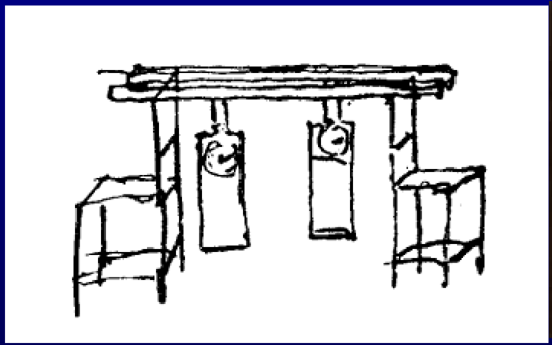
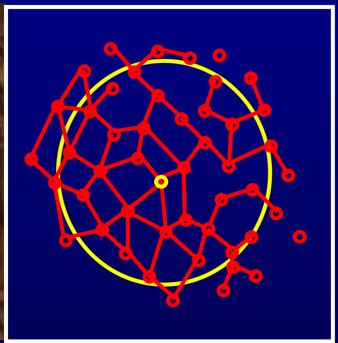
Phase synchronization of an ensemble of weakly coupled oscillators: A paradigm of sensor fusion





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15th ICCRTS

Santa Monica, CA, June 22-24, 2010

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Introduction

Why are we interested in synchronization ?

- Motivation: try to understand mechanisms of sensor fusion
- Emergent behavior: What is it and when does it occur?
- Simple oscillators provide a good model for studying these
- We focus on phase synchronization as a basic mechanism for inducing co-operative behavior
- Is it possible to extend the paradigm to real applications, *e.g.* in modelling military sensor networks ?

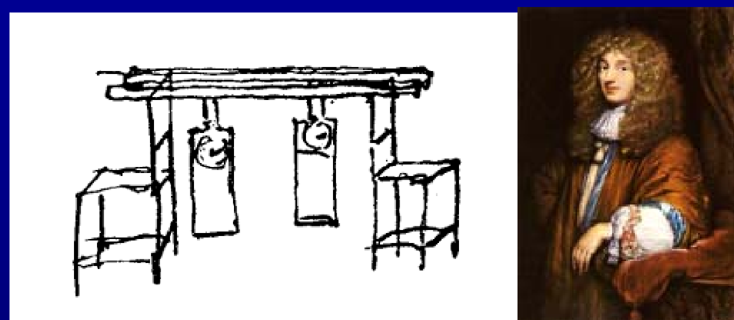
- Systems studied:
  1. Non-linear coupling of 2 linear oscillators
  2. Non-linear coupling between N linear oscillators
  3. Linear coupling of 2 non-linear oscillators

Examples of synchronization processes

- Biology: fireflies, yeast, algae, crickets
- Physiology: heart, brain, biological clocks, ovulation cycle
- Chemistry: chemical clocks
- Engineering: Power grids, distribution of time (UTC)
- Communication requires synchronisation at all OSI layers
- Physics: coherence of lasers and masers, phase transitions, ferromagnetism, superconductivity, spin waves

- SHOW physics demo: 3 metronomes synchronization
- presentatie\Synchronization of Three Metronomes.MP4

Synchronization



- History: Christiaan Huygens (Feb 1665) "Sympathie des horloges"
  - 2 pendulum clocks suspended from the same beam will in a relatively short period assume the same rythm if they are initially out-of-phase; they will eventually synchronize and lock in antiphase !
- Constant phase difference between 2 oscillations:

$$\Delta\dot{\varphi} = 0 \Rightarrow \Delta\varphi = constant$$

- Only possible if both have the same frequency:

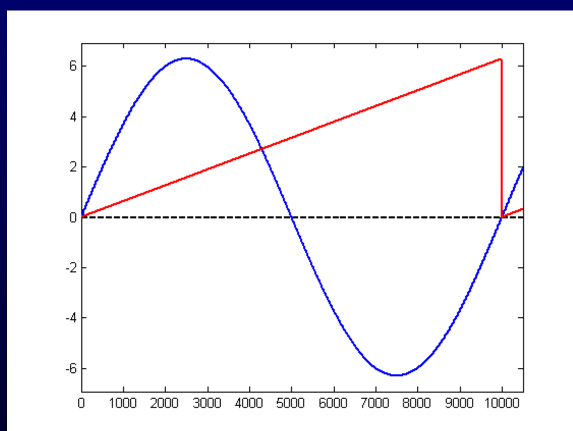
$$\Delta\dot{\varphi} = 0 \Rightarrow f_1 = f_2$$

- Amplitude of oscillator can be chaotic

The definition of phase

- How can “phase” be defined for an arbitrary, periodic signal?

