Evaluating dynamics of Organizational Networks via Network Entropy and Mutual Information

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Organizational performance & network structure

- Organizational performance depends on timely access to information and the ability to use this information to make appropriate decisions.
- The structure of organizational network (formal & informal) impacts communication patterns and thus information diffusion.

Organizational adaptation & Network dynamics

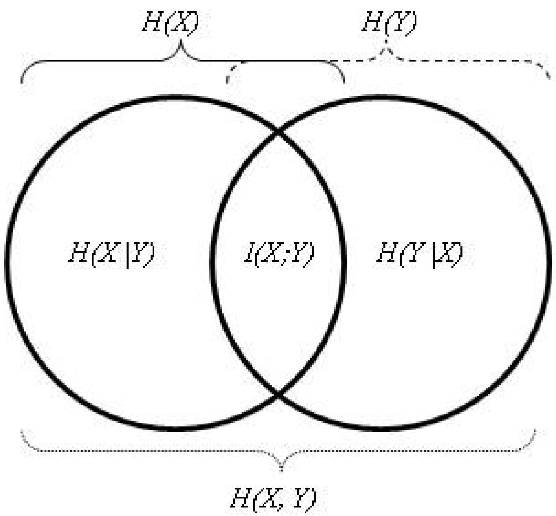
- Uncertain environment asks for continuous organizational adaptation
- Organizational adaptation depends on the structural agility of organizational networks
- Structural agility means conducting <u>intended</u> network evolution <u>efficiently</u>.
- **What is “intended” and what is “efficient” ?**

Research Question

- What measures can we use to evaluate network evolution in terms of effectiveness and efficiency?
- Prospective measures are expected to
  - Capture primary structural features as they are pertinent to organizational performance
  - Provide a lens on network evolution, viewing it as a process of related stages
  - Be easily implemented

Information Entropy & Mutual Information

- Shannon (1949)
- *Entropy* ***H*(*X*)**: the amount of uncertainty about a random variable (***X***), captured by a probability distribution over possible microstates.
- *Mutual information* ***I*(*X;Y*)**: change in the amount of uncertainty about the desired variable (***X***) by observing a related variable (***Y***).



Entropy & Mutual Information for networks

- Uncertainty in network structure: the degree distribution
- Node degree: the number of one-hop neighbors of the node.
- Network degree (probability) distribution
  - If the network has N nodes and N_i of them have degree i, then the probability that a node with degree i is $p_i = N_i / N$.

p_i

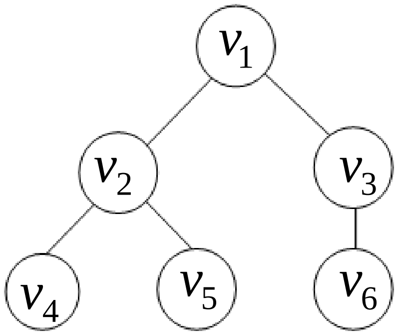
Network Entropy (*NE*)

- Definition:

Assume a network X. $NE(X) = -E[\log p(X)] = -\sum_i p(x_i) \log p(x_i)$, where $p(x_i) = N(x_i)/N(X)$. There are $N(X)$ nodes in X, among which $N(x_i)$ nodes has the degree of i.

- Example: a 6-node network X

i	0	1	2	3	4	5
$P(x_i)$	0/6	3/6	2/6	1/6	0/6	0/6
$\log_2 p(x_i)$	-	-1	-1.58	-2.58	-	-



$$NE(X) = -\sum_{i=0}^{5} p(x_i) \log p(x_i) = 1.46$$

Mutual Information (*MI*)

- Definition

Assume a network whose degree distribution changes from *X* to *Y*. $MI(X;Y) = \sum_i \sum_j p(x_i, y_j) \log \frac{p(x_i, y_j)}{p(x_i)\, p(y_j)}$, where $p(x_i, y_j) = p(y_j \mid x_i)\, p(x_i)$ is the joint probability of *X* and *Y*, when $X = x_i$ and $Y = y_j$. As previously defined, $p(x_i) = N(x_i)/N(X)$ $p(y_j) = N(y_i)/N(Y)$

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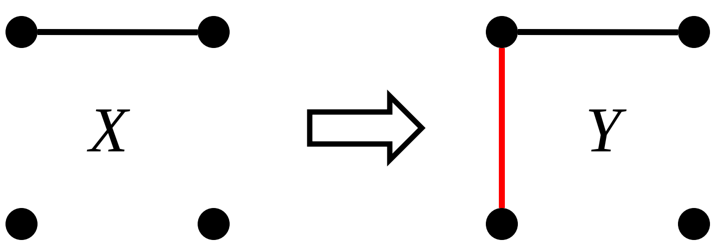
Mutual Information (cont.)

