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A New Paradigm for Dynamical Modelling of Networked C2 Processes

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Abstract
A new type of mathematical model for studying synchronisation of interacting C2 processes across complex networks is proposed. The approach surrenders the requirement to distinguish the various products of individual C2 processes (for example planning, execution of strategic, campaign or tactical activities to name a few), but does distinguish the *time-scales* of and *interactions between* individual processes. This enables representation of interacting C2-processes from across the strategic-operational-tactical spectrum of command within a mathematically elegant and compact model. More specifically, C2-processes *within* a given system must be self-synchronised while C2-processes for two adversaries must seek to *outpace each other*, as in Boyd's original OODA loop for air combat. It is argued that for C2-systems undertaking genuine Complex Endeavours this is an appropriate and manageable approach to modelling. The paper discusses how standard mathematical analysis enables the study of self-synchronisation and stability within the friendly C2-system in this approach. This will then enable assessment of different network structures and dynamical requirements for possible C2-systems of the future.

Strategy is the art of making use of time and space. I am less chary of the latter than the former. Space we can recover, time never. … I may lose a battle, but I shall never lose a minute.

Napoleon Bonaparte

Introduction

This paper formulates a theoretical framework for studying the interactions between various loops within and across military Command and Control (C2) systems that operate over disparate time-scales and across different networks but which need, at times, to be synchronised.

There is little doubt that Command and Control (C2) is about cycles in time: those of the enemy and one's own. C2 is about many more things too – technology, good leadership, clarity of concepts, appropriate management of people and resources, and so on. But at the core remain the cycles which technology facilitates the traversal of with greater speeds, within which communication is undertaken, assessment of the facts performed, decisions are made, promulgated and implemented. Boyd [1987] recognised this in formulating his classic C2 decision cycle: the Observe-Orient-Decide-Act (OODA) loop. Drawn from his experiences as a fighter pilot, the OODA loop remains the simplest paradigm for conceptualising adversarial – blue-against-red – C2 interactions: all things being equal, those who can outpace the adversary's OODA loop win the battle. However, *within* a blue C2 network there occur isomorphic decision cycles. Within a C2-system, decision cycles need to *synchronise* so that the outputs of one node become the timely inputs of another or that interdependent decision processes track one another appropriately. A classic example is the way Strategic, Operational and Tactical levels of command may go about planning. At the broadest level, each undertakes something akin to the Military Appreciation Process (MAP) with a sequence of Scoping → Mission Analysis → Course of Action Development → Course of Action Analysis → Decision → Execute. However, inputs and outputs at each MAP step would be at different levels of aggregation and time-scales depending on the Command Level at which the MAP is conducted. Such a distribution of MAPs across levels, at which different time-scales apply depending on political/social/adversarial/environmental constraints, nevertheless needs to be synchronised.

Considering that both the MAP and OODA loop can be mapped onto a continuous, circular decision loop, the finer detail of discrete milestones – or even subloops – in this cycle can be regarded as irrelevant. In fact, regardless of the *object* of decision-making – be it in relation to planning or execution or other activities – this paper posits that in C2-modelling for Complex Endeavours [Alberts & Hayes, 2007]:

- The *cyclic* nature of C2 processes is universal.
- The intrinsic *timescale* of these cycles is deeply dependent on the strategic/operational/tactical environment in which the C2-node acts.
- The *structure* of blue-blue and blue-red interactions plays a pivotal role in the ability of blue to synchronise within itself and to outpace red.

This supports an approach to C2-modelling in which, on the one hand, the specific goals of C2-processes do not need to be distinguished, but, on the other hand, the interconnectedness of C2-processes and their timescales becomes the important feature to model. This enables simultaneous representation of C2 at the tactical interface – where the object is the range of kinetic and non-kinetic effects in the battlespace – as well as C2 at the strategic level – where the timescales of negotiations of government and other civilian agencies are often justifiably different and intrinsically unavoidable. By stepping back in the fidelity of specific C2-goals being represented, one can legitimately model across the entire C2-spectrum, from strategic to tactical.