- Different ways to define momentary phase:
  - For a simple sine — $\varphi \in [0, 2\pi]$
  - For a periodic function — phase plane; Poincaré map
  - For a complex oscillator — Hilbert transform



$$x = x(t) \quad x \in \mathbb{R}$$

$$y(t) = \mathcal{H}[x] \equiv \frac{1}{\pi} P \int_{-\infty}^{\infty} \frac{x(\tau)}{t-\tau} d\tau$$

$$z(t) = x(t) + iy(t) = r(t) e^{i\varphi(t)} \quad z \in \mathbb{C}$$

$$\tan(\varphi(t)) = \frac{y(t)}{x(t)}$$

Two coupled linear oscillators (1)

- 2 oscillators, each with its own eigenfrequency $\omega_{1,2}$:

$$\frac{d}{dt}\varphi_1 = \omega_1 + U_{12}(\varphi_1 - \varphi_2)$$

$$\frac{d}{dt}\varphi_2 = \omega_2 + U_{21}(\varphi_2 - \varphi_1)$$

- with nonlinear interaction $U_{12}(\vartheta)$ dependent on the phase difference $\vartheta = \varphi_1 - \varphi_2$

so that

$$\frac{d\vartheta}{dt} = \Delta\omega + u(\vartheta)$$

with

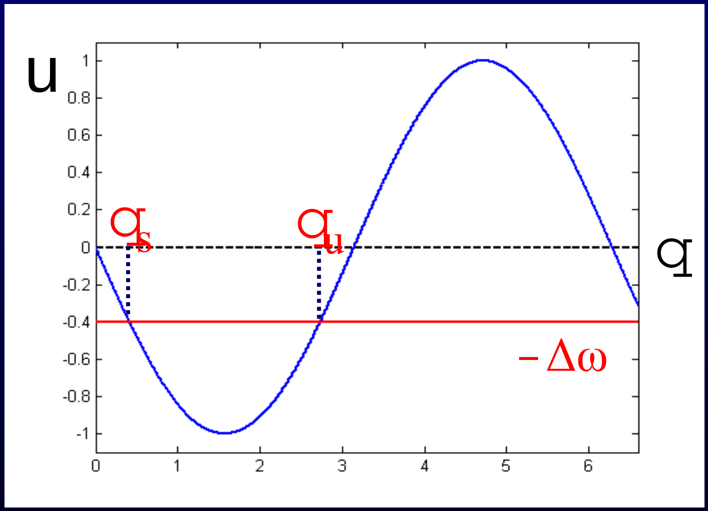
$$\vartheta = \varphi_1 - \varphi_2$$
$$\Delta\omega = \omega_1 - \omega_2$$
$$u(\vartheta) = U_{12}(\vartheta) - U_{21}(-\vartheta)$$

Two coupled linear oscillators (2)

- If synchronization occurs, we have: $\frac{d\vartheta}{dt} = 0 \Rightarrow \boxed{-\Delta\omega = u(\vartheta)}$
- So if real roots for this algebraic equation exist, we have found synchronous solutions !
- A an example we take $u(\vartheta) = -\varepsilon \sin\vartheta$ and find the graphical solutions of

$$\frac{d\vartheta}{dt} = \Delta\omega - \varepsilon \sin\vartheta = 0$$

- ϑ_s is stable and ϑ_u unstable



Two coupled linear oscillators (3)

- Synchronization occurs when $\frac{d\vartheta}{dt} = 0 \Rightarrow \boxed{-\Delta\omega = u(\vartheta)}$

