



# Distributed Auction Algorithm for the Assignment Problem with Partial Information

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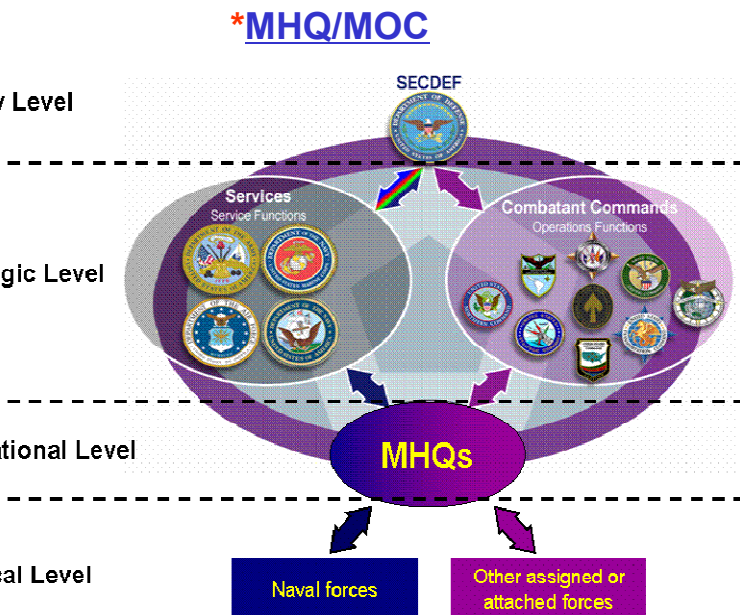
# Introduction



## Objective & Motivation

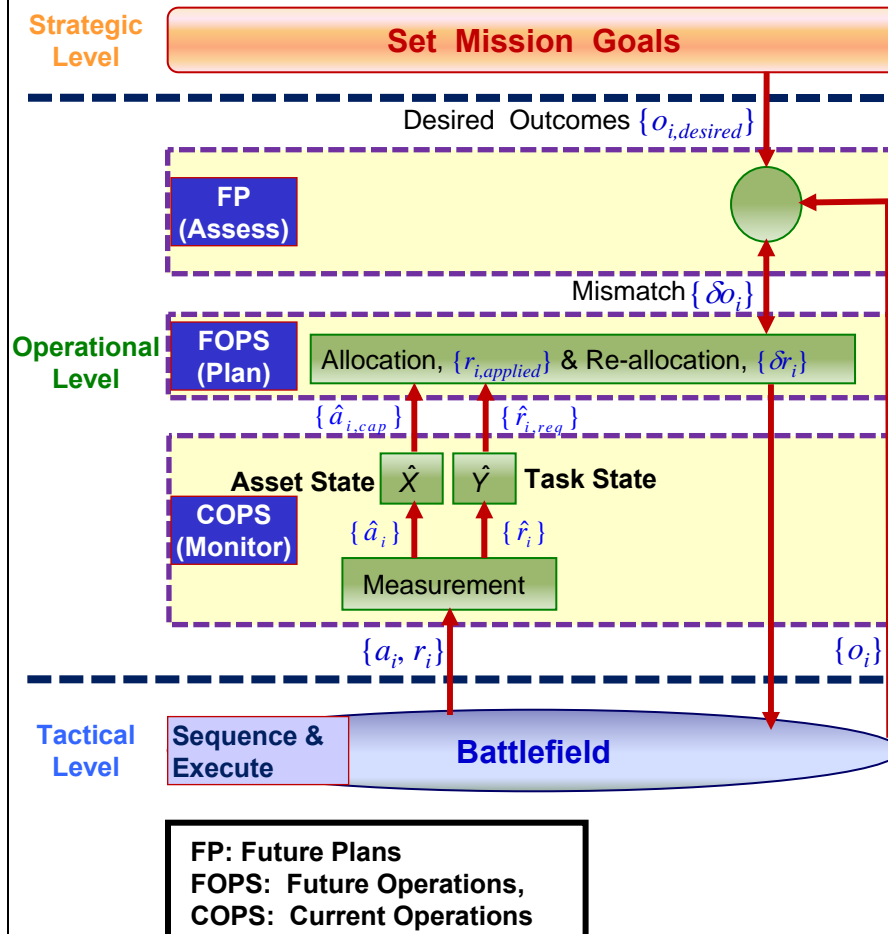
Develop analytical and computational models for multi-level **collaborative mission planning** and monitoring processes associated with Maritime Operations Centers (MOC\*)

