A Computationally-Feasible Algorithm for Estimation of Opponent Strength in Urban Combat

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Estimating Opponent Troop Levels and Position

- Increasing numbers of (semi-)autonomous vehicles are increasing pace of decision-making.
- Would like a semi-automated means for estimating the opponent's ground forces levels and positions.
- Automated estimators (filters) have proven indispensable in tracking/targeting aircraft, missiles, etc...
- Troop movements are much slower, but much more complex.
  - Aircraft state is type, position, velocity, possibly attitude.
  - Opponent ground-force state consists of fire-teams (or individuals), their locations, equipment, health, and more.
  - May be **many** fire-teams.

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Complexity

- Kalman filter for air-vehicle state estimation.
- Must propagate observation-conditioned probability distribution over state space.
- In order to make this practical, we assume linear dynamics, and then only need to propagate the mean and covariance.
- Simplified ground-force estimator: Assume homogeneous fire-teams with associated strengths.
- Strengths will subsume health and equipment.

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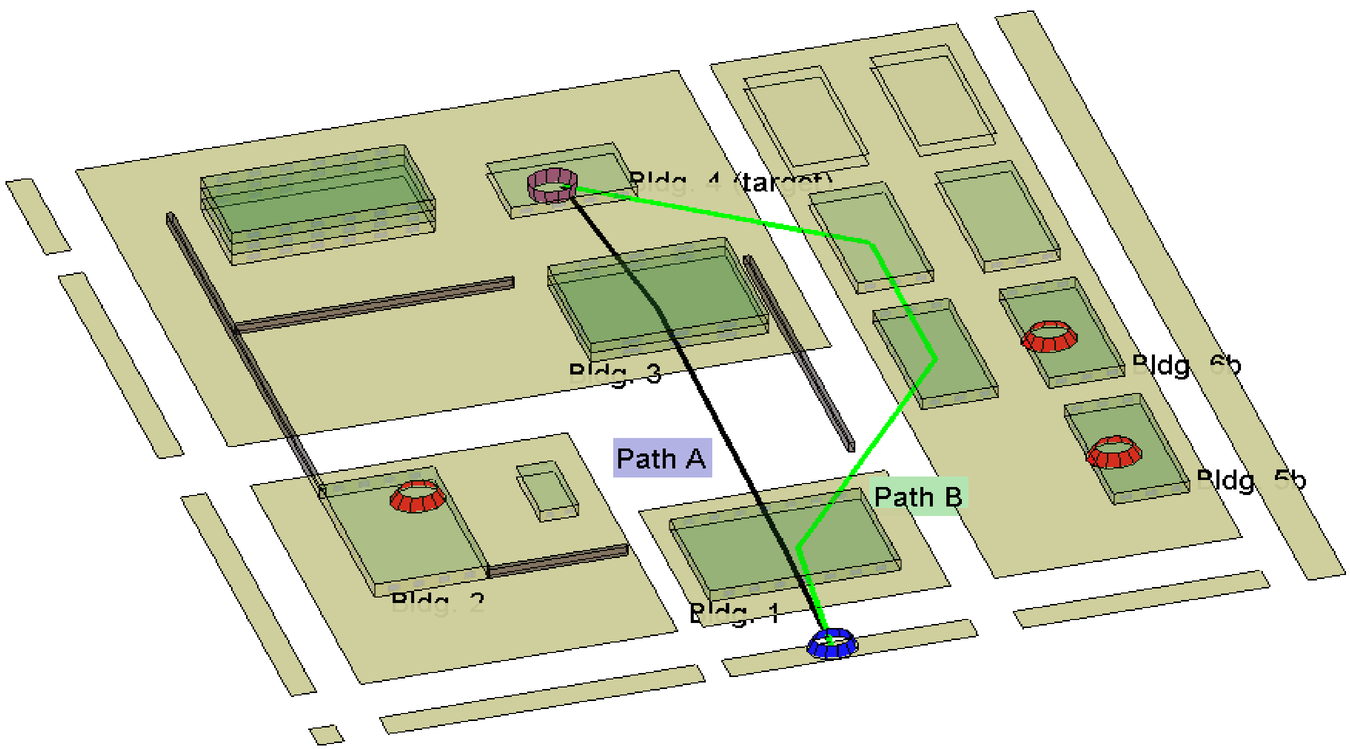
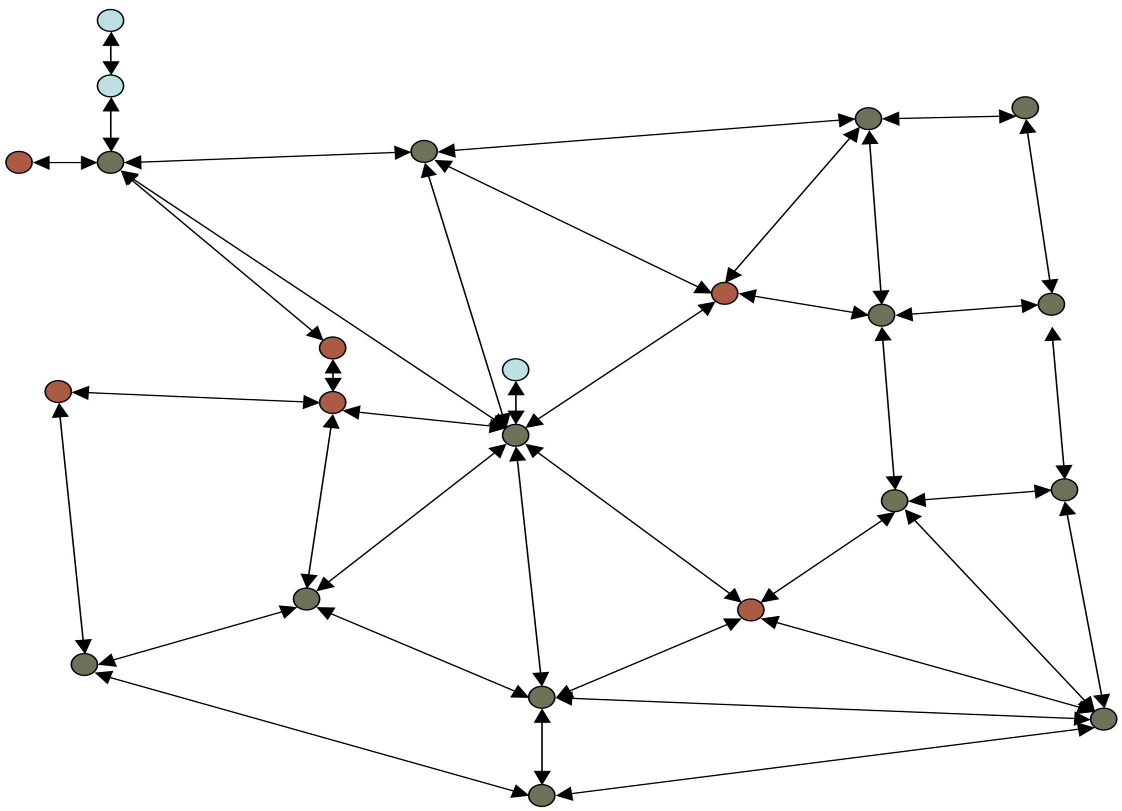
Complexity (Model Issues)

- Each fire-team has location and strength.
- Positions are modeled as discrete locations on a graph (the movement graph).
- Fire-teams move can from one node to another adjacent node on this graph.
- Each fire-team has an associated (discrete) strength.
- Assume fire-teams do not re-combine (although may occupy identical locations).