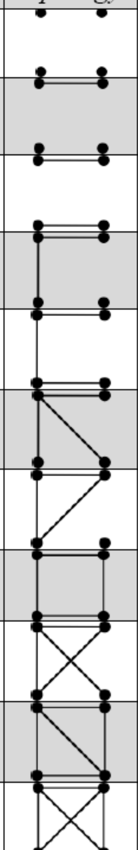
- Example: a 4-node network changes from Stage X to Stage Y

i	0	0	1	1
j	0	1	1	2
$P(x_i)$	2/4	2/4	2/4	2/4
$P(y_j)$	1/4	2/4	2/4	1/4
$p(y_j \mid x_i)$	1/2	1/2	1/2	1/2
$p(x_i, y_j)$	1/4	1/4	1/4	1/4



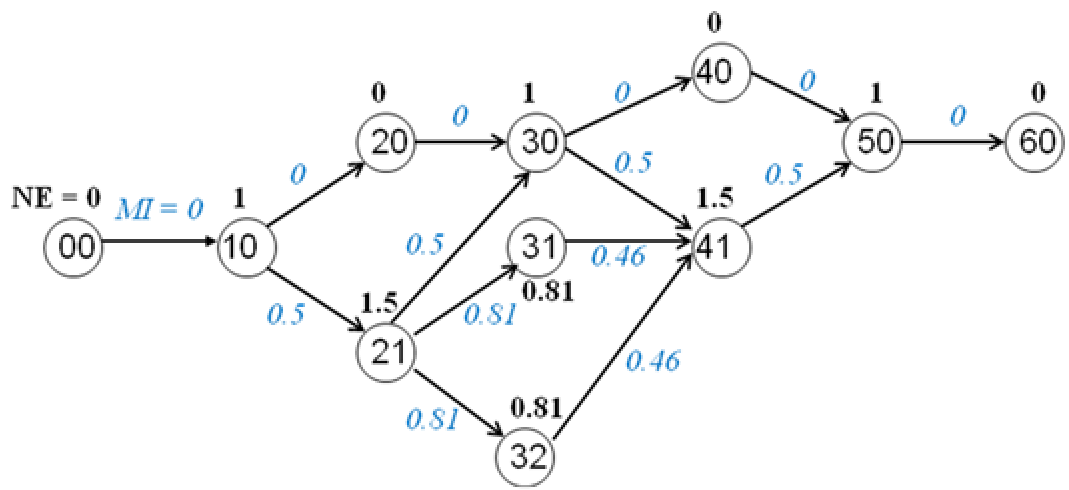
$$MI(X;Y) = p(x_0, y_0)\log\frac{p(x_0, y_0)}{p(x_0)p(y_0)} + p(x_0, y_1)\log\frac{p(x_0, y_1)}{p(x_0)p(y_1)} + p(x_1, y_1)\log\frac{p(x_1, y_1)}{p(x_1)p(y_1)} + p(x_1, y_2)\log\frac{p(x_1, y_2)}{p(x_1)p(y_2)} = 0.5$$

Measuring Network Evolution



#	topology	m	p(0)	p(1)	p(2)	p(3)	NE
00		0	4/4	0	0	0	0
10		1	2/4	2/4	0	0	1
20		2	0	4/4	0	0	0
21		2	1/4	2/4	1/4	0	1.5
30		3	0	2/4	2/4	0	1
31		3	0	3/4	0	1/4	0.81
32		3	1/4	0	3/4	0	0.81
40		4	0	0	4/4	0	0
41		4	0	1/4	2/4	1/4	1.5
50		5	0	0	2/4	2/4	1
60		6	0	0	0	4/4	0

