the constraints expresses a series of shades of meaning. By varying the network topology between team members, we hope to explore the relationship between decision times and average distance, and shed light on the question raised above.



Figure 8. Distributed team decision and chat tools for the verbal assignment problem.

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