At the core of this approach are networks and at the nodes of these networks are entities – oscillators – which undertake a cyclic change of state, or loop. The rate of progress through this loop of one oscillator at a particular node depends on the point in the loop of another oscillator at a connected node. On this basis, then, adversarial C2-systems can be represented by two complex networks[1] *within each* of which adjacent (node-to-node, according to hierarchy or network connectivity) loop cycles must *locally* synchronise, but *across* networks loops must seek to outpace the other, in classic Boydian terms. A mathematical dynamical model for this framework is presented herein. Within a C2-system, the intra-node interaction is described by a model used successfully for studying

[1] Note that the networks here are patterns of *interaction* and not just communication (as in NCW communication networks).

synchronisation in a diversity of physical, chemical and biological systems: the Kuramoto model [Kuramoto, 1984]. Here, the loop cycles of oscillators at each node complete on a time-scale determined by a distribution of frequencies. It is well-known [Strogatz, 2000] that, for a critical value of the coupling strength across the network, loop cycles synchronise without the intervention of any master controller: the behaviour is *self-synchronisation*. For adversarial interactions, a similar mathematical model is proposed whose entities seek to "get inside the opponent's OODA loop".

A framework for analytical and simulation study of such a model is described below. The aim of such a program is to explore how the network *structure* of a C2-system figures in the C2 dynamics to enable self-synchronisation, stability, and fulfilment of the true aim of military C2: the defeat of the enemy.

This paper presents a high-level mathematical formulation of the model (sufficient for the non-mathematical experts), explains its relevance to military-strategic/operational C2-systems and provides analytical approaches to solving the system.

Mathematical model for self-synchronisation on a network

The literature on self-synchronisation in mathematically encoded cooperative systems is vast, going back to Wiener [1961] and Winfree [1967] and scattered across mathematical, physical, biological and computational scientific journals. The basic idea of such models is that linking up nodes which separately undergo cyclic behaviour can lead a mass effect whereby a large part of the system locks itself into a collective cyclic behaviour. This can be truly called self-synchronisation since it is a consequence of interactions and not the manipulation of the system by a master-controller. It was Kuramoto [1984] who succeeded in distilling the bare essentials of such models with the first order differential equation:

$$\dot{\theta}_i = \omega_i + \sigma \sum_{j=1}^{N} \sin(\theta_j - \theta_i)\,. \qquad (1)$$

Here θ_i represents a time-dependent *phase* associated with node *i* of a complete network of *N* nodes, $\dot{\theta}_i$ is the derivative of the phase with respect to time *t*, ω_i represents a "natural" or "intrinsic" frequency" and σ is a coupling constant. The role of θ_i as a phase is seen when it is reinserted in the complex variable

$$\chi_i = e^{i\theta_i}\,. \qquad (2)$$

With the coupling switched off, $\sigma = 0$, solutions are straightforward: $\theta_i = \omega_i t$ so that each node undergoes a rotation seen through the complex variable χ_i through the angle 2π radians (or 360°) in time-period $2\pi / \omega_i$ or correspondingly there will be $\omega_i / 2\pi$ rotations of the unit circle per unit time. The values of these frequencies in this context are usually selected from some statistical distribution. Assumptions about the shape of this distribution – for example, symmetry about some single mean value at which the distribution peaks ("uni-modal") – lead to interesting results that are discussed below.

With the coupling switched on matters become more complicated – and perhaps even complex. For small coupling over short periods of time one can intuitively see that each node seeks to continue its separate motion about the circle – at its own pace, governed by ω_i – however over longer periods of time the coupling will begin to distort this behaviour. For example, for small differences between phases the angular speed is modified by the perturbation

$$\sigma(\theta_j - \theta_i).$$

Thus if node *i* lags behind node *j*, $\theta_j > \theta_i$, the correction is positive and there will be an angular speed up. Correspondingly if *i* is ahead of *j* the correction is negative and there is a slow down. So *locally* the interaction is such that nodes seek to synchronise. What is not clear is what should happen *globally*: how does the overall system behave? Another feature of the model is the periodicity of the interaction: if one node out-laps another by an entire cycle then the interaction resets back to zero.

An interpretation of these features in terms of C2 concepts is given below. Here we review the analysis by Kuramoto which exposes its key behaviours.