In MOC, multiple DMs with **partial information** and **partial control over** assets are involved in the development of operational level plans



## Collaborative Mission Planning Problem

- In the collaborative planning problem, each DM “owns” a set of assets and is responsible for planning certain tasks



# Task-Asset Assignment Problem



## Task and Asset States

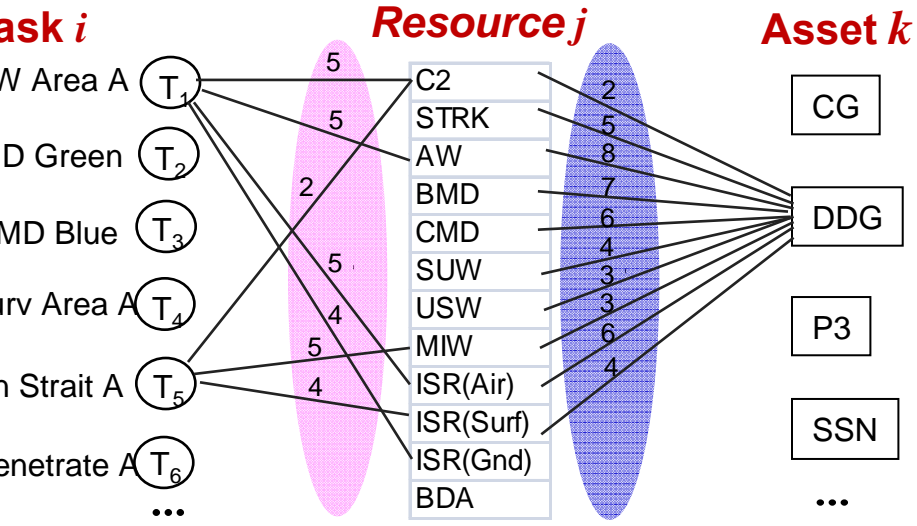
Task Requirements

$$r_{ij}(\underline{\hat{y}}(t))$$



Asset Capabilities

$$a_{jk}(\underline{\hat{x}}(t))$$



- $r_{ij}$ : Requirement  $j$  of task  $i$ , given the task state  $\underline{\hat{y}}(t)$  at time  $t$
- $a_{jk}$ : Asset Capability  $j$  of asset  $k$  given the asset state  $\underline{\hat{x}}(t)$  at time  $t$

- The degree of match between the task-resource requirement vector and asset-resource capability vector determines the accuracy of task execution
- The distributed assignment problem with partial information considered next is a simplified and abstracted version of the collaborative planning problem

Legend	Description	Legend	Description
AEW	Airborne early warning	USW	Undersea warfare
AMD	Theater air/missile defense	BDA	Battle damage assessment
MIW	Mine warfare	ISR	Intelligence, surveillance and reconnaissance
C2	Command and control	CVN	Nuclear aircraft carrier
STRK	Strike	CG	Guided-missile cruiser
AW	Air warfare	DDG	Guided-missile destroyer
BMD	Ballistic missile defense	P3	Ant-submarine aircraft
CMD	Command	SSN	Nuclear submarine
UW	Surface warfare		