Bldg. 4 (target)
Bldg. 3
Bldg. 6b
Path A
Path B
Bldg. 5b
Bldg. 2
Bldg. 1



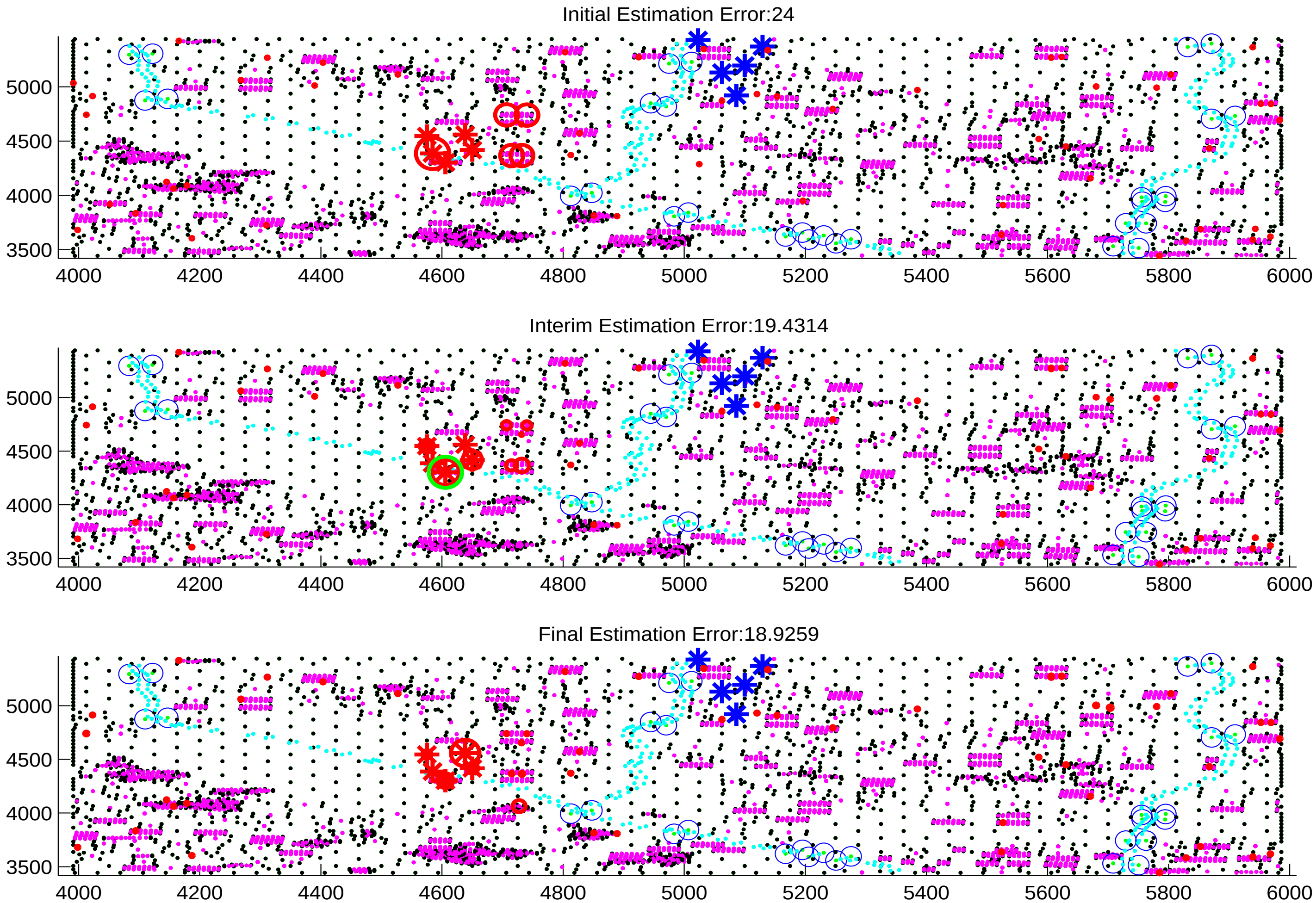
ASSOCIATED MOVEMENT GRAPH

Complexity

- If used a probabilistic filter, we would need to propagate an observation-conditioned probabilty distribution over the set of possible opponent-states.
- Assume $L = 1000$ locations (*very* small, unrealistic).
- Assume up to $N = 30$ enemy fire-teams.
- Then the number of possible laydowns (location possibilities, neglecting strength!) is

$$\binom{L+N-1}{N} \simeq 10^{60}.$$

- Even neglecting strength, one would be working with probability distributions over a set of size roughly 10^{60}.
- Propagating such a distribution is completely unrealistic.