It is useful to introduce an order parameter: a quantity which shows markedly different behaviour in one dynamical regime compared to another, so that the two distinct behaviours can be thought of as separate "phases". Kuramoto considered:

$$re^{i\phi} \equiv \frac{1}{N}\sum_{j=1}^{N}\chi_j = \frac{1}{N}\sum_{j=1}^{N}e^{i\theta_j} \qquad (3)$$

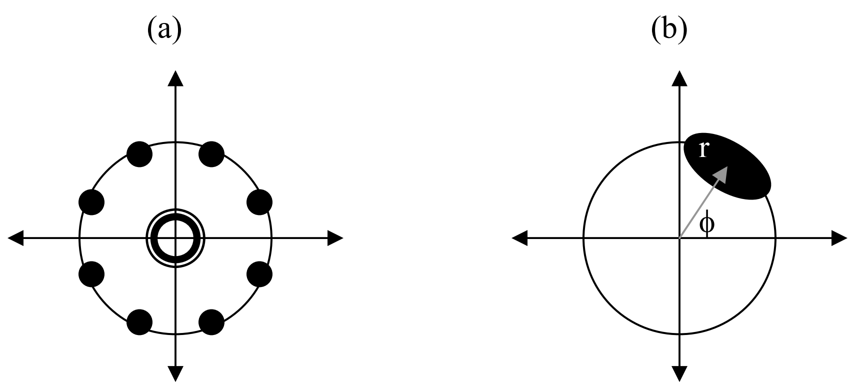

Figure 1 Geometrical interpretation of Kuramoto's order parameter. For case (a) with individual phases (solid blobs) distributed uniformly about the circle the centre of gravity (open circle) is at the origin and $r \approx 0$ while for case (b) with phases clustered closely (oval blob) the centre of gravity is close to the circle circumference and $r \approx 1$ with ϕ the phase of the centre of gravity.

Geometrically, the left hand side of Eq.(3) can be regarded as giving the radius and angle, with respect to the unit circle, of the *centre of gravity* (technically, the *centroid*) of the collection of points on the unit circle representing the individual positions of the phases. The one extreme of completely unsynchronised individualised behaviour (Figure 1(a)) will have points distributed uniformly about the circle. Thus, the centre of gravity will be at the origin and so *r* will be approximately zero. At the other extreme, with all phases locking and moving in synchrony, the points will be concentrated in a cluster (Figure 1(b)) at angle ϕ and the variable *r* is approximately 1. Thus *r* is the order parameter distinguishing two main modes of behaviour: incoherence $r \approx 0$ and synchrony $r \approx 1$. So there are at least two phases. Does the transition from one to the other represent a phase-transition, like the ice-water transition?

Kuramoto exploited the simplicity of the complete network coupling to demonstrate a transition for the case of $N \to \infty$ with symmetric, uni-modal distribution $f(\omega)$ of frequencies about a mean value $\overline{\omega}$, and a coupling that scales with the number of nodes, $\sigma = K/N$. Firstly, at a critical value of the coupling K_c the system dramatically changes its behaviour from incoherence ($K < K_c$) to synchrony ($K > K_c$). Let r_∞ be the equilibrium value of $r(t)$. Then, in the synchronous regime, the nodes whose frequencies satisfy the bound $|\omega_i - \overline{\omega}| < Kr_\infty$ lock into collective mode while nodes for which $|\omega_i - \overline{\omega}| > Kr_\infty$ randomly drift with respect to the cluster. Finally, and most remarkably, Kuramoto determined analytically the value of the critical coupling:

$$K_c = {}^{2}\!/_{\pi f(\overline{\omega})}.$$

Despite the artificiality of the assumptions, in particular the infinite size of the network in Kuramoto's case, the basic property that the change of behaviour from incoherence to synchrony at some critical coupling is akin to a phase-transition has been observed in numerical simulations. How this differs when the network is no longer complete is discussed below. It suffices to mention at this stage that, whereas for the complete network a single locked cluster emerges, for complex networks typically many clusters of locally locked oscillators can occur and one can speak of "partial synchronisation".

Interpretation in terms of Command and Control

It is worthwhile at this stage clarifying how the various quantities and behaviours in the Kuramoto model can be reinterpreted in the context of Command and Control.

The key variable of the model, θ_i, represents progress in time through a continuous decision cycle. Certainly, the discrete steps of Boyd's OODA loop or the MAP are not at odds with the continuous nature of the human decision making and execution process. Indeed, the steps merely mark significant milestones in the process, which may also be points of reference for others in negotiating their own decision cycles.