# Auction Algorithm for Assignment Problem



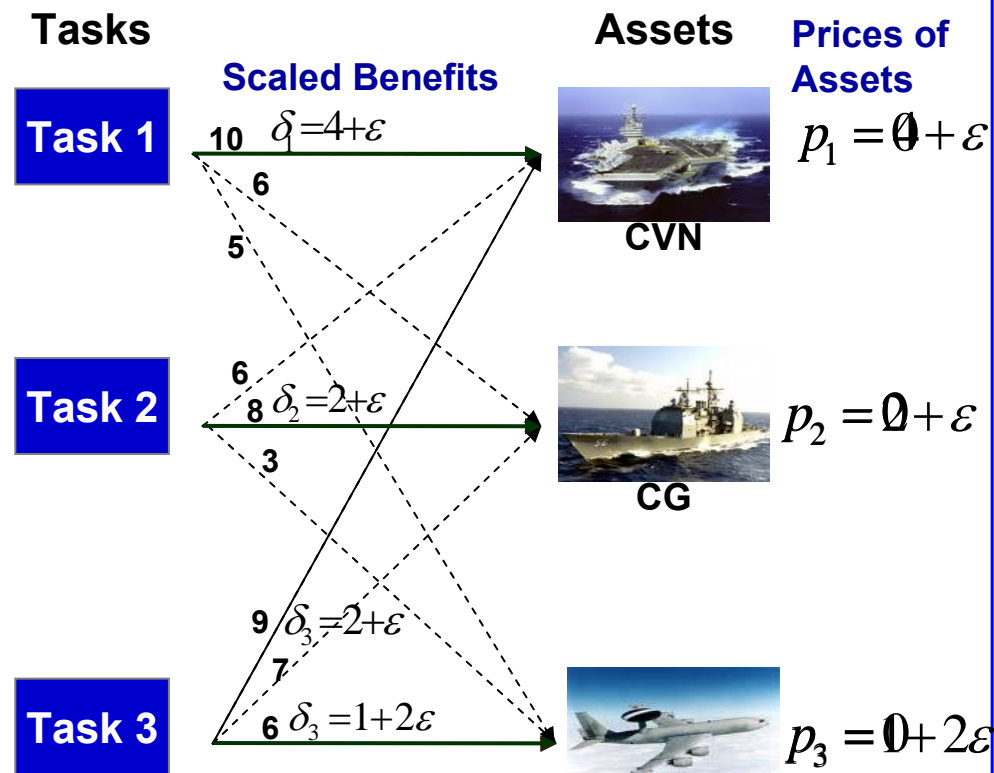
## Auction Algorithm for Assignment Problem

- Objective:** Match  $n$  tasks to  $m$  assets to maximize a benefit function
- There is a benefit matrix  $A = [a_{ij}]$ ;  $a_{ij}$  = benefit of assigning asset  $j$  to task  $i$
- When  $n=m$ , it is called a symmetric assignment problem. Otherwise, asymmetric

## Auction Algorithm Process

**Bidding phase:** Each unassigned task  $i$  bids for an asset  $j$  with the best profit ( $=a_{ij} - \text{price of asset } p_j$ ) with a bid  $d$  equal to  $(a_{ij} - 2^{\text{nd}} \text{ best profit} + \epsilon)$ ;  $\epsilon < 1/n$

**Assignment phase:** Each asset is assigned to the highest bidder (task) and adjusts its price to the highest bid

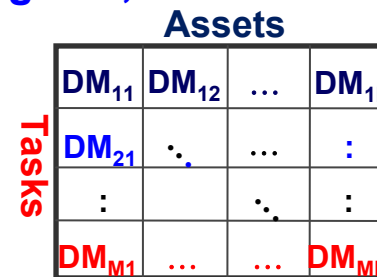
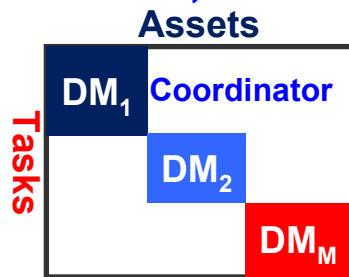
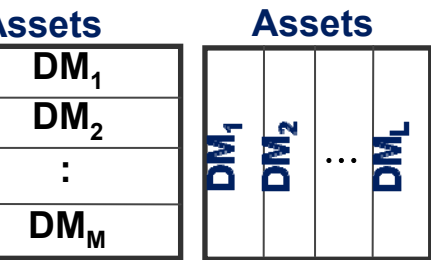