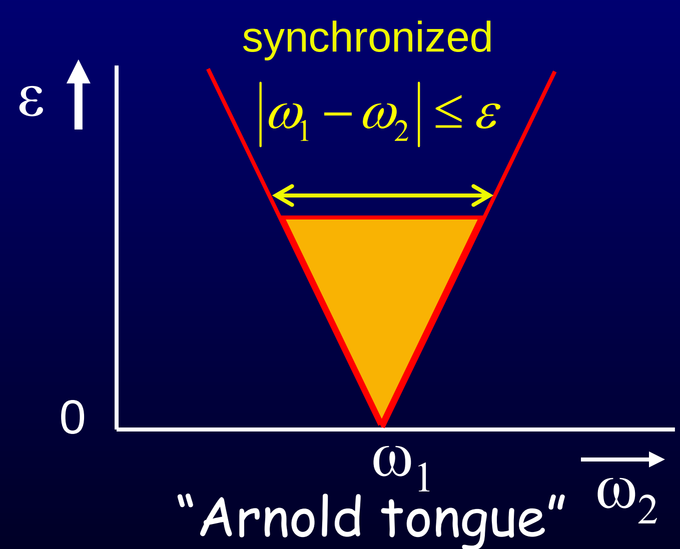
- This algebraic equation only has real roots *iff* the difference in eigenfrequencies $\Delta\omega$ lies within the interval of values of the function $u(\vartheta)$:

$$u_{\min} \le \Delta\omega \le u_{\max}$$

- In our example we have:

$$u(\vartheta) = -\varepsilon \sin\vartheta$$

$$|\Delta\omega| \le \varepsilon$$



Two coupled linear oscillators (4)

- The common synchronization frequency Ω of the two coupled oscillators follows from:

$$\Omega = \omega_1 + U_{12}(\vartheta_0) = \omega_2 + U_{21}(-\vartheta_0)$$

where ϑ_0 is the phase difference from the stable graphical solution

- Outside the entrainment region the motions are <u>not</u> synchronous, but they can still influence each other significantly. (-> phase slips)

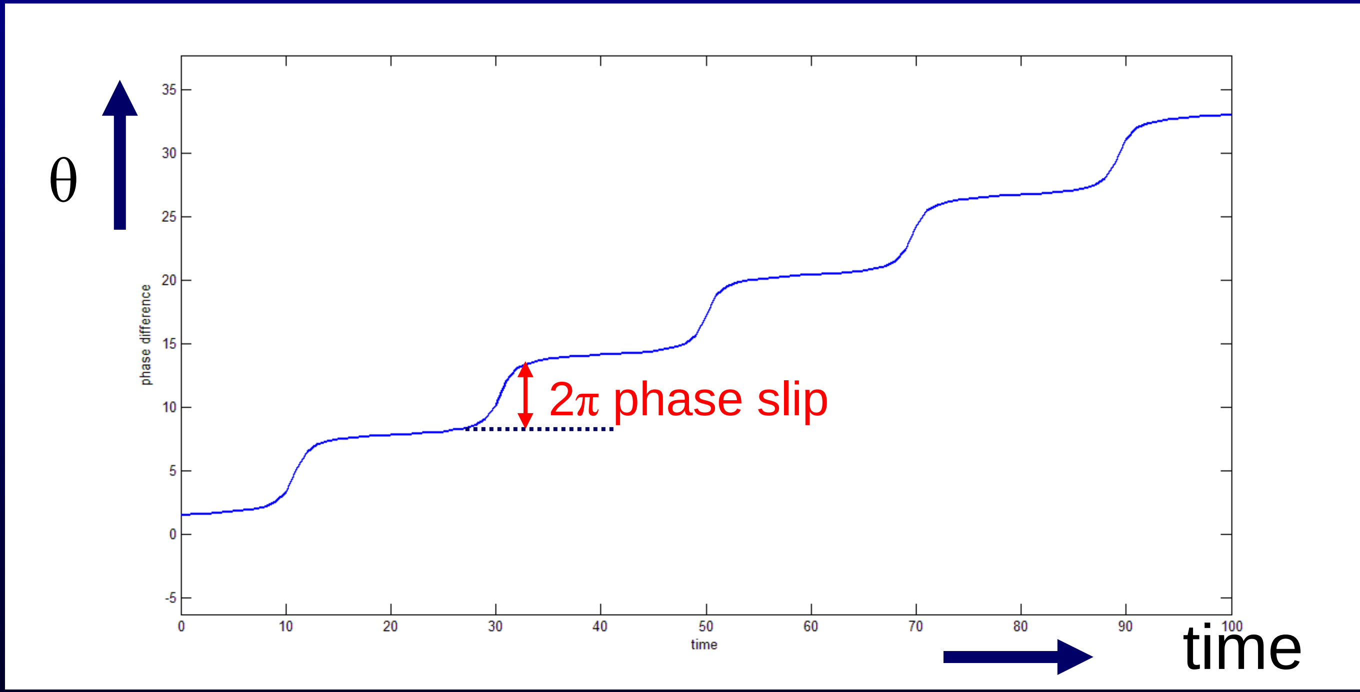
As an example take $U_{12}(\vartheta) = -U_{21}(-\vartheta) = -\frac{1}{2}\varepsilon \sin\vartheta$