Initial Estimation Error:24
Interim Estimation Error:19.4314
Final Estimation Error:18.9259

Strength Distribution Form

- Let N be the maximum total enemy troop strength.
- Let the graph be denoted by $(\mathcal{L}, \mathcal{E})$ where $\mathcal{L}$ is the set of nodes, and $\mathcal{E}$ is the set of edges (pairs of nodes that are connected).
- The Strength Distribution, at any given time, maps locations on the graph to strength levels, $S_t : \mathcal{L} \to [0, N]$.
- Let $[S_t]_l$ denote the estimated strength at node l at time t.
- Let $[H_t]_l$ be the (unknown) true strength at node l at time t.
- Note S_t is a set of $L = \#\mathcal{L}$ values; much, much smaller than a probability distribution over the set of possible true strengths, $P : \{H_t\} \to [0, 1]$.

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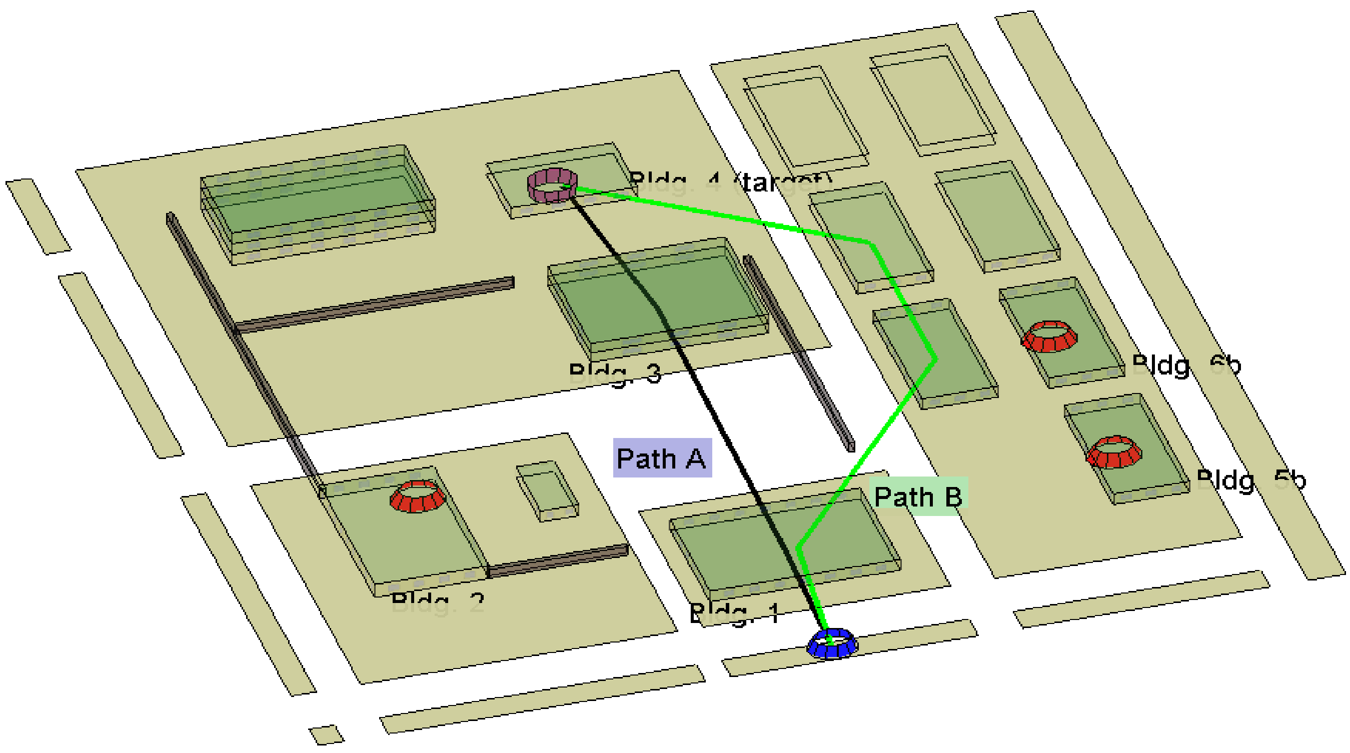
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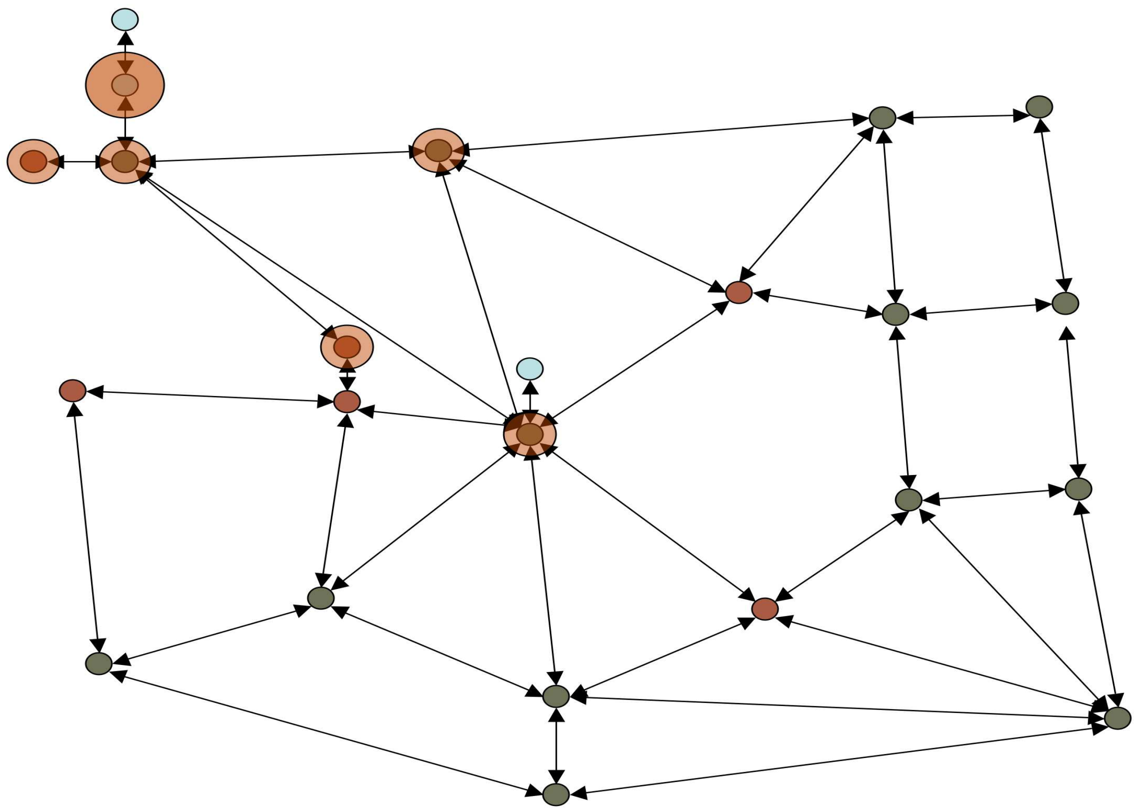