# Novel Distributed Assignment Problems

**Objective:** Match  $n$  tasks to  $m$  assets to maximize a benefit function when DMs have only **partial information** of benefit matrix,  $A$

**Information Structures:** Horizontal, Vertical, Block diagonal, Checkerboard



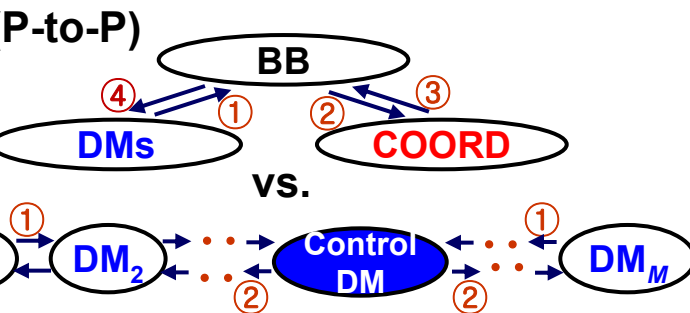
**Column info:** Each DM has a set of tasks (assets), but knows benefits for all assets

**Block diagonal info:** Each DM knows benefits for own task-asset pairs; Coordinator knows the rest

**Checkerboard info:** Each DM knows benefits for own task-asset pairs

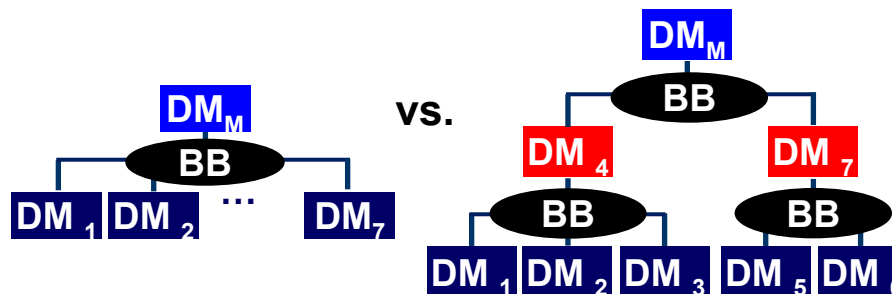