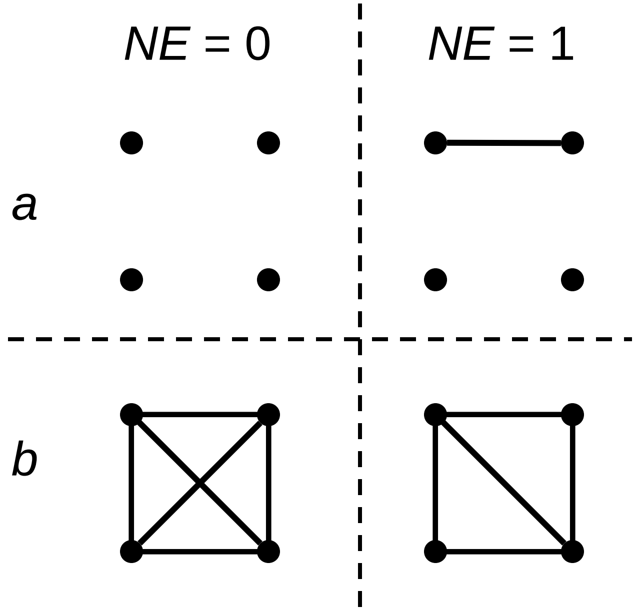
Example: Adding links to a 4-node empty network until it becomes fully connected, one link at a time



Graph of Network Evolution

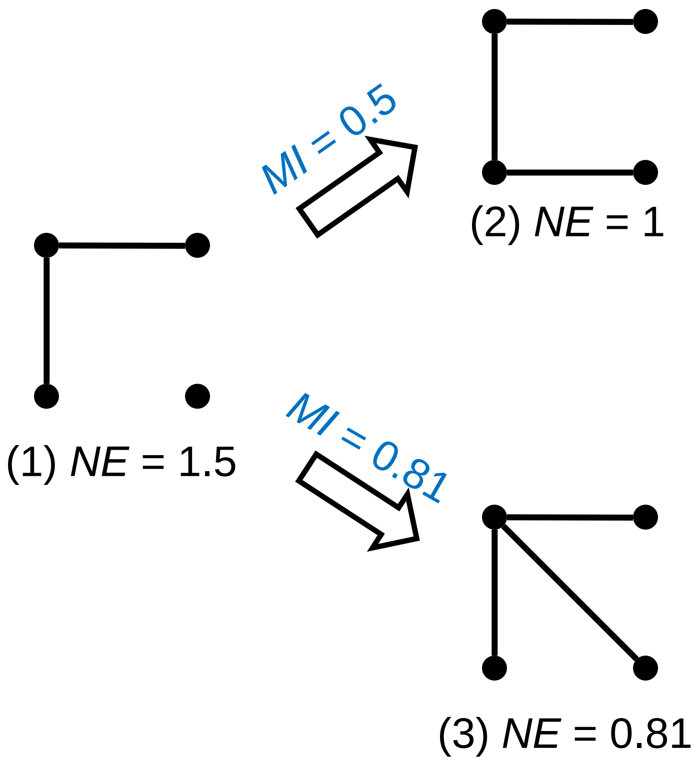
Measuring Network Evolution

- *NE* measures the start and end network states (**effectiveness**).
- Small *NE* implies most nodes are similar in degree. Yet there are two probabilities:
  a. *Centralized structure*: Most nodes connect to a few hubs and are thus separated from each other. There are relatively fewer links in the network
  b. *Decentralized structure*: Most nodes connect to each other. There are relatively more links in the network.



Measuring Network Evolution

- *MI* measures the changing process (**efficiency**)
- Large *MI* implies more changes in network degree distribution, which can be interpreted as
    a. Agility (bigger step to intended structure)
    b. High change cost



