Shannon Entropy as a Metric of Situational Awareness in C^2 Structures

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Entropy

- Origins: Thermodinamics (Clausius;Kelvin,1850)
- Boltzmann: microscopic states

$$S \equiv k \ln \Omega$$

- (Shannon,1948):

$$E[-\log p_i] = -\sum_{i=1}^{n} p_i \log p_i = H(X)$$

Entropy

- **Properties**
  - **H(X) is limited**
  - **Joint Entropy**

$$H(X,Y) = -\sum_{i,j} p(x_i, y_j) \log p(x_i, y_j)$$

  - **Conditional Entropy**

$$H(X|Y) = E_Y[H(X|y_j)] = \sum_j p(y_j) H(X|y_j)$$

  - **Mutual Information**

$$I(X;Y) = H(X) + H(Y) - H(X,Y)$$

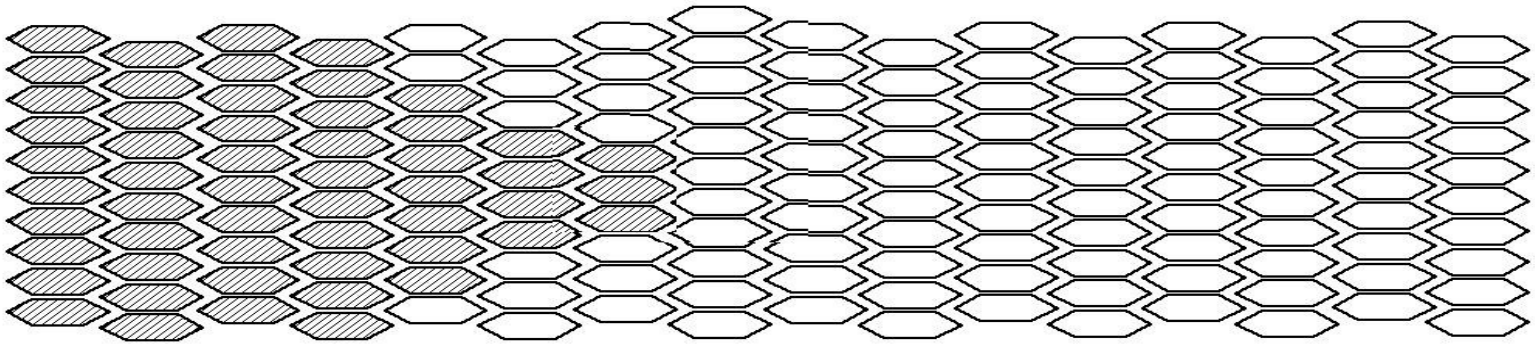
Entropy and Information in the Military Domain

- **(Sherrill;Barr,1997)**

$$P[T(j)|\neg I(j)] = \frac{(1 - p_D)p_j}{1 - p_D p_j}$$

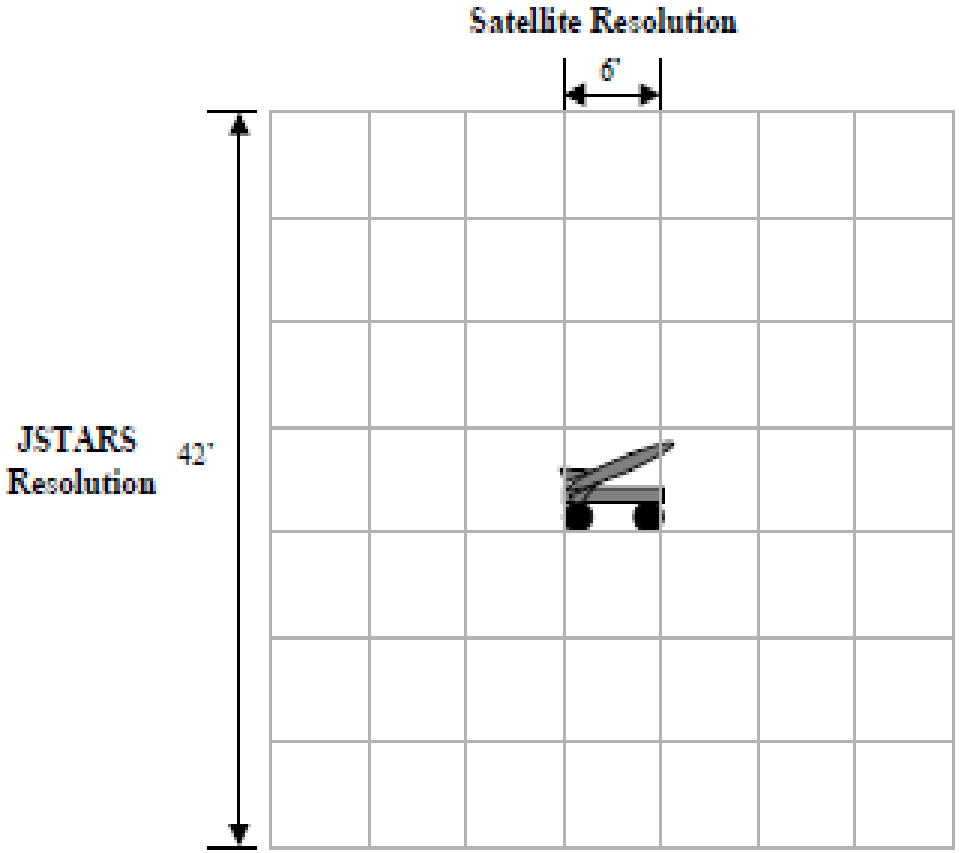
$$P[T(i)|\neg I(j)] = \frac{p_i}{1 - p_D p_j}, i \neq j$$

$$H(T, t) = -\sum_C p_t \log p_t.$$



Entropy and Information in the Military Domain

- (Beene,1998)