**Communication Structures:**

**Blackboard (BB) vs. Point-to-point**



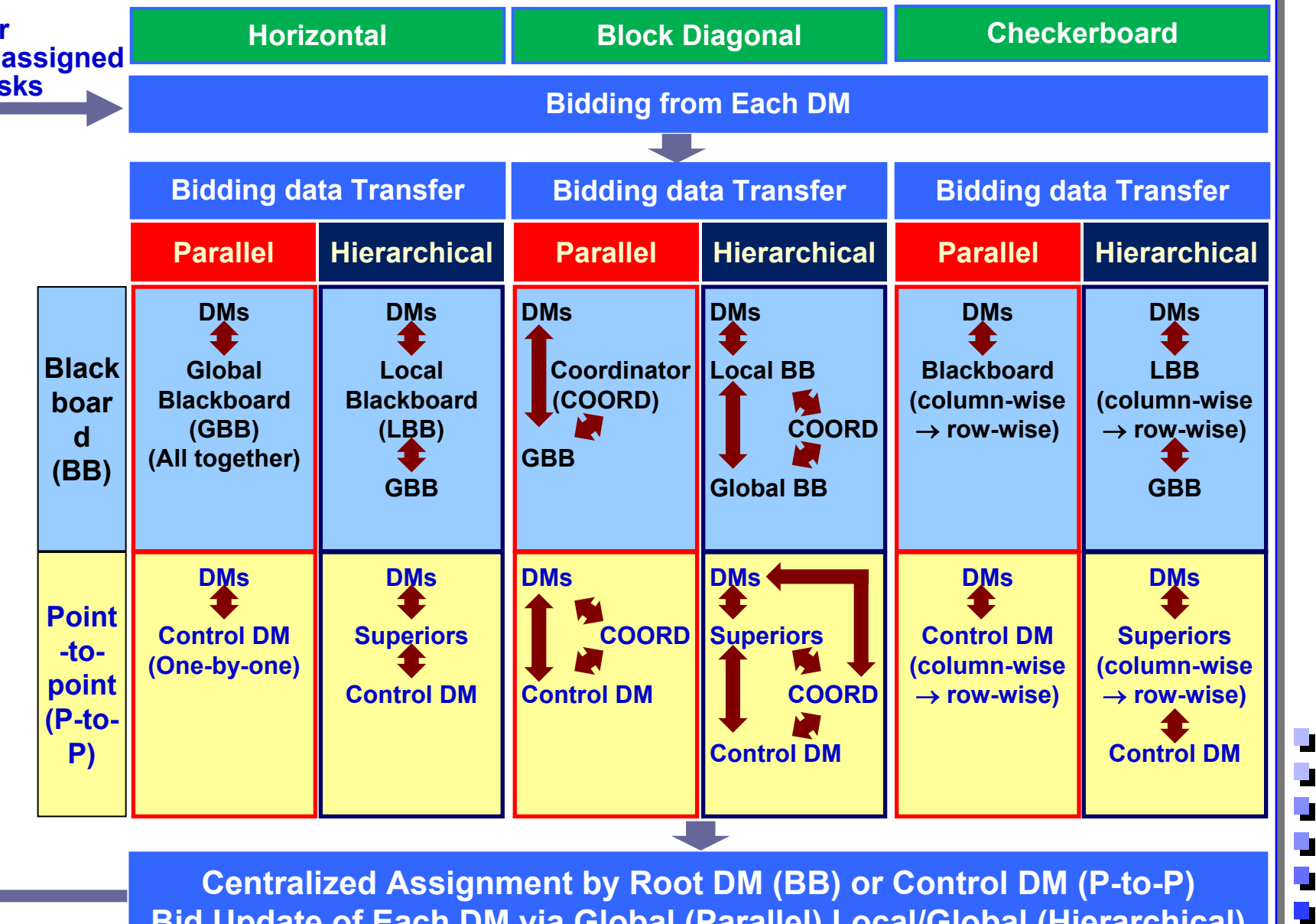
**Organizational Structures:**

**Parallel vs. Hierarchical tree**



**Key Idea:** Can construct the centralized assignment solution by transferring bids and the best as well as 2<sup>nd</sup> best profits toward the root DM (for BB) or the control DM (for P-to-P) even if DMs have **partial information**

# Distributed Auction Algorithms



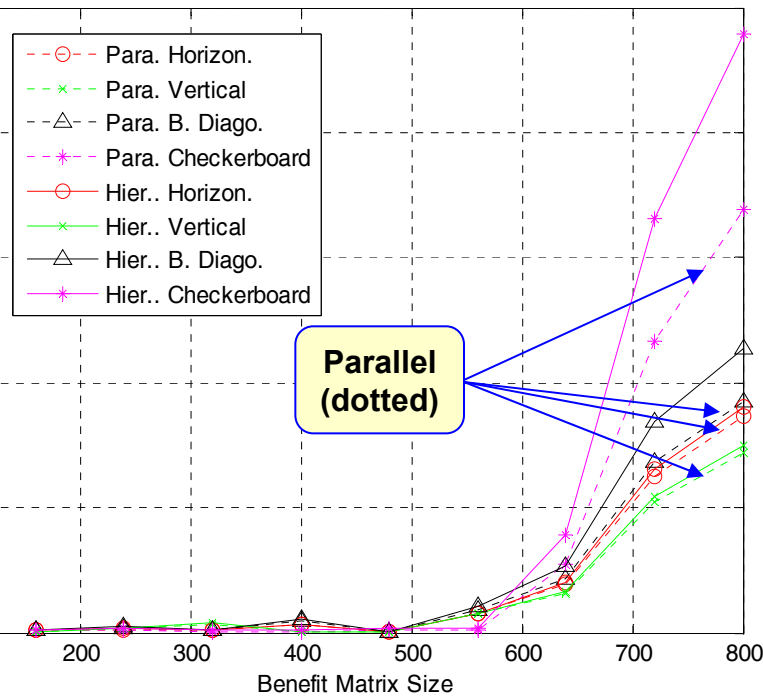
# Performance of Distributed Auction Algorithm