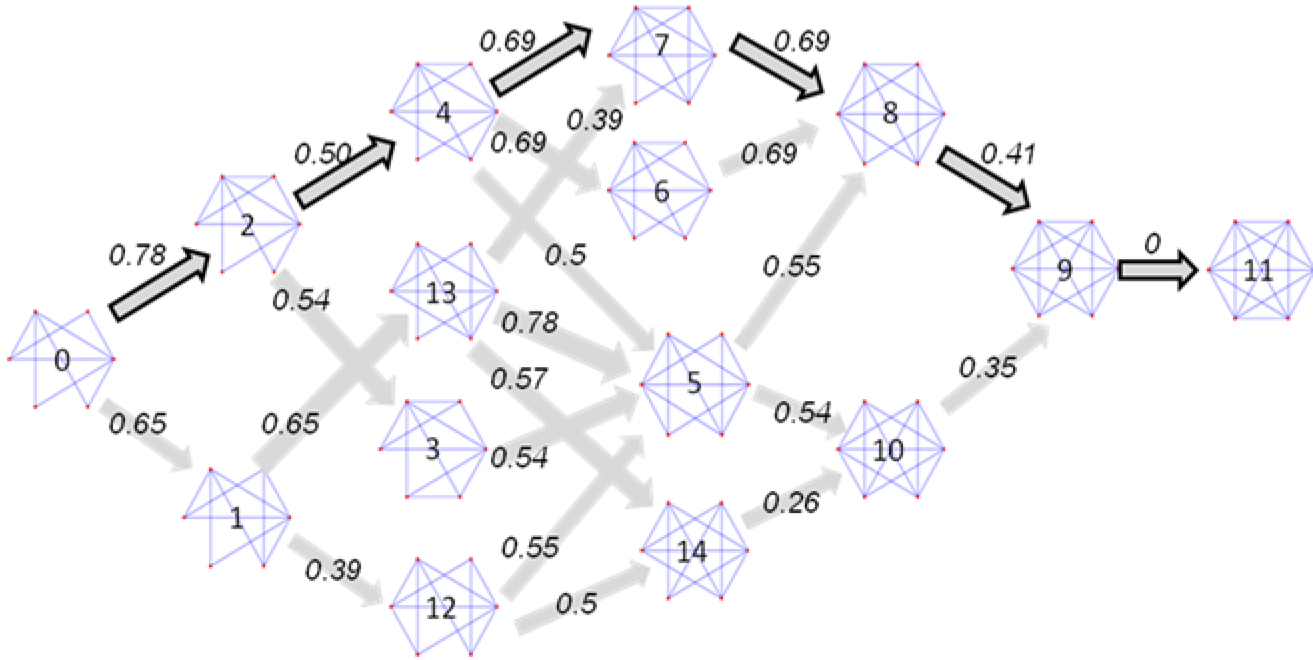
Level of centralization: (1) < (2) < (3)

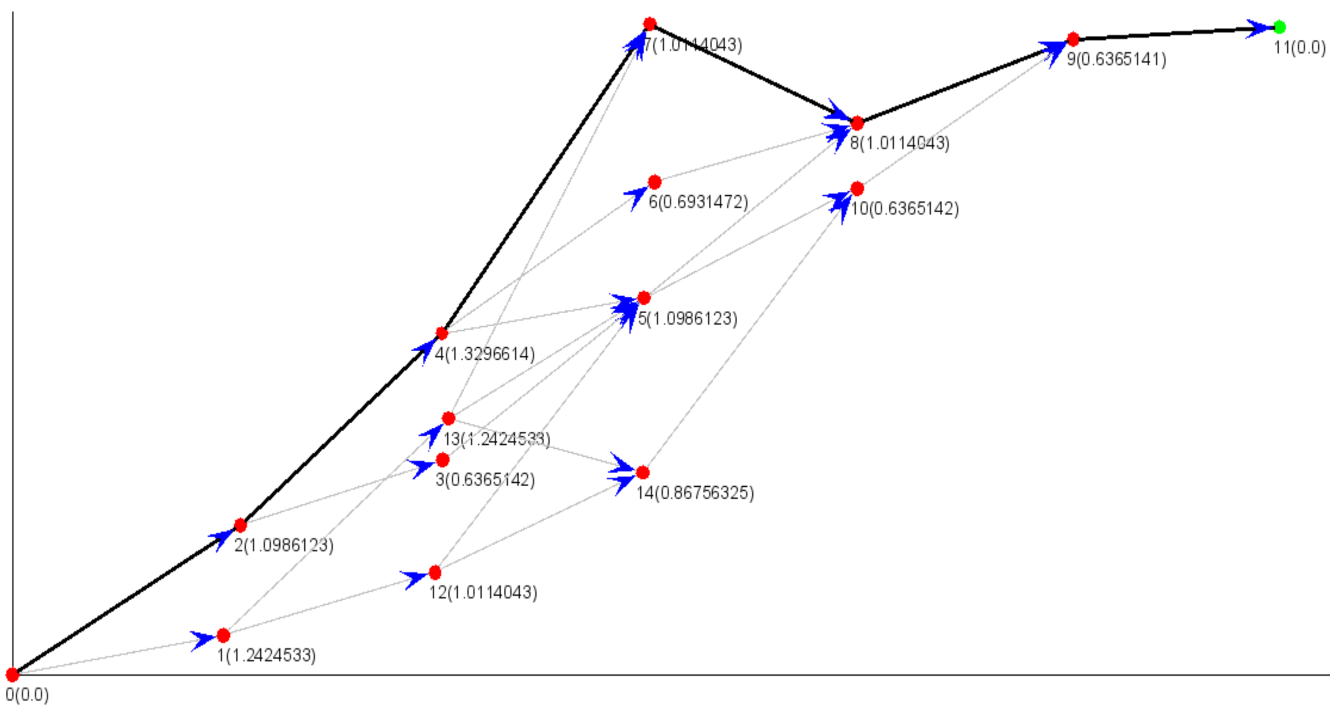
The Best Path & the Agility of Organizational Network