Further dynamics flow/diffusion

Dynamics

- A filter has two components: Dynamics propagation and observation updates.
- Let time be discretized, with time-step indexed by $t \in \{0, 1, 2, \ldots\}$.
- Suppose have estimate S_t at time t, and need to propagate forward in time (without observations).
- We let $S_{t+1} = \mathcal{F}^T S_t$, where $\mathcal{F}$ is referred to as the flow matrix.
- Proportion of strength flows from node i to node j in one step by amount $\mathcal{F}_{i,j} \in [0, 1]$
- Conservation of mass implies $\sum_{j \in \mathcal{L}} \mathcal{F}_{i,j} = 1$.

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Dynamics (continued)

- $\mathcal{F}_{i,j}$ represents average proportion of strength we expect to flow from i to j in one step.
- If half the time, we'd expect a force to move from node 177 to node 214 and half the time to node 35 instead, one would have $\mathcal{F}_{177,214} = \mathcal{F}_{177,35} = 0.5$ and $\mathcal{F}_{177,j} = 0$ otherwise.
- In deterministic-model case, $\mathcal{F}$ consists only of 1's and 0's.
- To allow for unanticipated opposing-commander input, we allow for dynamics according to $S_{t+1} = \left[\mathcal{F}^T + \mathcal{U}_t^T\right] S_t$, where $\mathcal{F} + \mathcal{U}_t$ is the commander-augmented flow matrix.
- $\mathcal{U}_t$ is presumed unknown, with no associated probability distribution (as in robust control methodology).