$$H(X) = -\sum_i p_i \ln(\frac{p_i}{A})$$

Entropy and Information in the Military Domain

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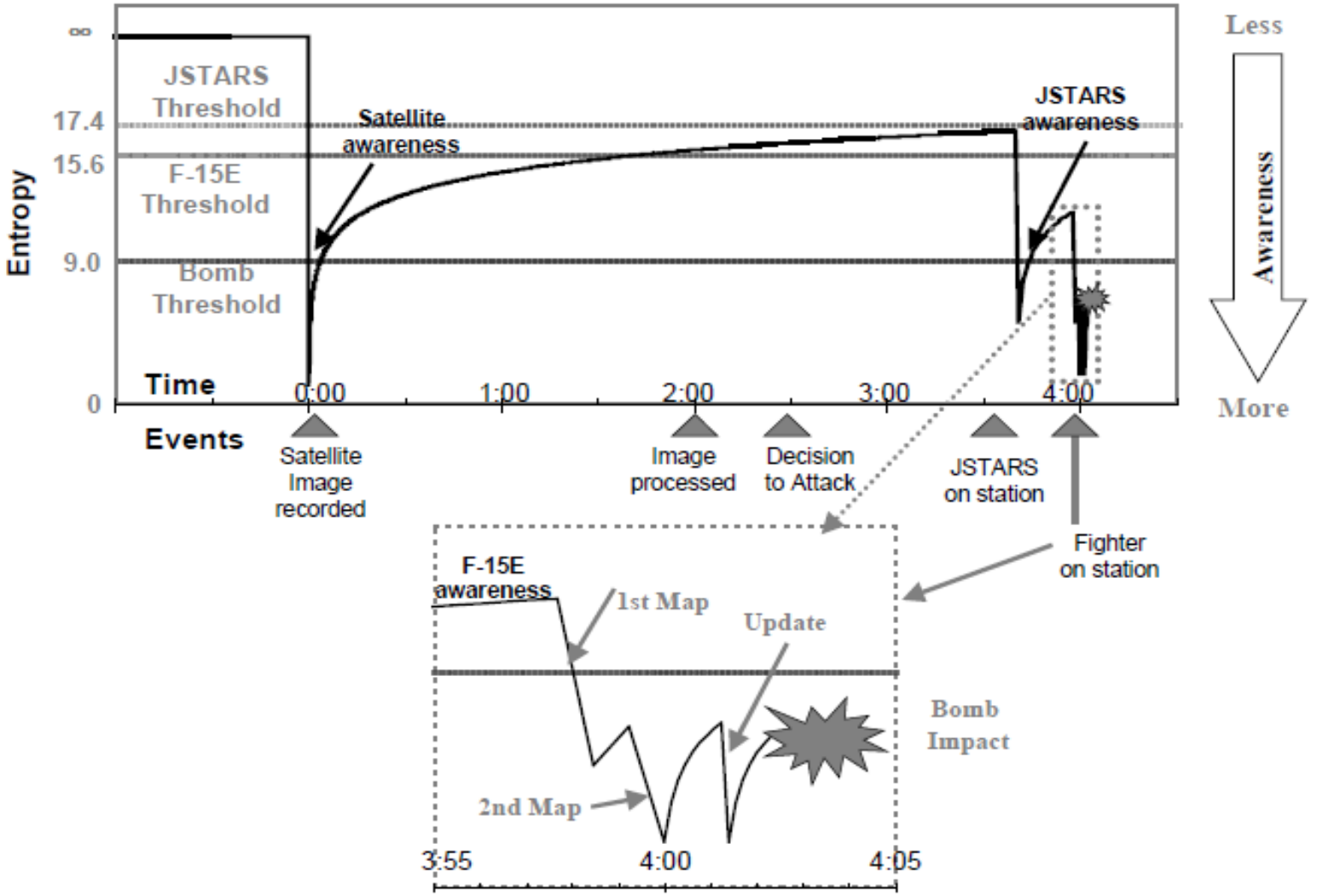

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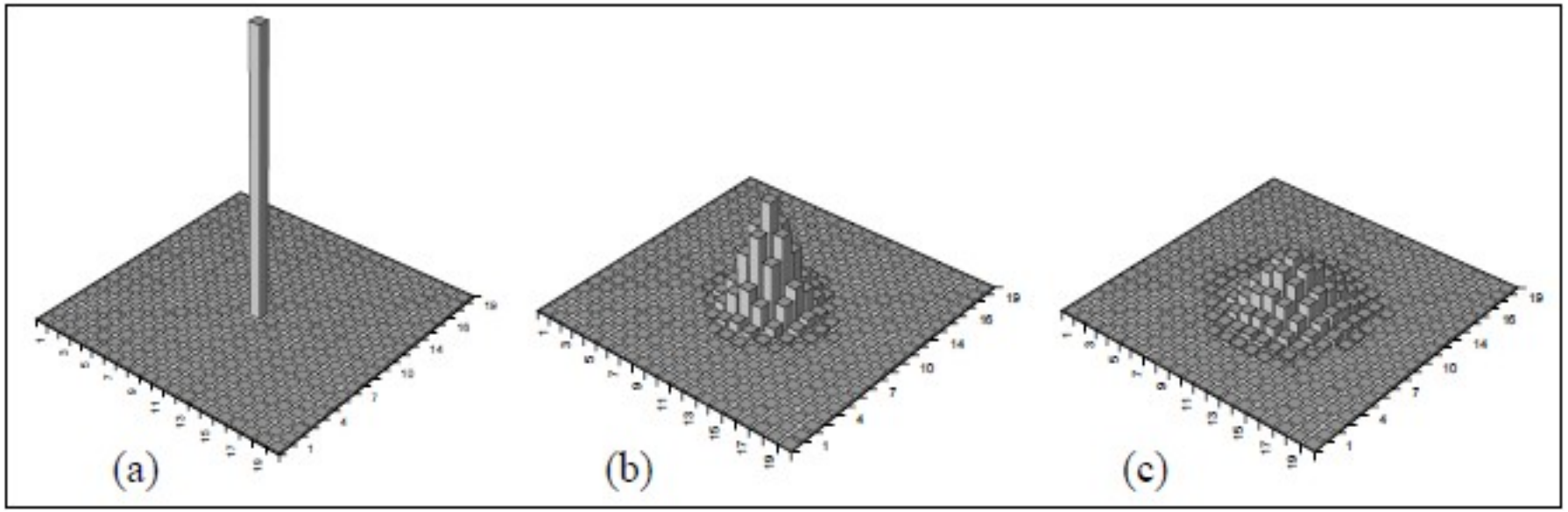
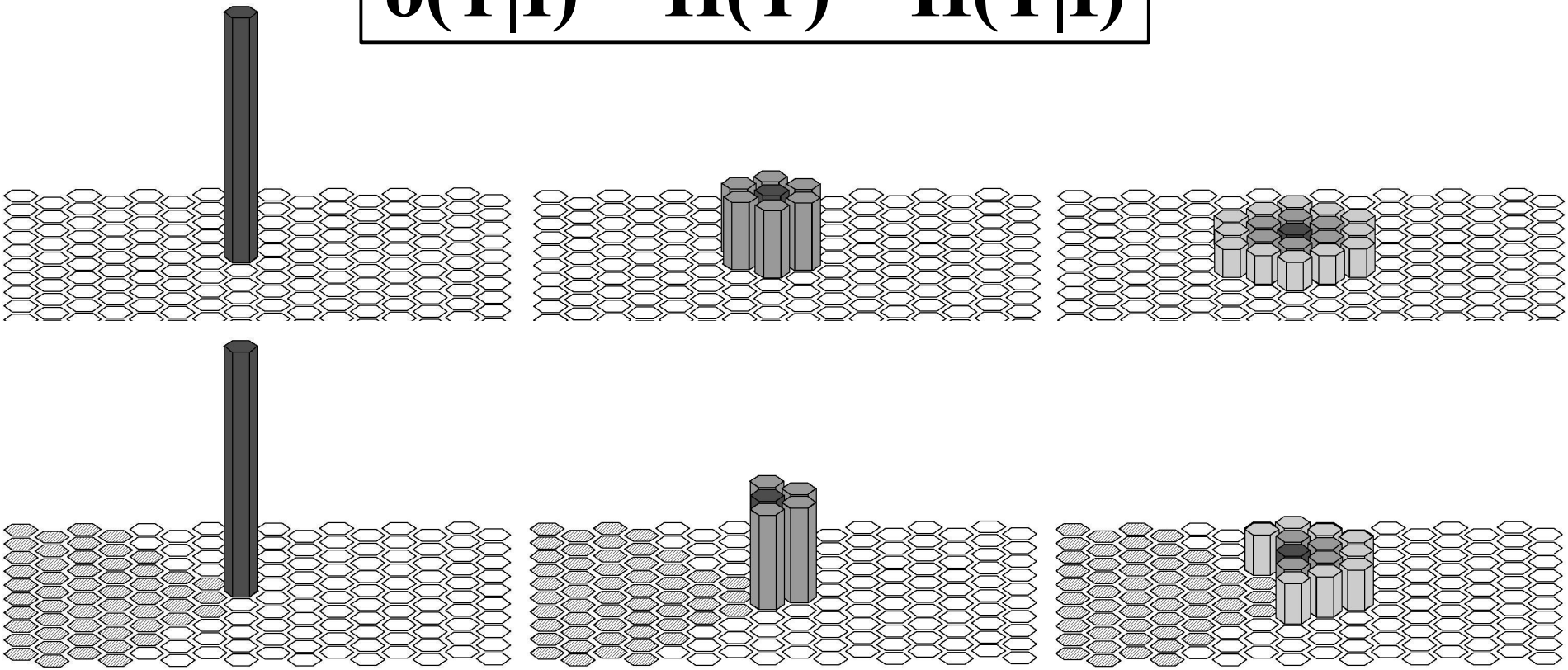