- Given the same type of network evolution (e.g., link addition), a path with large sum of *MI* indicates an agile organizational network, which moves between centralization and decentralization in the biggest magnitude
- The best path: the <u>longest</u> path in terms of *MI* in the graph of network evolution
- Find the best path
  - Construct the graph of network evolution
  - Associate each link in the evolution graph with the opposite number of *MI*
  - Find the shortest path using Bellman–Ford algorithm

Example 1

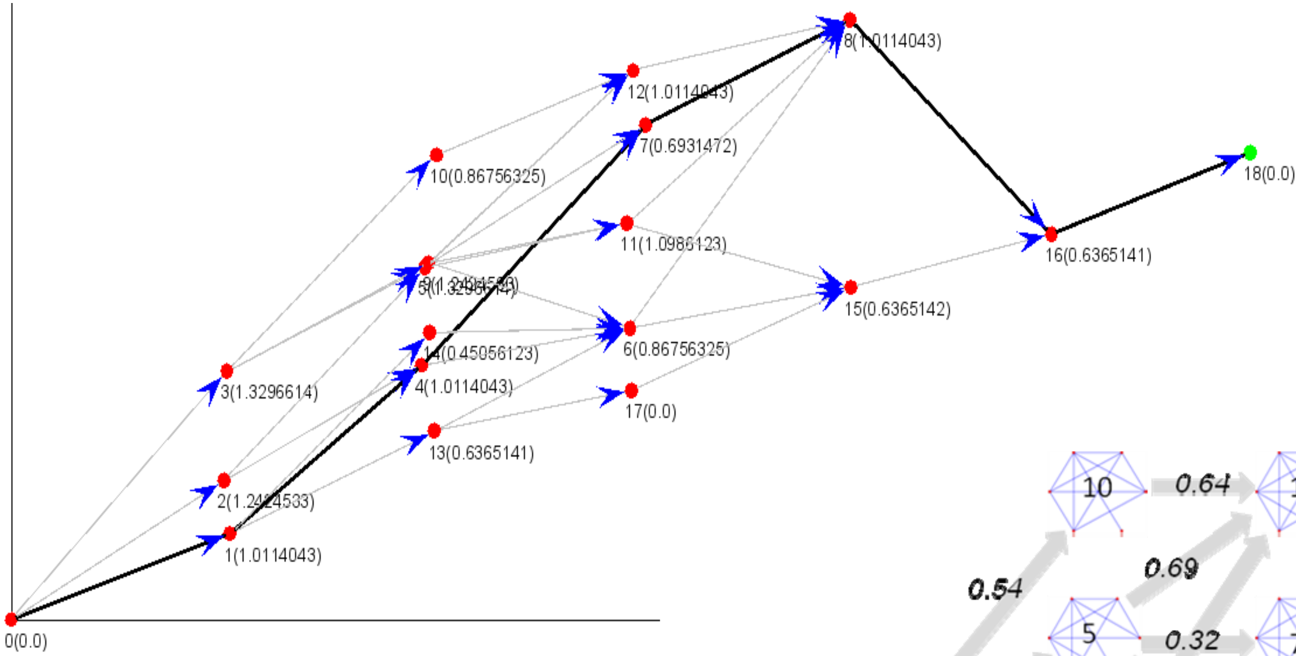
Adding 6 links to a 6-node, 9-link random network