The coupling constant has a slightly less direct interpretation. At one level it can be seen as the degree of responsiveness required of one node's decision progress to the behaviour of another. For example, this can be understood as the degree of Mission Command [van Creveld, 1985]: how closely should neighbouring C2-agents track their progress in making decisions, what degree of autonomy is permitted in the system? Of course, in the Kuramoto model this constant is assumed to be uniform across the complete network. This strong assumption is retained in discussing other networks below, but it may also be a function of where in the command chain (seen as a hierarchy) the node resides, or even as a function of time subject to certain dynamical conditions as in Adaptive Control [Stewart, 2006]. Mathematically, all these possibilities are easily modelled but become the realm of simulation models and thus beyond the scope of this initial study.

Next we come to the specific nature of the interaction. There is actually some freedom here in representing how one node outputs its information about its progress in the cycle to an adjacent node. The Kuramoto model as given above is consistent with *complete transparency* amongst adjacent C2 agents in their progress through the decision and execution process. This is a strong assumption with implications that any agent can adjust continuously within its own process. However the other extreme is also artificial, that only at certain milestones – say "Decision" or "Act" – can one agent determine the state of progress of another agent and adjust accordingly. How such interactions can be modelled I shall treat later in representing Boyd's adversarial OODA interaction. However, within a C2-system the reality lies somewhere in between, where, within certain echelons of human organisations, one person can readily know of another's progress with respect to shared products or processes through formal update briefs, technologically enhanced communication and collaboration media and/or less tangible social/informal interactions. In this respect, much of an analyst's labour in capturing informal networks in military organisations can be subsumed into the degree of transparency between nodes of the actual formal command structure or process. The periodicity of the interaction term, encoded as the sine of the phase differences, is also sensible in light of the diminishing relevance of "stale" information. A C2 agent who has dropped out of a particular decision cycle must typically adjust to the state of adjacent partners within the *current* cycle.

The two types of behaviour seen in synchronisation, locking and drifting, have their counterparts within given military C2 systems, although it is hard to make a direct comparison based on the complete network. Recall the brief comment concluding the previous section on clustered behaviour. A typical military C2 system will have many localised hubs of activity integrating several specialist products, especially in the world of Joint Operations. For example, a team within, say, the J5 planning function whether at strategic, operational or tactical levels must be able to mutually lock into a common cycle when focussed on a particular operation. But there are other teams corresponding to the various functions, for example, the intelligence J2, logistics J4 or communications systems J6 functions. Each of these teams needs to be able to internally synchronise. This is also why more complex networks are required. For example, a Joint Task Force may not directly interact with specific J-functions in an operational headquarters. Contrastingly, within the headquarters a Chief of Staff (COS) must be able to respond to the progress of the J-function teams in order to integrate them into an overall Joint Concept of Operations (CONOPS). The ability to "drift" from one team to another is vital to the COS being able to herd the separate clusters into one collective cycle. While the term "drifter" is pejorative in the military C2 context, the point being made is that it is not necessarily consistent with the mission of a military C2 system that all nodes lock into the one collective. But the system ideally should be designed so that those who need to lock are enabled to do so, and those who must oversee multiple clusters of locked activity are not dynamically driven exclusively into the orbit of one or another cluster.

The dynamical linking of cycles overcomes some of the criticisms levelled at the original Boyd OODA loop, namely that because it does not include the effects generated at the "Act" stage it cannot represent the delays that every real C2 system encounters [Brehmer, 2006]. The drive to represent effects in C2 models therefore leads variously to models useful for combat or headquarters processes and products with varying levels of fidelity. However, it is difficult then to link such representations in reasonably computable models: the data representing kinetic effects, non-kinetic effects (EBO) and internal headquarters processes that direct and respond to such battlespace activity are extremely disparate in nature. We argue that what is important in the space of effects is the timescales of decision cycles associated with them and not the detailed products that they generate. By focussing less on the specific products – be they effects in the battlespace, incident reports in a tactical headquarters, planning documents in an operational headquarters or broad guidance at the strategic level – and by focussing more on the time-scales within which these products are used or created, one is able to integrate into a single description the C2 activity of all – from a tactical commander or subordinate up to senior commanders and the elected political authority to whom they must answer. Therefore this paradigm enables C2 across all its levels to be brought together into a unified description.