Time Steps	Four-direction model		Eight-direction model		Hexagonal cell model		Actual Area	
	Area	Entropy	Area	Entropy	Area	Entropy	Area	Entropy
100	20201	9.91	40401	10.61	26241	10.17	31416	10.36
1000	2.00×10^6	14.51	4.00×10^6	15.20	2.60×10^6	14.77	3.14×10^6	14.96

Metric 1: Information Gain

$$\delta(T|I) = H(T) - H(T|I)$$



Metric 2: Information Superiority

$$\delta(\text{BLUE}) = H_{MAX} - H(\text{BLUE}),$$
$$\delta(\text{RED}) = H_{MAX} - H(\text{RED})$$

$$S(\text{BLUE},\text{RED}) = \delta(\text{BLUE}) - \delta(\text{RED})$$

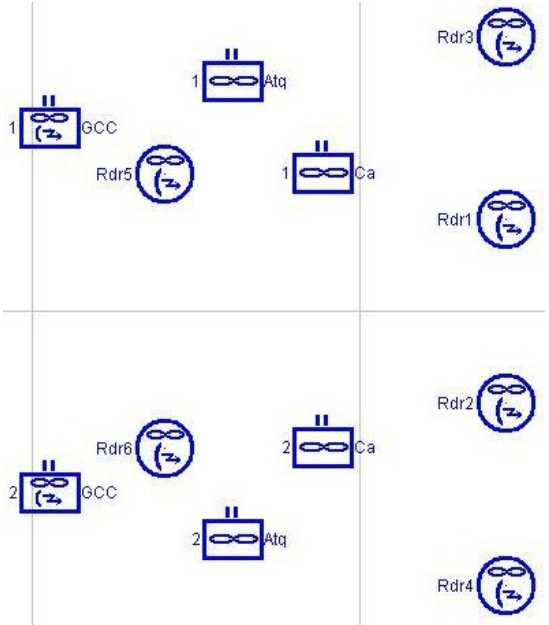
$$\boxed{S(\text{BLUE},\text{RED}) = H(\text{RED}) - H(\text{BLUE})}$$

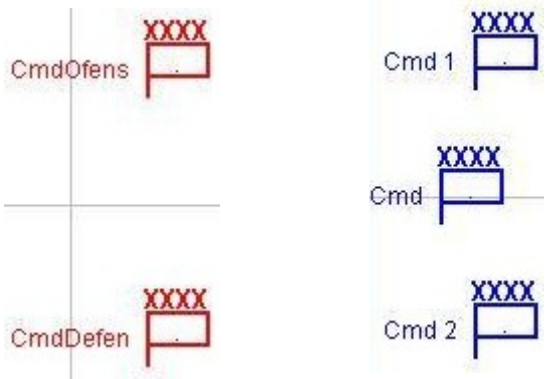
Example: Air Campaign

- Operations Area: 600NM x 300NM
- 2 Opposing Air Forces
  - Fighters and Strikers
  - Sensors
  - Comm Stations and
  - Command Centers
- Goal:
  - To study the relationship between performance, organizational structure and uncertainty.

Conceptual Model

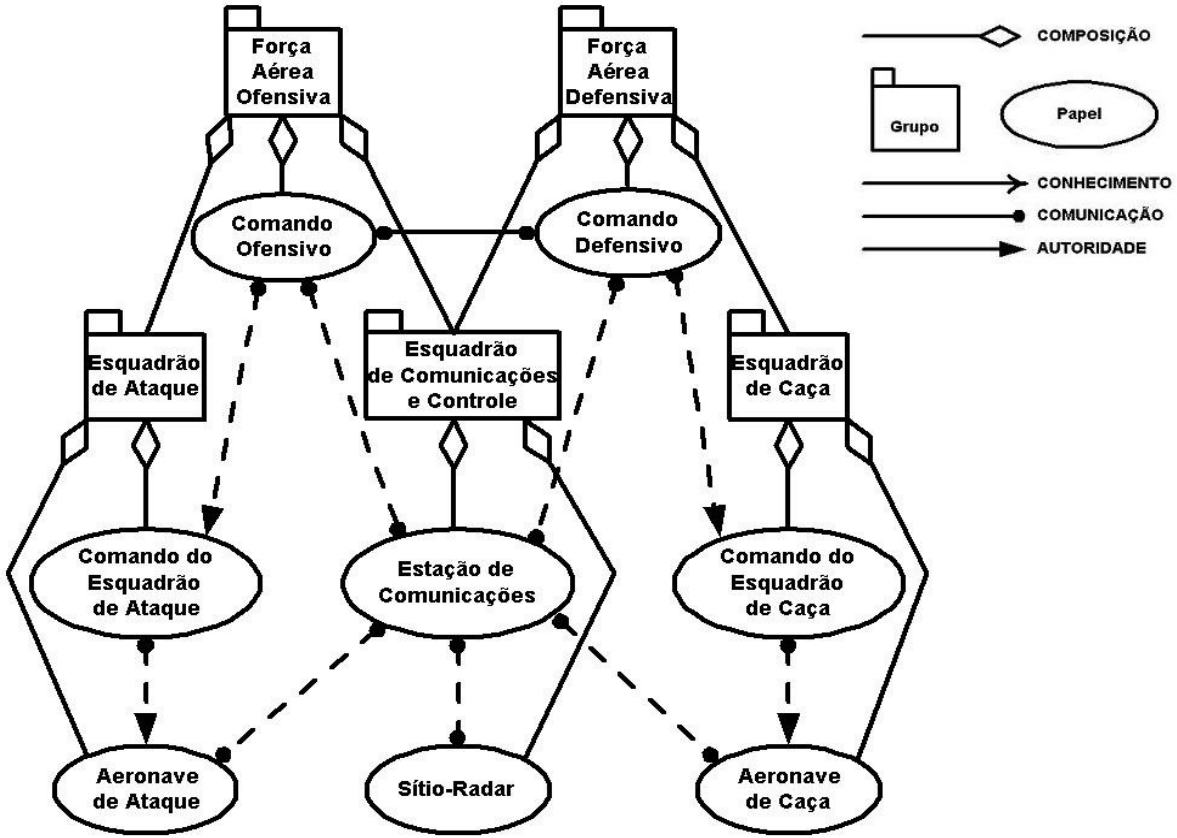
- Common Entities
  - Fighter Squadron Commands
  - Strike Squadron Commands
  - Comms Stations
  - Fighter Aircrafts
  - Strike Aircrafts
  - Radar Sites
- Centralized Organizational Structure
  - Attack Command
  - Defense Command
- Regional Organizational Structure
  - Regional Commands