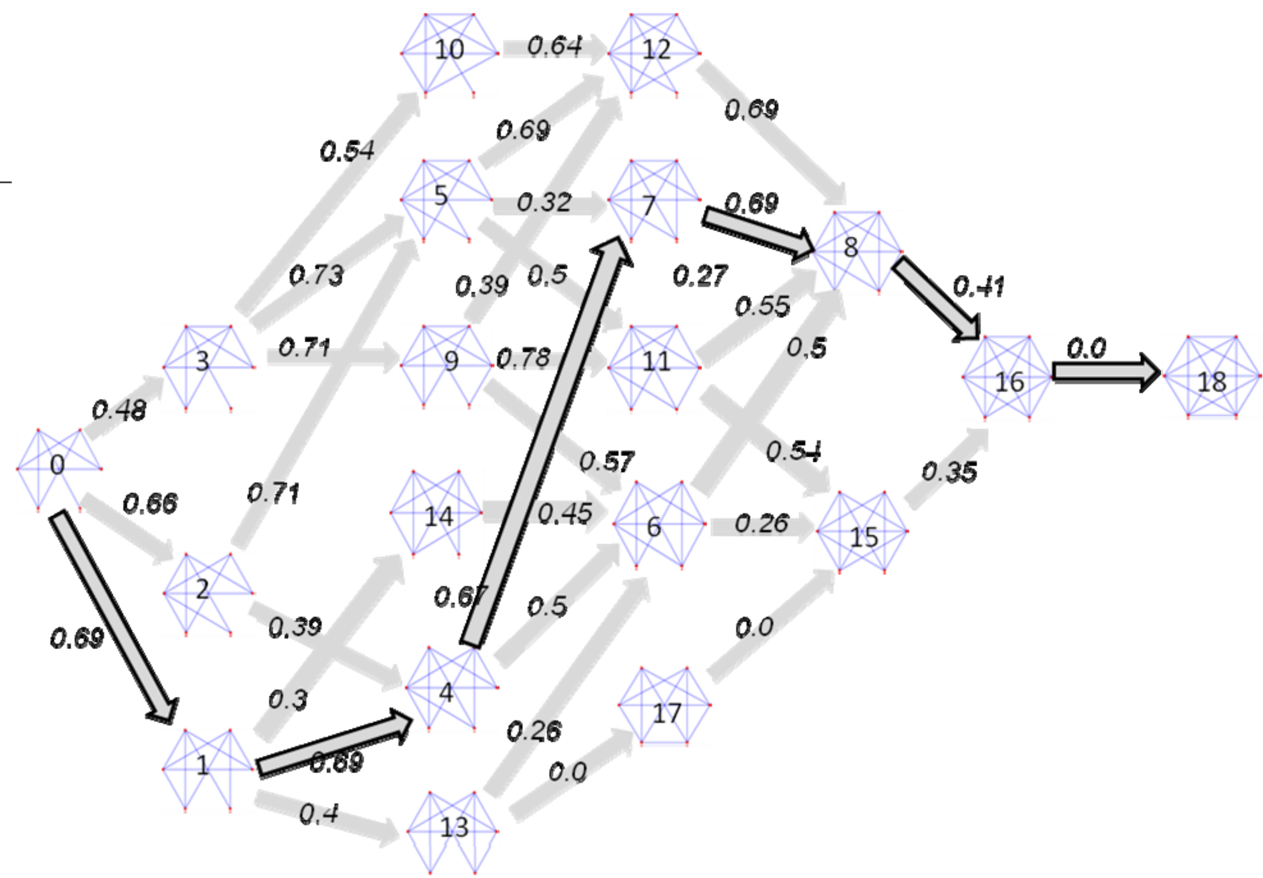




Example 2

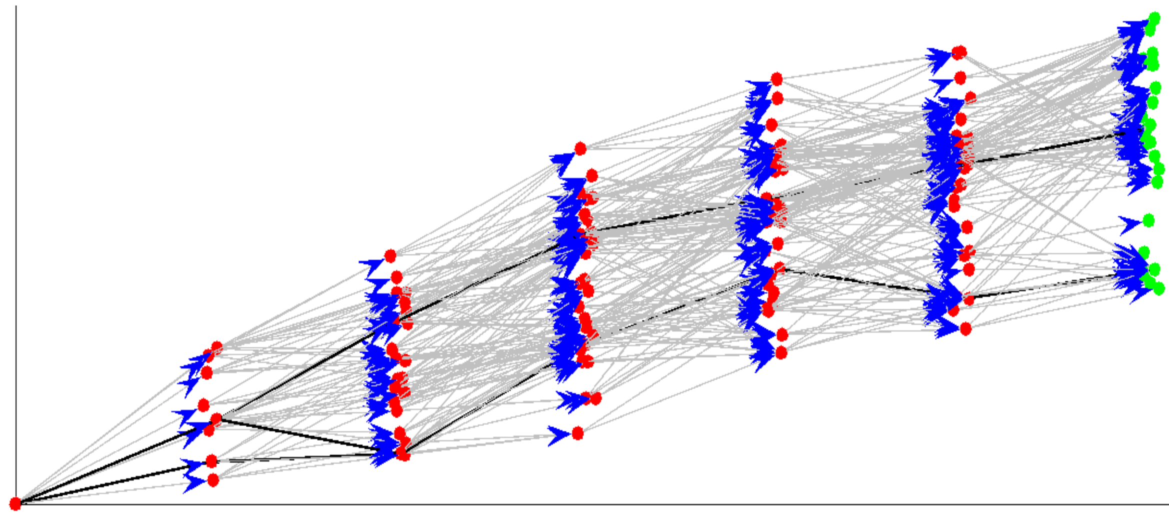
Adding 6 links to a 6-node, 9-link scale-free network