so that $$\frac{d\vartheta}{dt} = \Delta\omega - \varepsilon \sin\vartheta$$

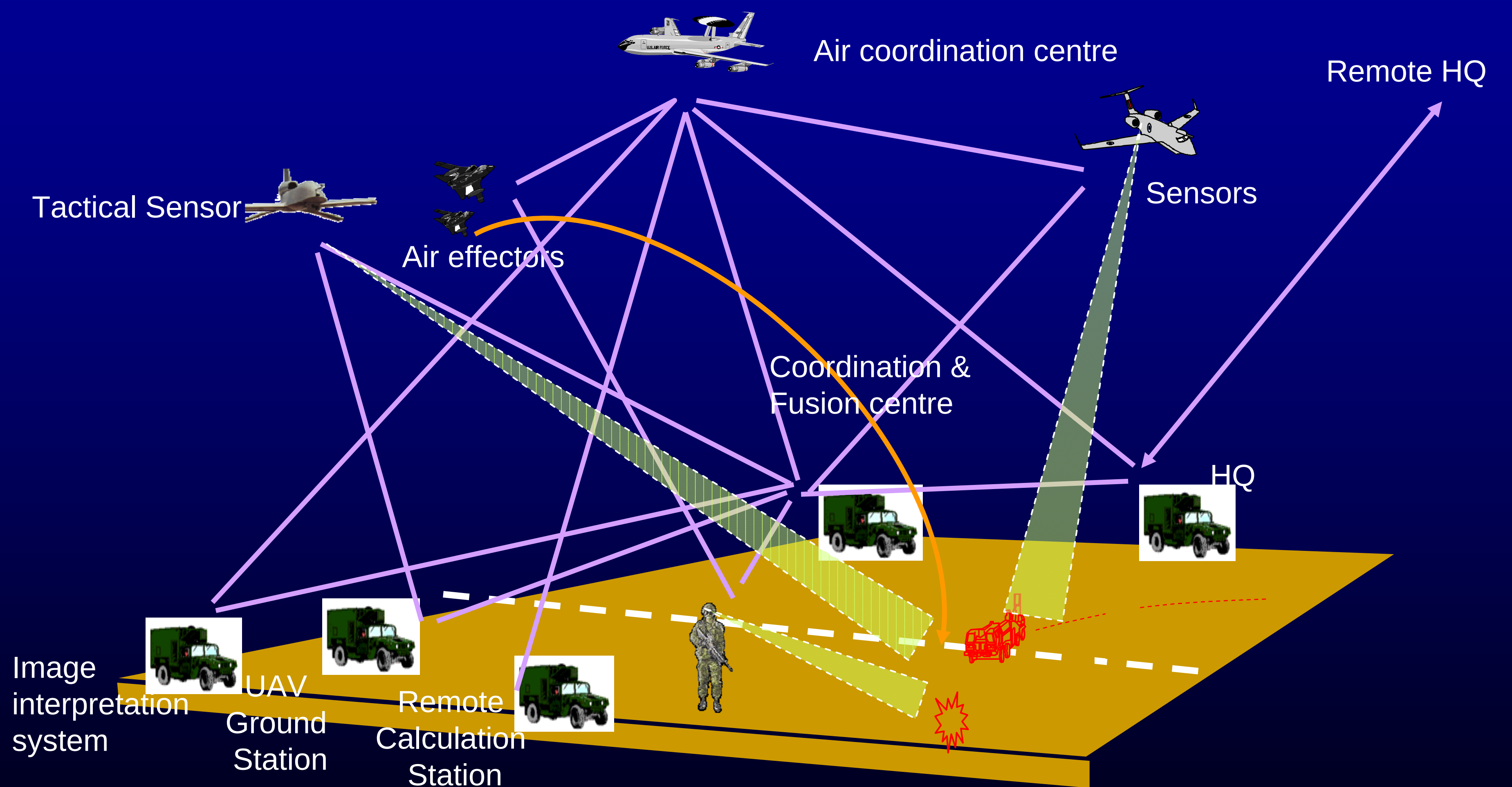
with solution $$\vartheta(t) = 2\arctan\left[\frac{\varepsilon}{\Delta\omega} + \sqrt{1 - \frac{\varepsilon^2}{\Delta\omega^2}}\tan\left(\tfrac{1}{2}\sqrt{\Delta\omega^2 - \varepsilon^2}\,t\right)\right]$$