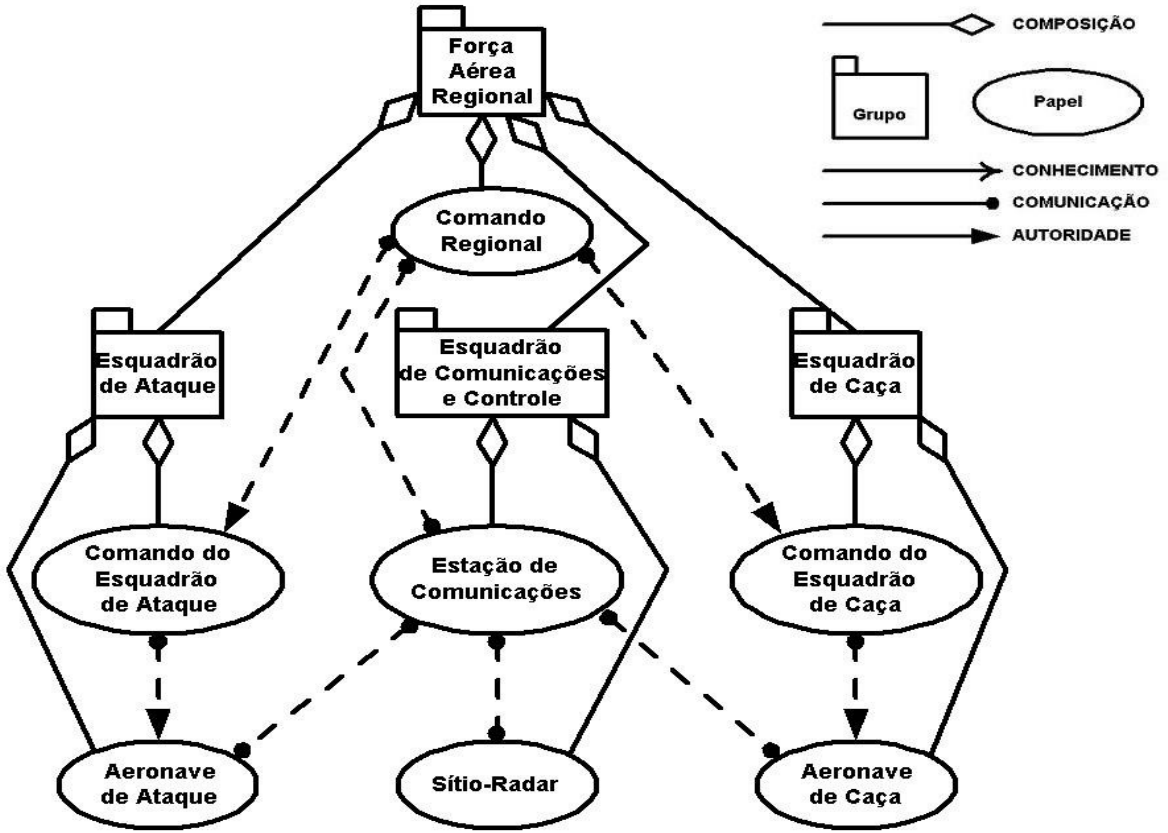
Communicated Model

- **Structural Specification**
  - Centralized Structure



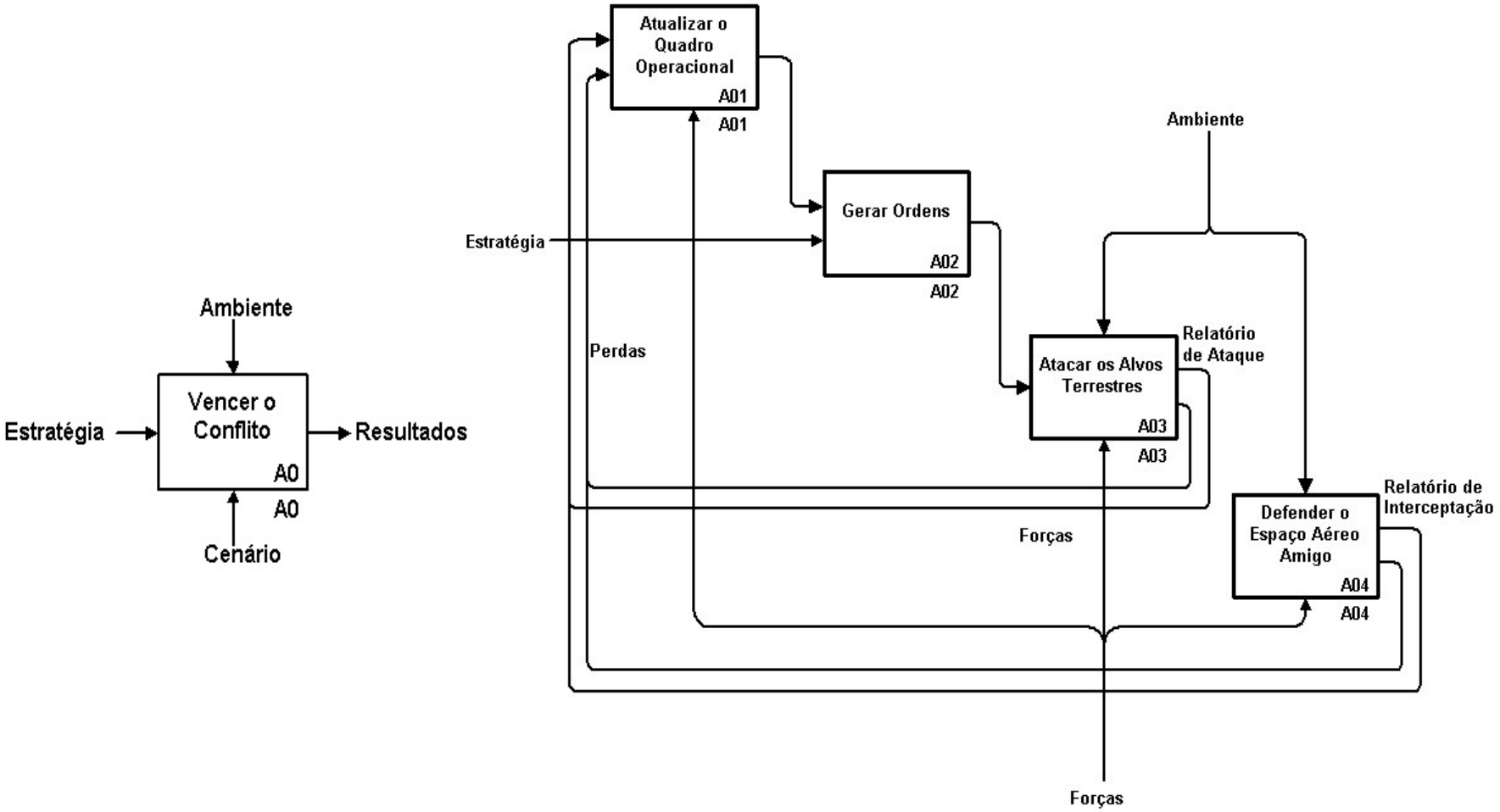
Communicated Model

- **Structural Specification**
  - Regional Structure

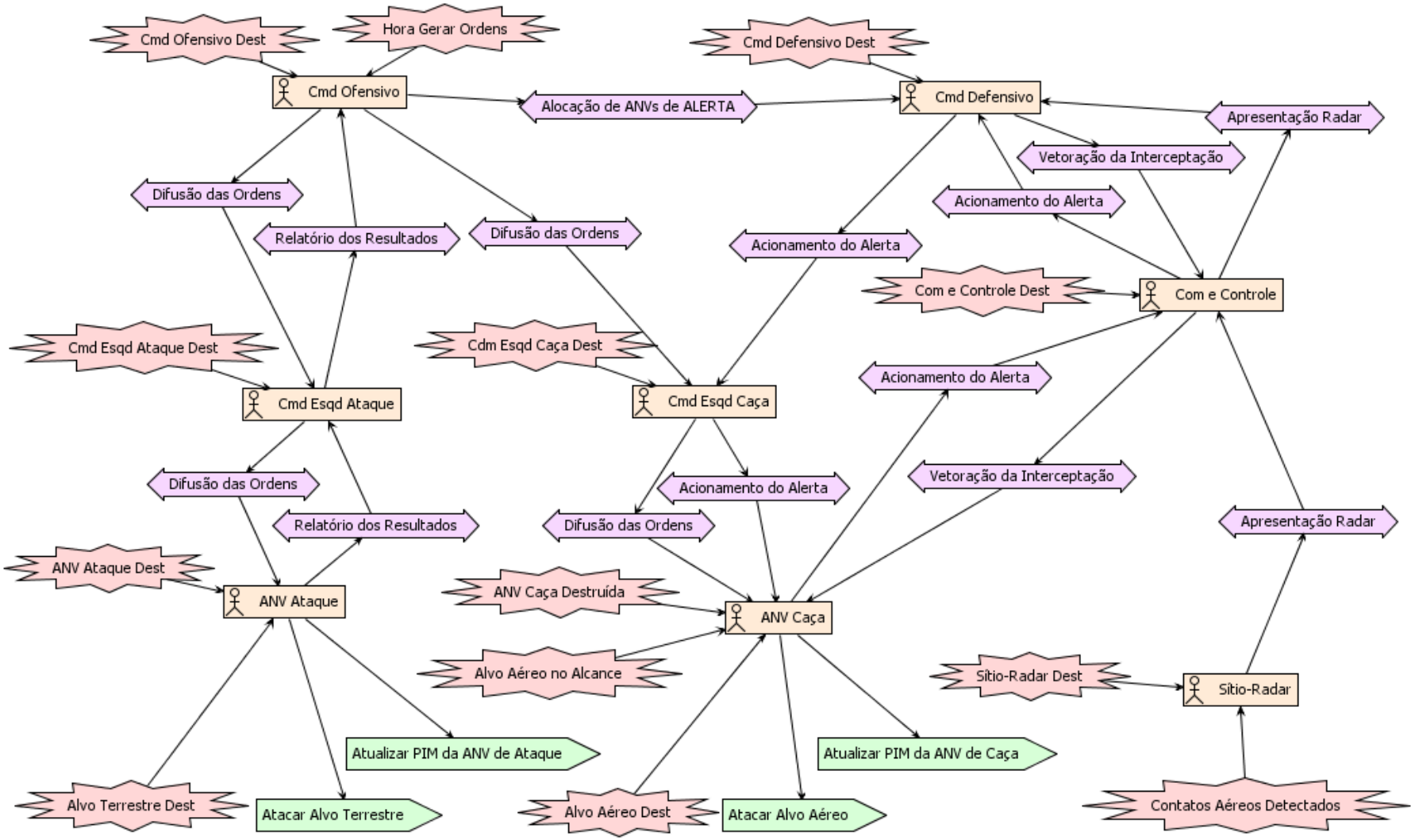


Communicated Model