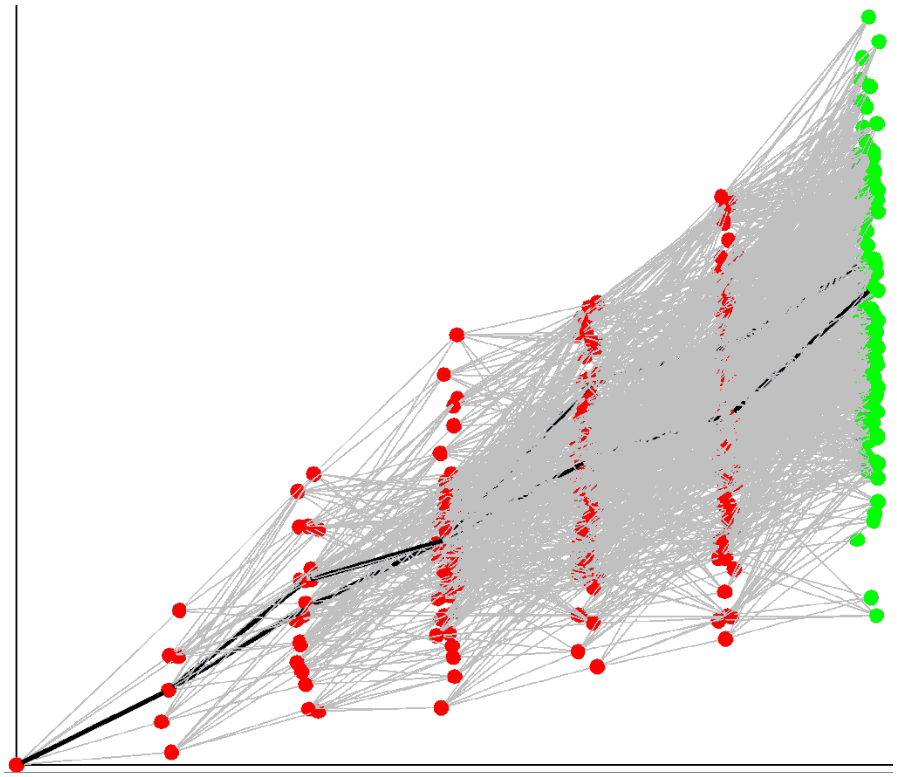




Example 3

Adding 6 links to a 10-node, 33-link real-data network (data adapted from Knoke & Kuklinski,1982)