Two coupled linear oscillators (5)

- Time dependence of phase difference $\vartheta = \varphi_1 - \varphi_2$ outside the region of synchronization. $\varepsilon = 1 \quad \Delta\omega = 1.05 \quad \Delta\omega > \varepsilon$



NEC in inhomogeneous networks



Modeling a homogeneous sensor network

- Homogeneous network of N nodes ("agents") acts as a distributed sensor (or detector)
- Homogeneous networks are part of typical NEC networks
- Node composition: (analog) sensor, memory, decision taking
- Mathematical modelling: Node = oscillator
- Observable determines the oscillator frequency: Node = parametric oscillator (or VCO ?)
- Contact between nodes through non-linear coupling K
- Study the dynamic behavior of the ensemble of N coupled oscillators

Why is synchronization important ?

- Sensor network: each member of the population is represented by a phase oscillator
- Synchronization on physical layer, not on protocol layer: faster and more accurate
- Greater robustness, fault tolerance, scalability, small complexity self-synchronization
- Ultimate goal: local information storage, propagation of information, distributed, “soft” decision taking
- Redistribution of mobile sensors to more effectively sample the environment in presence of measurement noise
- Propagation and fusion of analog information without a central fusion master

N linear oscillators

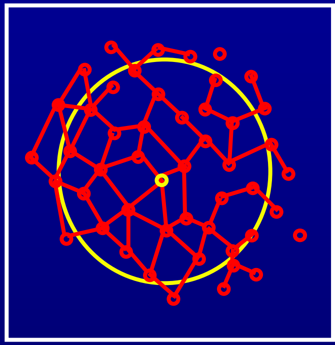
- Kuramoto model
  - ensemble of N nearly identical oscillators
  - symmetric distribution of eigenfrequences $g(\omega) = g(-\omega)$
  - global coupling strength $K \geq 0$
  - evolution of oscillator phase given by

$$\frac{d\vartheta_k(t)}{dt} = \omega_k + \frac{K}{N}\sum_{j=1}^{N}\sin(\vartheta_j(t) - \vartheta_k(t)) \quad (k = 1,\ldots,N)$$

- Stationary synchronization (mean-field approximation)
  - complex order parameter r: $re^{i\varphi} = \frac{1}{N}\sum_{k=1}^{N} e^{i\vartheta_k}$

$$\frac{d\vartheta_k(t)}{dt} = \omega_k + Kr\sin(\psi - \vartheta_k) \quad (k = 1,\ldots,N)$$