❖ Functional Specification



Programmed Model



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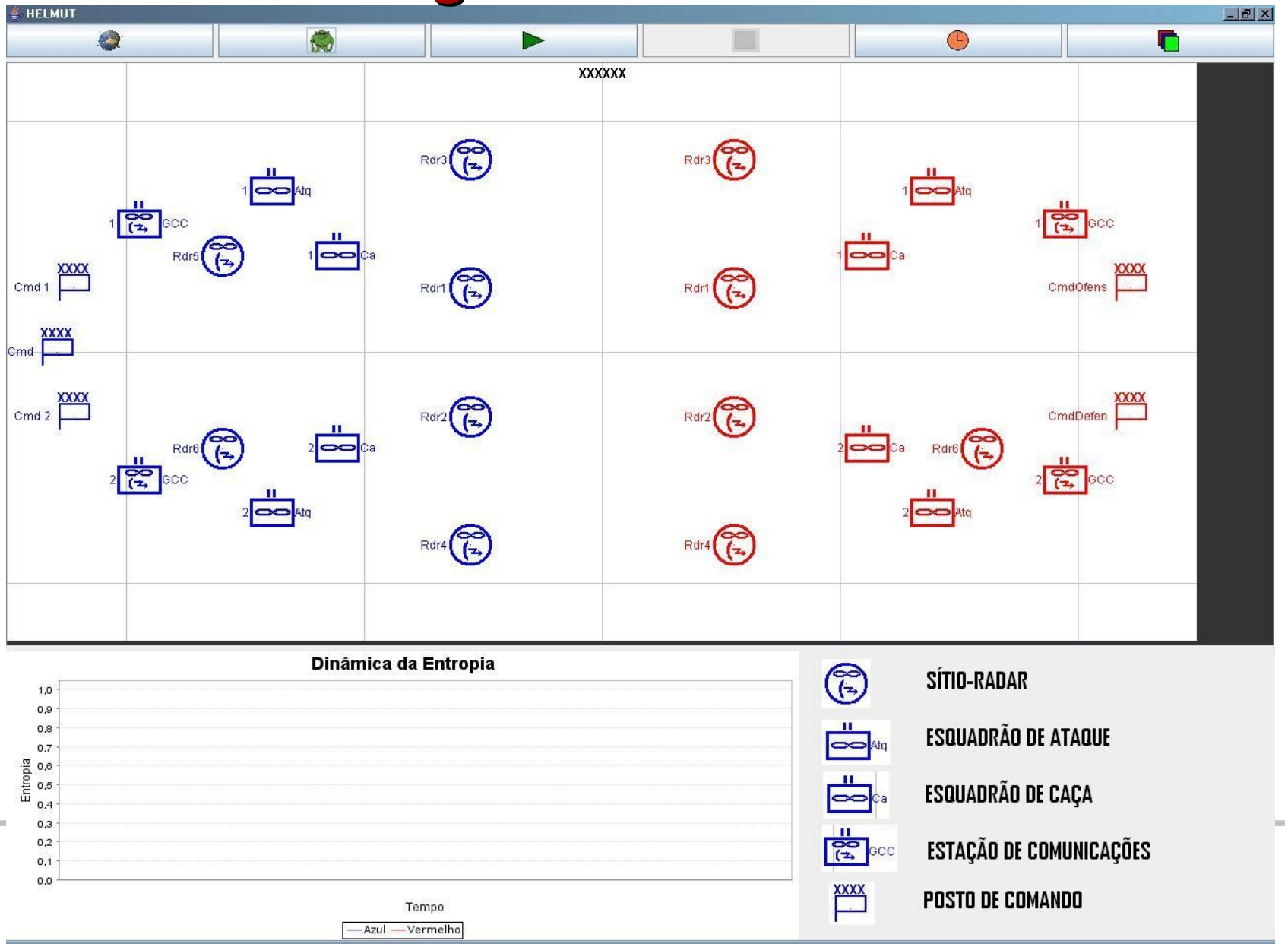


Experimental Model

- **Parameters:**
  - **Organizational Structure**
    - { Centralized, Regional }
  - **Sensors Range**
    - { 155km, 217km, 279km }