Some characteristic time scales are in order. A fighter pilot engaged with an aerial enemy has some seconds within which to progress through their decision cycle. On the ground a deployed land combat unit engaging with a similarly deployed enemy may have seconds, to minutes or even a few hours to formulate and execute decisions. More precise data from various sources on engagements in Gulf War I with surface targets and theatre ballistic missiles are summarised in [Moon *et al.*, 2002] giving numbers from minutes up to a few hours. Of course in these cases the aim is not to synchronise with the enemy but to outpace their decision cycle. However, none of these C2 nodes are free to engage with the adversary in isolation. The pilot or combat commander may have to synchronise their decision through a Task Force Commander (TFC). Pulling the TFC in the other direction is their responsiveness to outputs from higher levels of command, for example the issuing of a Branch or Sequel to a current CONOPS from a higher level operational headquarters. Coyle [1987] has already noted the "vibrational", namely oscillatory, character of a headquarters C2 system. Pertinent is the broader scale of information which a headquarters must process. As a consequence, the time-scales over which activity must be planned are longer. The experience of the author's team in work with Australian Defence Force higher C2 [Kalloniatis and Wong, 2007, Hanlon *et al.*, 2008] suggests that tasks undertaken by operational level J3 staff within execution of an operation may occur on the scale of a few hours to a day. But, the J3 function must work within the campaign view owned by the J5. The time-scales are longer still for all the same reasons of the breadth and depth of information.

As one approaches the pinnacle of the C2 hierarchy in any representative democracy one encounters the national strategic level which is characterised both by quite short and long time-scales: short because of the rapid time scales of media reporting ("the CNN War") which impact on the political will of the national strategic level; long because of the strategic nature of a government's view of how a nation's military should be used. This effect is amplified from the tactical up to the national strategic levels by the interagency, non-governmental and coalition interactions implicit with modern military operations. We argue that it is this diversity of time-scales of decision cycles across disparate structures which must still somehow synchronise their activity which gives Complex Endeavours [Alberts & Hayes, 2007] that fall within the theme of this conference their intrinsically unpredictable character.

It should be clear from this discussion that, however simply it mixes them, the Kuramoto model contains the essential elements to be found in military C2 systems.

Self-synchronisation on general networks

We now return to the mathematical formulation. A general network is introduced straightforwardly using the adjacency matrix A_{ij} which has value one if a link (or edge) exists between nodes *i* and *j* and is zero otherwise; for simplicity we remain within the bounds of undirected graphs, though further generalisations are possible. The governing time evolution equation is then:

$$\dot{\theta}_i = \omega_i + \sigma \sum_{j=1}^{N} A_{ij} \sin(\theta_j - \theta_i) \,. \quad (4)$$

Kuramoto's method of analytic solution is no longer available, precisely because the adjacency matrix, present now in the evolution equation, is not present in the definition of the order parameter *r* which remains unchanged. The method of Kuramoto's solution has however inspired approximations and mean-field approaches which, consistent with numerical simulations, continue to exhibit the characteristic transition behaviour from incoherence to synchrony. What these show, as hinted earlier, because of the incompleteness and finiteness of the network is an intermediate type of behaviour, is partial synchrony: local clusters of locked phases form with their own frequencies averaged over the participants in the cluster, while other nodes drift randomly with respect to these various hubs. As coupling is increased some networks such as Erdős-Rényi (a class of random) graphs will transition to total synchrony – namely with a single locked core – but only via partial synchrony. Others, such as scale free networks which have a spectrum of connectivity from a small number of highly connected hubs to a large number of sparsely connected nodes, will transition directly to total synchrony from incoherence bypassing the partial state altogether. These results, reported in [Gómez-Gardeñes, 2007] are intuitive based on the degree of randomness of the respective networks. Finite lattices synchronise very poorly [Dekker, 2007]. Small world networks (recall the popular notion of "6 degrees of separation"), which can be generated by randomly rewiring lattices, tend to synchronise at very low couplings even with small rewiring probability [Hong, 2002]. It emerges that the criterion determining whether a node will lock or drift with respect to some hub involves a delicate interplay between the coupling constant, the difference in natural frequency of the phase oscillator at that node from the mean frequency of the potential hub and the degree of the node,

$$d_i = \sum_{j=1}^{N} A_{ij} . \qquad (5)$$

Simulations have been useful in enabling study of dependences of key quantities such as the critical coupling in terms of an analyst's favourite network theoretic metric. However, they invariably depend strongly on how in a given simulation a particular network has been generated and which metric one prefers to see the world in terms of. The average degree of a network, clustering coefficient and average distance are three popular choices which have seen some attention [Dekker, 2007].