## Simulation Results (Delays) for Combination of Structures

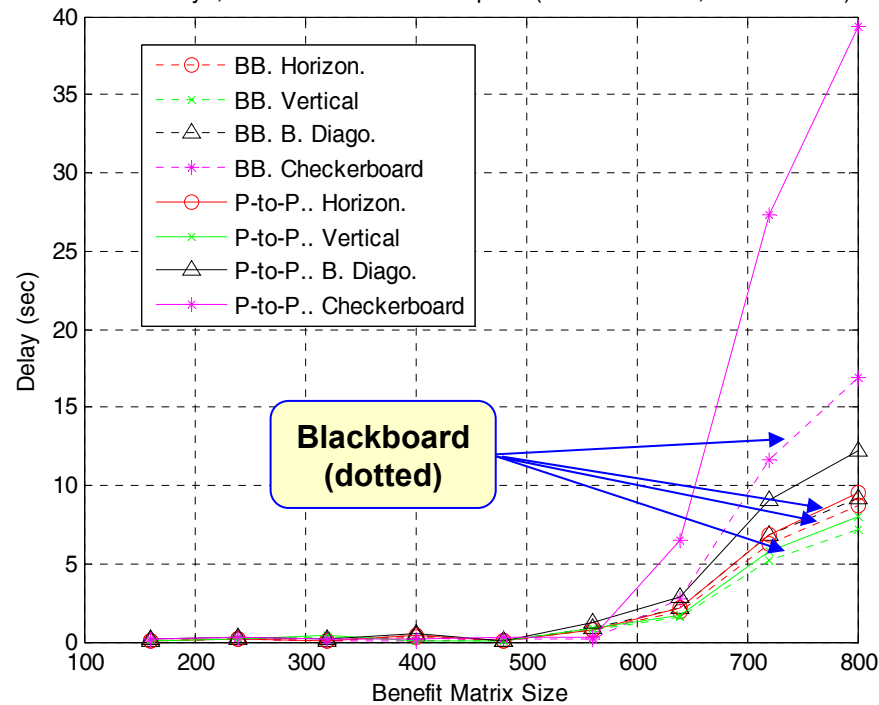
### Parallel vs. Hierarchical

Total Delays, Parallel vs. Hierarchical (No. of DMs = 5; 100 MC runs)



### Blackboard vs. Point-to-point

Total Delays, Blackboard vs. Point-to-point (No. of DMs = 5; 100 MC runs)



Parallel structure is 1.1 ~ 1.5 times faster than Hierarchical tree structure

Blackboard structure is 1.2 ~ 2.3 times faster than Point-to-point structure

Horizontal/Vertical structures have better performance than other structures

Checkerboard structure (akin to a matrix structure) shows the worst performance: Significant coordination delays and overlap among DMs

Block diagonal structure shows reasonable performance: Coordinator resolves row-wise (divisional) & column wise (functional) conflicts

Parallel structure with horizontal/vertical information structure is consistent with edge organizations





# Summary and Future Work



## Summary

- Distributed auction algorithms with various information (“who knows what”), communication (“who communicates with whom”) and organizational (“who controls whom”) structures
- By posting the bid, the best and the second best profits to the blackboard, the DMs can reconstruct the centralized assignment solution
- The performance of various information structures was evaluated by comparing the delays involved in converging to a centralized solution

## Future Work

- Collaborative planning algorithms with partial information and partial control of assets
- Information and coordination structures to maximize organizational efficiency and be robust to a range of missions
- Multi-objective optimization techniques for resource allocation