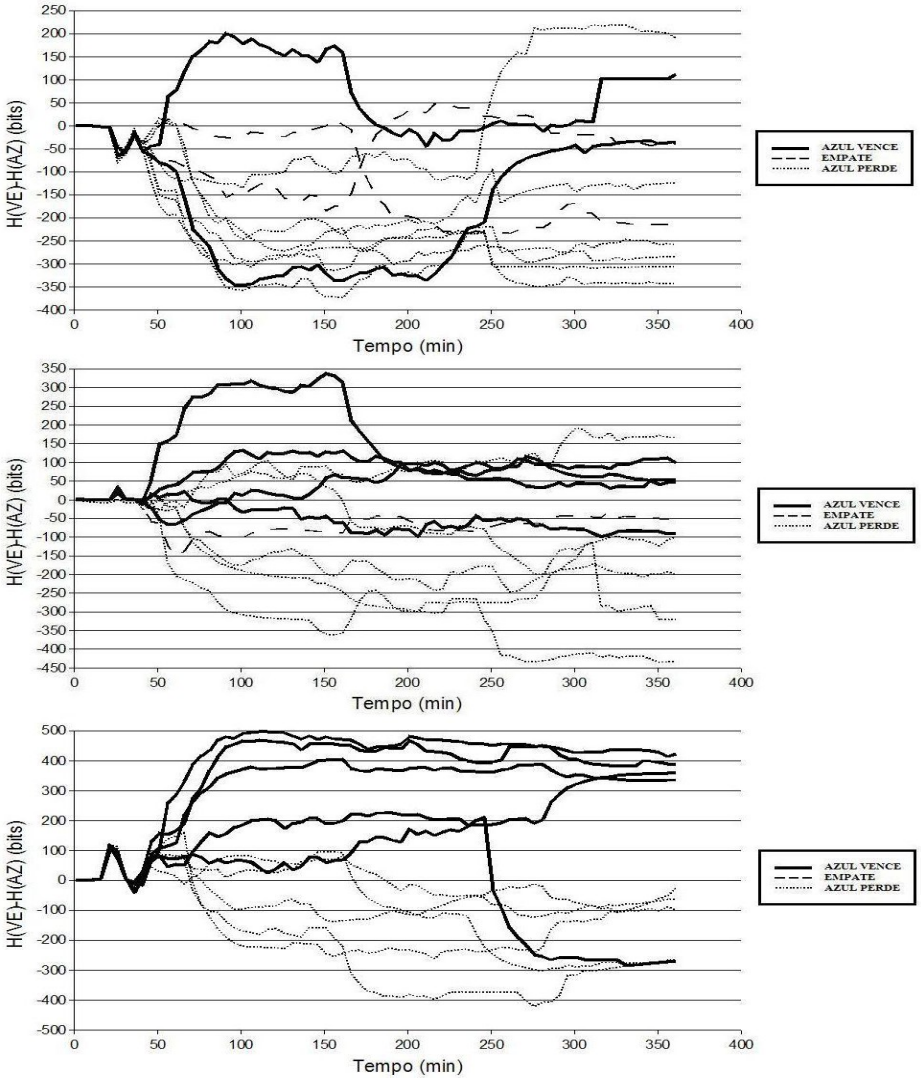
Experimental Model

- **Phase 1 – Centralized Scenario:**
  - BLUE Radar Range ={ 155km, 217km, 279km },

    RED Radar Range = 217km, P_{KILL}=40%, 10X
- **Phase 2 – Regional Scenario:**
  - BLUE Radar Az={ 155km, 217km, 279km },