Sensors acting as detectors



- Distributed, dense sensor network
- Detection as a stochastic process:

  $\omega_i = \Omega_1$ if an event is detected

  $\omega_i = \Omega_0$ if no event is detected
- Probability of detection p_0
- If the network is sufficiently large the phase rate converges to ω^* :
- $\vartheta_j(t) = \omega^* t + \vartheta_j(0)$

$$\frac{d\vartheta^*}{dt} = \omega^* = \frac{\sum_{k=1}^{N} c_k \omega_k}{\sum_{k=1}^{N} c_k} \equiv p_0 \Omega_1 + (1 - p_0)\Omega_0$$

$$\vartheta_j(0) = \begin{cases} \Theta_0 = \arcsin\left(\dfrac{\omega^* - \Omega_0}{Kr}\right) & with \quad probability = 1 - p_0 \\ \Theta_1 = \arcsin\left(\dfrac{\omega^* - \Omega_1}{Kr}\right) & with \quad probability = p_0 \end{cases}$$

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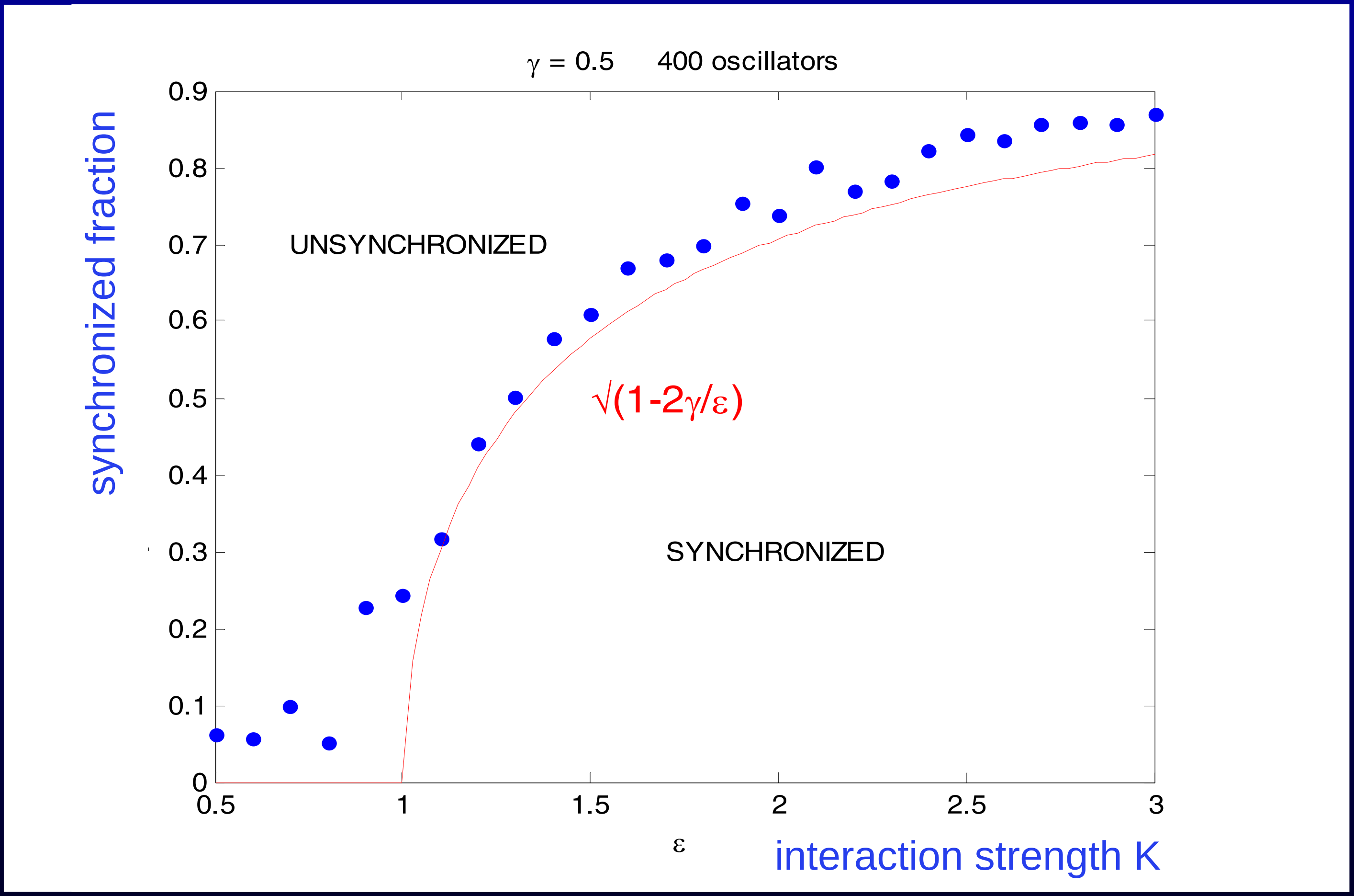
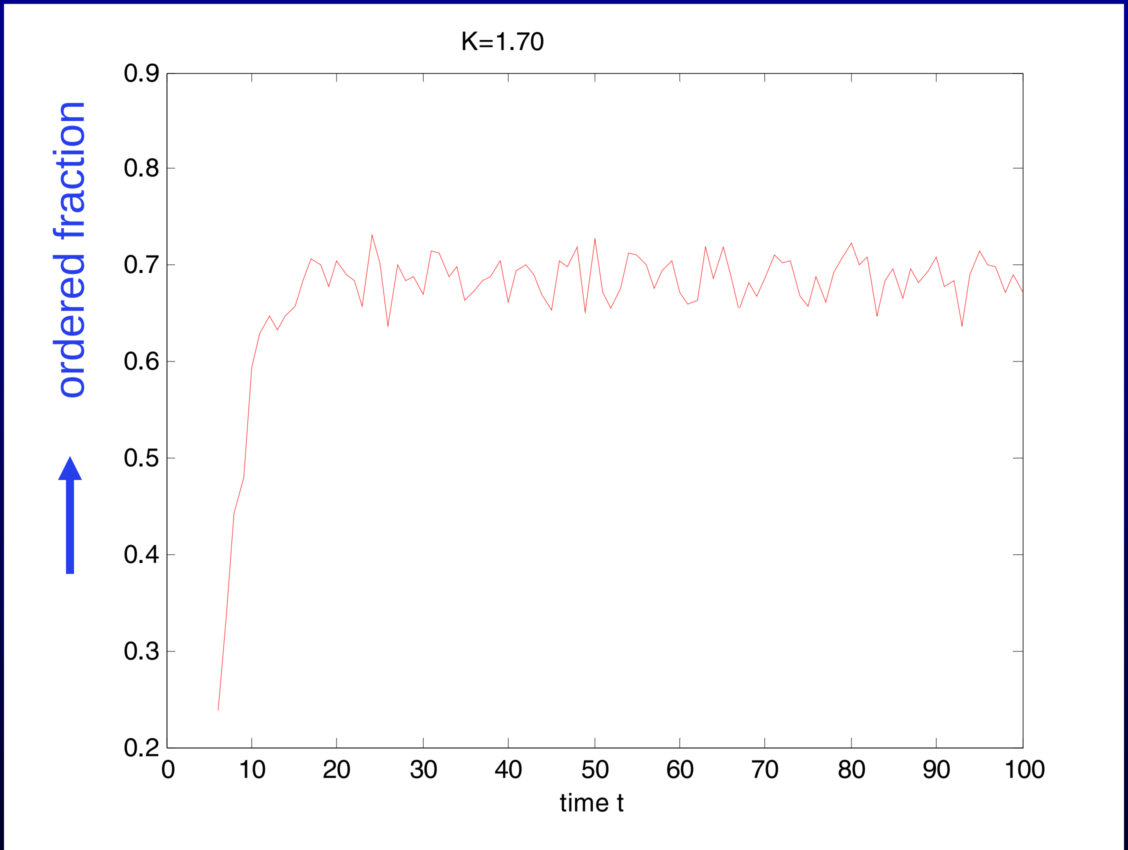
Synchronisation phase diagram N=400 oscillators