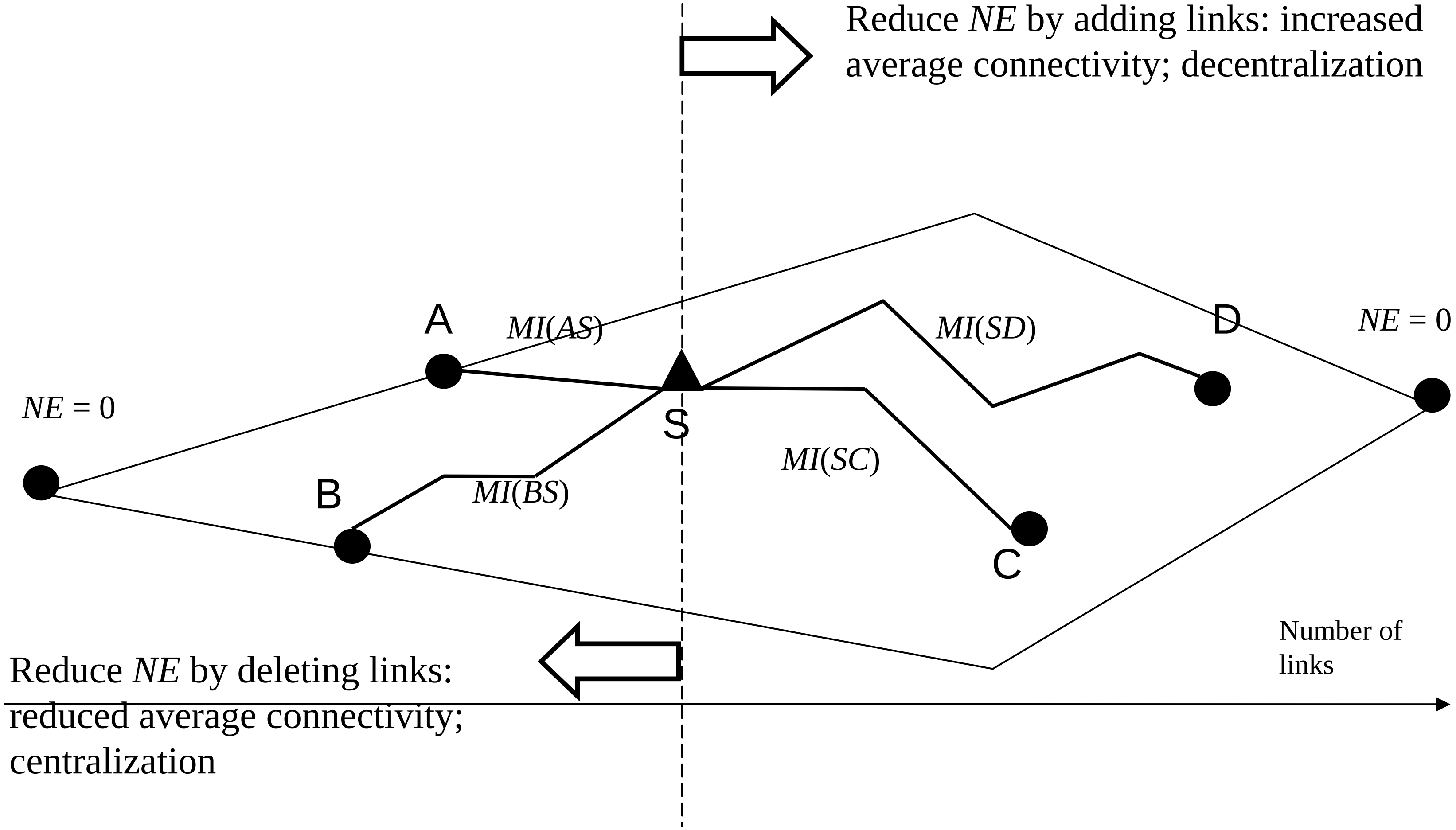




Example 4

Adding 6 links to an 11-node, 32-link real-data network (data adapted from Hlebec,1993)

Future Work : Combination of *NE* & *MI*



Conclusions

- Two measures—*NE* & *MI*—for evaluating the dynamics of organizational networks
  - Built on network degree distribution
  - See network evolution as a process of related stages
- The evolution path with large sum of *MI* indicates an agile organizational network
- Together they show the relative advantage of different organizational adaptation strategies, regarding the intended topological state and the evolution path an organization should take

Thank You

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