The valuable role played by simulations is being supplemented by the insight of analytic approaches, which has been the focus of our own research into dynamical processes on networks. This author is seeking to make progress on this front by using an *ansatz*[2] for the solution to the dynamical equations in the synchronised regime

$$\dot{\theta}_i(t) = \dot{s}(t) + \eta_i(t) \qquad (6)$$

where $s(t)$ represents a collective component contributing to part of the underlying behaviour at every node while $\eta_i(t)$ is a "noise" function [Reichl, 1998]. Structure is encoded in the noise, and is controlled by a minimal number of parameters whose values are determined by the localised interactions of the system. The *ansatz* is then inserted in the equations; an average taken over the noise functions and consistency constraints on the parameters extracted such that the *ansatz* solution is in fact a true solution. The method should recover firstly that the steady state behaviour of the collective mode corresponds to the average frequency:

$$\dot{s} \to \overline{\omega} .$$

The Holy Grail is the extraction of a relationship between the key properties attributed to a node *i*:

- The difference between its frequency and the mean frequency, $\delta\omega_i = \overline{\omega} - \omega_i$;
- The coupling constant, σ;
- The parameters characterising the noise at the node;
- The connectivity between *i* and its adjacent partners, *j*;
- The difference between initial conditions (*t*=0) for the oscillator at the node *i* and that at a connected node *j*, $\delta\theta_{ji}(0) = \theta_j(0) - \theta_i(0)$.

Currently the author is exploring different structures for noise in order to derive such a fundamental relationship.

To summarise the main point of this section, we have seen that more complex networks – as are appropriate for military C2 – can be analysed within the framework of the Kuramoto model to expose the conditions for synchronisation. Simulation and analysis are powerful methods working hand in hand to quantify the structural and dynamical relationships such that a network overall synchronises and that a particular node, given its intrinsic properties and its connection to its neighbourhood, can participate in the collective behaviour or drift independently. Using such results in application to C2, networks can be constructed or enhanced according to whether a particular C2 node

[2] An *ansatz* can be thought of as an initial informed guess to the solution of an equation which is then refined or constrained such that a true solution is arrived at.

needs to be able to lock or drift according to its military function. When it comes to fruition, the approach can be applied to analyse existing networks as they are encountered in the military C2 world.

Generalisation of Kuramoto's model to Boyd's context

Thus far, interactions between decision cycles within a C2 system have been discussed. We now turn to the case of blue-on-red interactions, the regime of Boyd's original OODA concept. Instead of θ_i, we now use β_i and ρ_i to represent the phases of oscillators at nodes of blue and red networks, with intrinsic frequencies ω_i and ν_i respectively. Let B_{ij} and R_{ij} represent the adjacency matrices of the *decoupled* blue and red networks, of size N_B and N_R respectively. The interactions between blue and red systems will be represented by the adjacency matrix M_{ij} with $N_{BR} < N_B + N_R$ nodes. Finally let $\sigma_B, \sigma_R, \varsigma_B$ and ς_R represent coupling constants for intra-blue, intra-red, blue-to-red and red-to-blue interactions. The model can now be given in terms of the coupled differential equations:

$$\begin{aligned}\dot{\beta}_i &= \omega_i + \sigma_B \sum_{j=1}^{N_B} B_{ij} \sin(\beta_j - \beta_i) + \varsigma_B \sum_{j=1}^{N_{BR}} M_{ij} f(\rho_j - \beta_i) \\ \dot{\rho}_i &= \nu_i + \sigma_B \sum_{j=1}^{N_B} R_{ij} \sin(\rho_j - \rho_i) + \varsigma_R \sum_{j=1}^{N_{BR}} M_{ij} f(\beta_j - \rho_i).\end{aligned} \tag{7}$$

What has not been specified is the output function f for the blue-red interaction. If we were to follow the Kuramoto model, these could also be sine functions; the adversarial nature of the interaction would be reflected in the different coupling constants and formal distinction between blue and red networks. In other words, a blue node connected to a red node must also, in the first instance, seek to synchronise with the adversary in order to defeat them. However, this requires some modification. The assumption of complete transparency justifying the sine function for intra-C2 interactions is inapplicable here. Intelligence may expose some of the inner workings of the adversarial decision process, but it is likely to be noisy (which can also be modelled). The simplest alternative is that one can only take the "Act" step in the adversary's OODA loop as input. Let that point in the cycle correspond to a phase angle κ, some point on the unit circle arguably in the fourth quadrant (Figure 2).