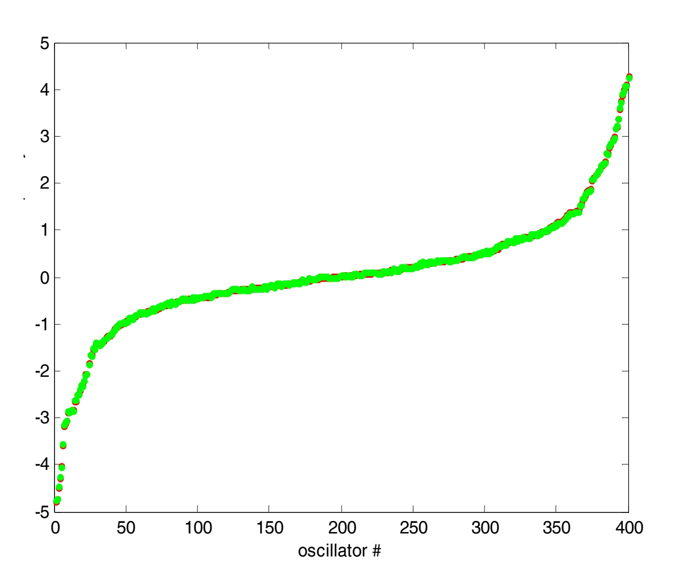
How fast is synchronization for N=400 ?

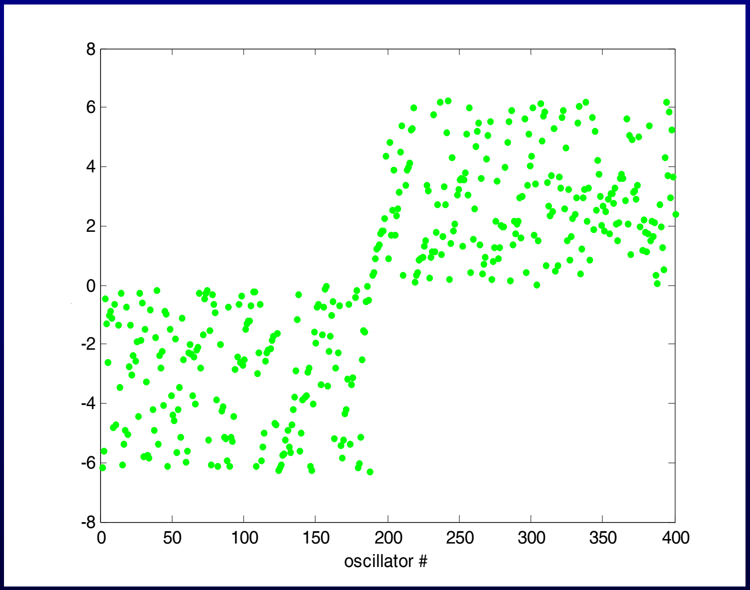
Integration time step = 0.01



Results of simulations N=400

- presentatie\freq N=400.avi
- presentatie\phase N=400.avi





frequency

phase

Synergy

- The basic notion of synergy is:

$$g(A \cup B) \geq g(A) + g(B)$$

- or at least:

$$g(A \cup B) \geq \max(g(A), g(B))$$

- Non-lineartity is an essential ingredient for understanding sensor data fusion !
- Classical approach via Bayesian networks, DS theory and/or fuzzy (belief and plausability) measures
- In the present study we focus on a different approach: the paradigm of phase transitions in physics

$$\exists \lambda > -1 \;\; g(A \cup B) = g(A) + g(B) + \lambda g(A) g(B)$$

Conclusions

- Nonlinear coupling of 2 linear oscillators results in very fast phase synchronization, provided that the interaction is strong enough.
- Synchronization of 2 nonlinear oscillators occurs already at very weak coupling.
- In a non-linear globally interacting many-particle system we observe spontaneous (partial) synchronization above a critical interaction strength.

The fast and spontaneous synchronization of globally interacting systems is a form of emergent behavior and may be exploited as a mechanism for military smart sensor networks.