    RED Radar=217km, P_{KILL}=40%, 10X

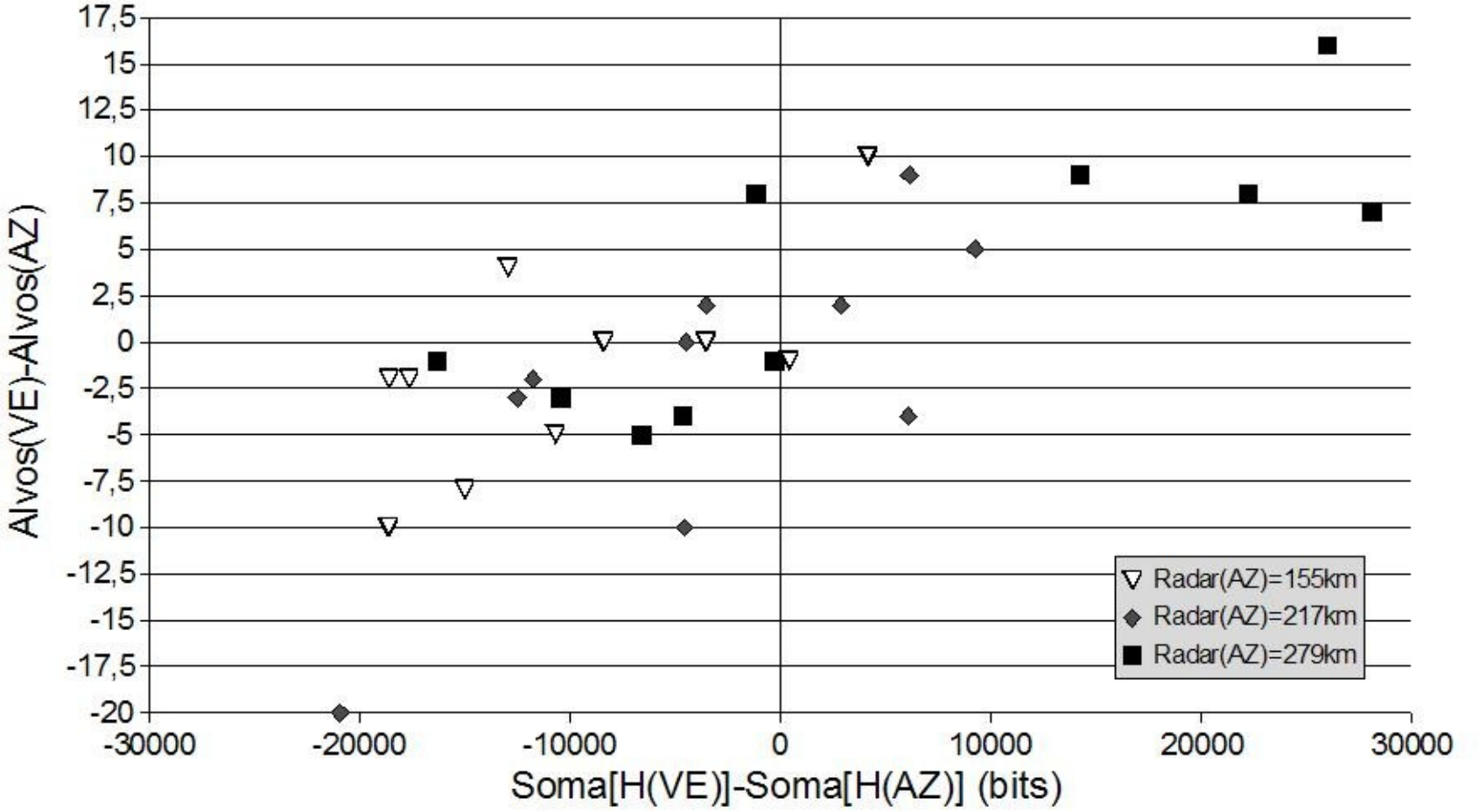
Results



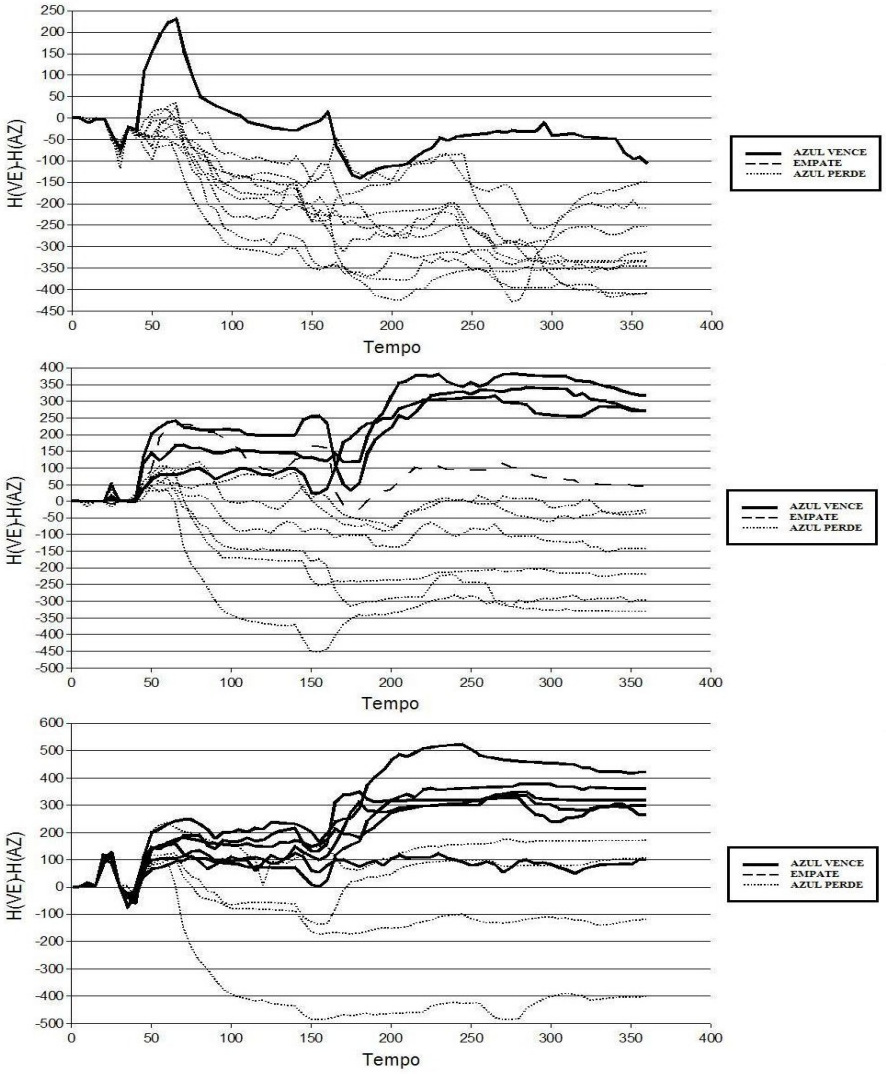
- **BLUE Radar={ 155km, 217km, 279km }**
- **RED Radar=217km,**
- **PKILL=40%,**
- **10X,**
- ***Centralized Scenario***

Results

- **BLUE Radar={155km,217km,279km}**
- **RED Radar=217km, PKILL=40%, 10X, *Centralized* Scenario**



Results



- **BLUE Radar={ 155km, 217km, 279km }**
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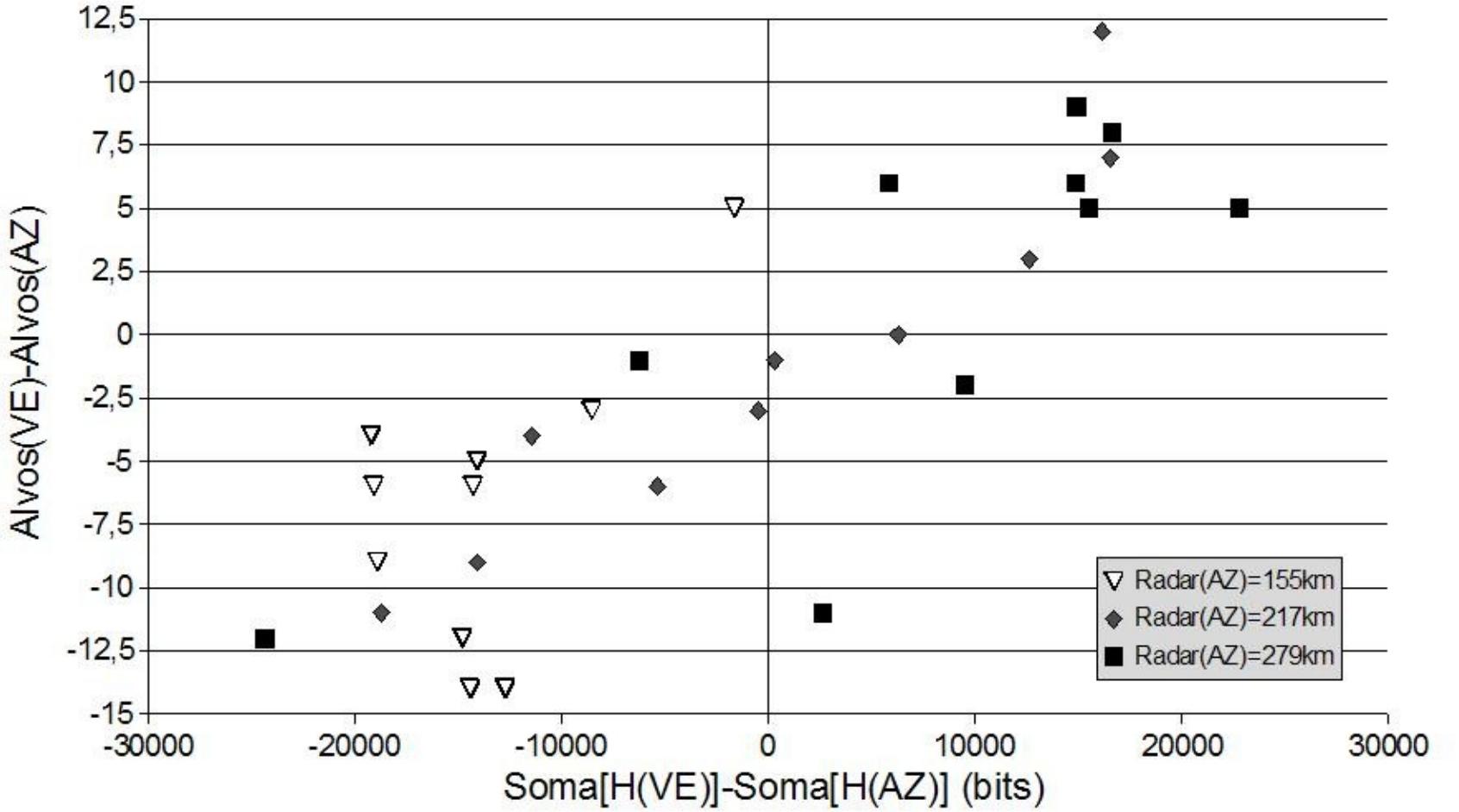
- ***Regional Scenario***