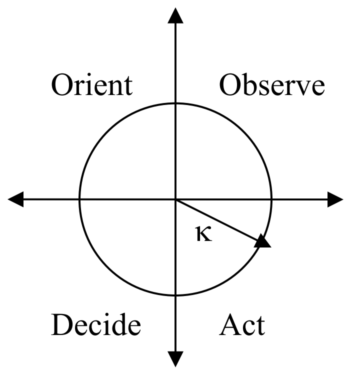

Figure 2 Choosing a point in the Act stage of the OODA loop to represent Boyd's interaction mathematically.

Then one could choose, in the spirit of the Kuramoto model,

$$f(\rho_j(t)-\beta_i(t)) = \sin(\rho_j(t)-\beta_i(t))\delta(\rho_j(t)-\kappa)$$

where we have used a "Dirac delta function" so that the output is sensitive only to the Act phase angle at the time that ρ_j has reached that point in its OODA loop. This transforms the continuous steady Kuramoto interaction into a type of "pulsed" interaction. In this manner, left to itself the phase β_i will seek to track the red agent's "Act": the interaction kicks in only when ρ_j has progressed to "Act". However, Boyd's concept is that blue must "get inside" red's OODA loop. This is easily achieved with an additional correction factor λ in the argument of the function:

$$f(\rho_j(t)-\beta_i(t)) = \sin(\rho_j(t)+\lambda-\beta_i(t))\delta(\rho_j(t)-\kappa)\,. \qquad (8)$$

This ensures that blue seeks to track ahead of red by some constant amount λ. Of course, the two parameters κ and λ can be combined into one but there is value in separately tuning them in future numerical studies. The delta function in Eq.(8) can also be smeared out by Gaussian functions.

The above model Eq.(7) is most likely not exactly soluble even were the networks to be taken as complete. But some things can be said, most importantly that the new interactions introduce additional instability into the dynamics of the oscillators β_i due to the competing pull by diverse hubs on a node straddling both blue and red networks. In combination with self-synchronisation within the blue C2 network this will have the effect of demoting some blue nodes from being "locked" to "drifting". Correspondingly, the critical coupling σ_B at which the blue network is able to synchronise will increase. In fact, the most likely insightful quantity is the ratio of the two couplings σ_B / ς_B.

The key quantitative questions which need to be addressed are of a local and global nature. Locally:

- Are nodes which *need* to cooperate *able to* dynamically lock?
- Correspondingly, are nodes which should engage with the adversary free to do so, or do they lock into intra-C2 clusters?

- With more complex behaviour, is there a regime where nodes can lock for appropriate amounts of time, drop out, engage in the adversary OODA loop and then return to the original hub?
- Can λ be tuned to achieve such behaviour?

While globally some of the questions are:

- Is there a critical value of σ_B / ς_B at which synchronisation can occur?
- What degree of heterogeneity of frequencies is consistent with synchronisation?
- How do two networks of different structure fare against each other?

These questions may require a combination of analysis and simulation to answer.

Final statements

We argue that this approach represents a "new paradigm" because it manages to move beyond the study of static network metrics and static diagrammatical "protomorphs" [Harré, 1970] of dynamical processes without falling into the trap at the other extreme of becoming immersed in the minutiae of computer simulation models. The Kuramoto model is simple and elegant, structural and genuinely dynamical and thus speaks to the essence of Command and Control. Nevertheless it offers a framework for higher fidelity embellishment and simulation studies.

Apart from such work to extract further insights, the process of validation and application to real world C2 systems beckons. Validation is most usefully done in the context of a given C2 system or organisation with a study of its hierarchy and other networked processes and the time-periods associated with them. On the one hand, the model allows for mathematical determination of the ideal self-synchronisability of the system and the pattern of locked and drifting nodes. On the other hand, this needs to be compared to the observations of participants in the given system with respect to questions such as: Do their duties require them to synchronise in a team or to oversee many disparate activities? Are they able to synchronise or move with sufficient flexibility across their areas of responsibility? With success in this respect, the program can move on to generating improvements to the system. But the bold prospect which this approach may provide is that, armed with the time-period of key C2 processes, appropriate networks can be designed *ab initio* such that their overall mission is achievable and the individuals who serve within them are able to achieve their individual mission effectively and efficiently.

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