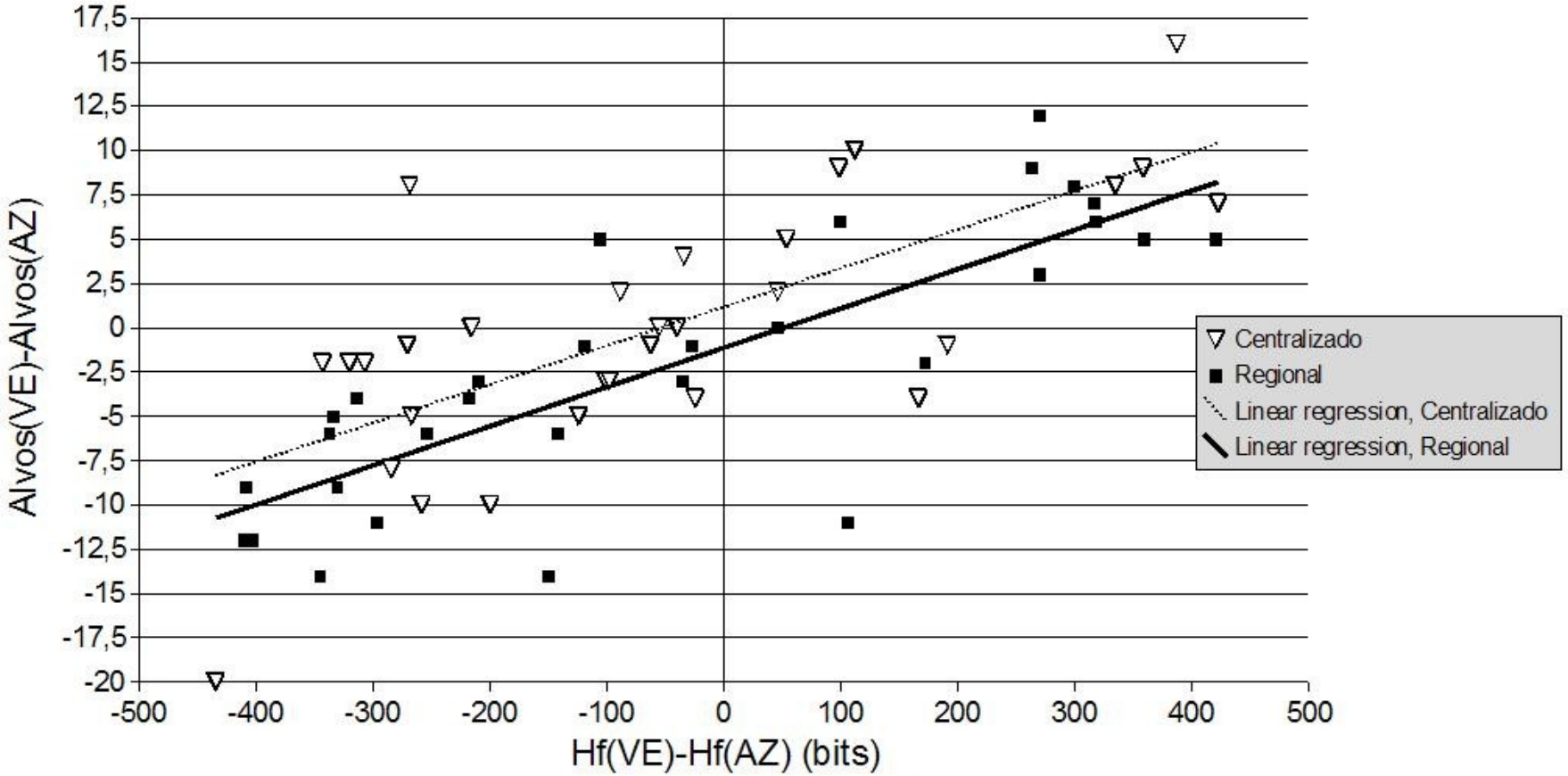
Results

- **BLUE Radar={155km,217km,279km}**
- **RED Radar=217km, PKILL=40%, 10X, *Regional* Scenario**



Results

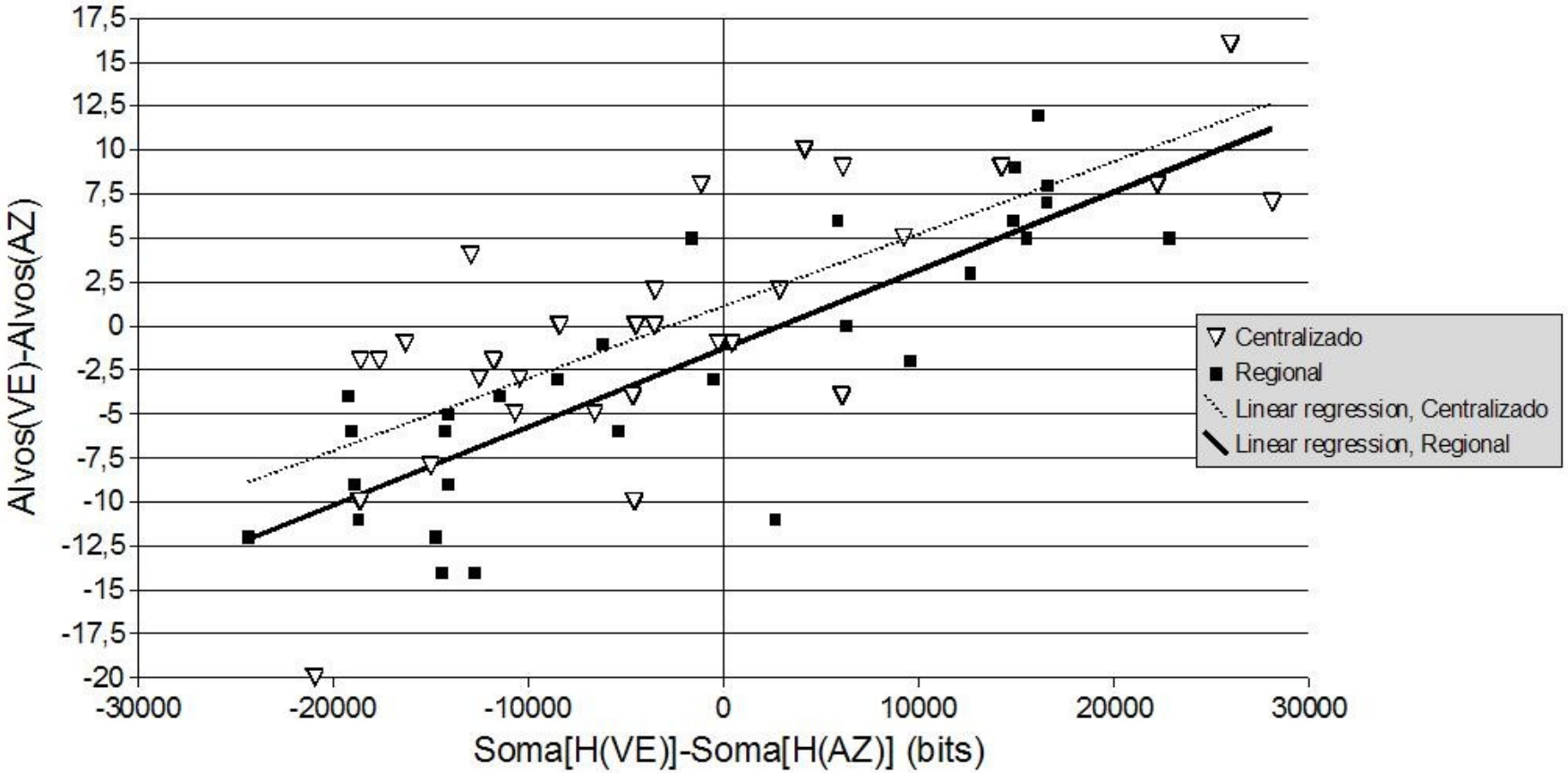
- Comparing centralized and regional structures on *final* values of entropy



Correlation R^2: Centralized 0,50 Regional 0,65

Results

- Comparing centralized and regional structures on *total, cummulative* values of entropy



Correlation R^2: Centralized 0,55 Regional 0,70

Conclusions

- **Two metrics based on Shannon entropy**
  - Information Gain
  - Information Superiority
- **Moderate correlation between information superiority and performance (as defined)**
- **Higher correlation: Sum[Sup] > Sup_{final}**
- **Small advantage to centralized structure**
- **Entropy metrics are dynamic**
- **Organizational and Systemic structures were considered**

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