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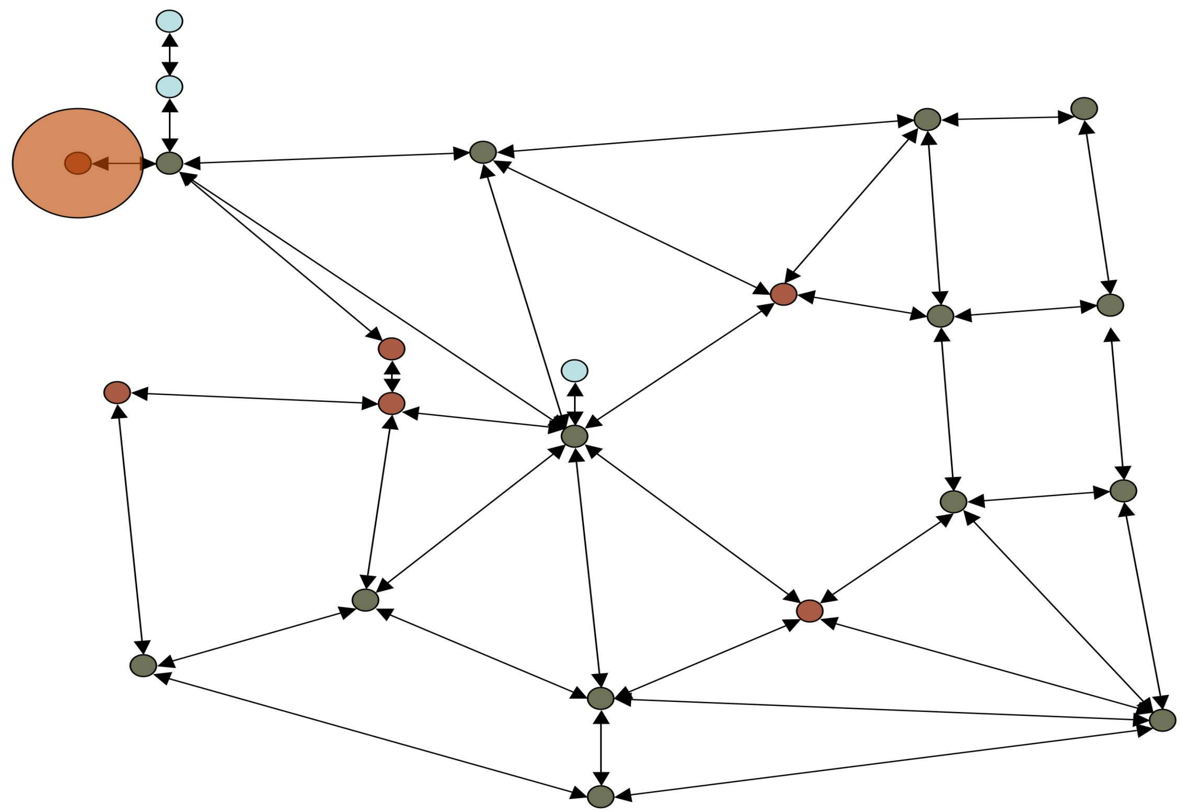
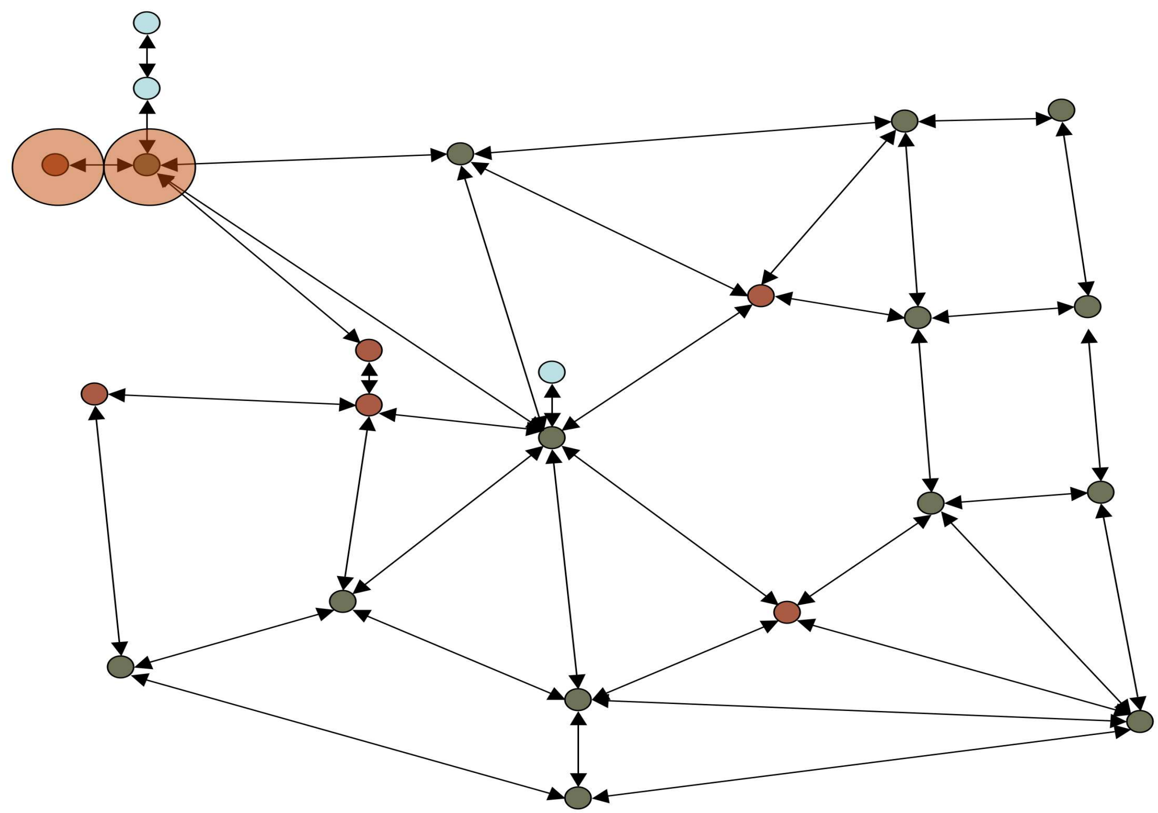
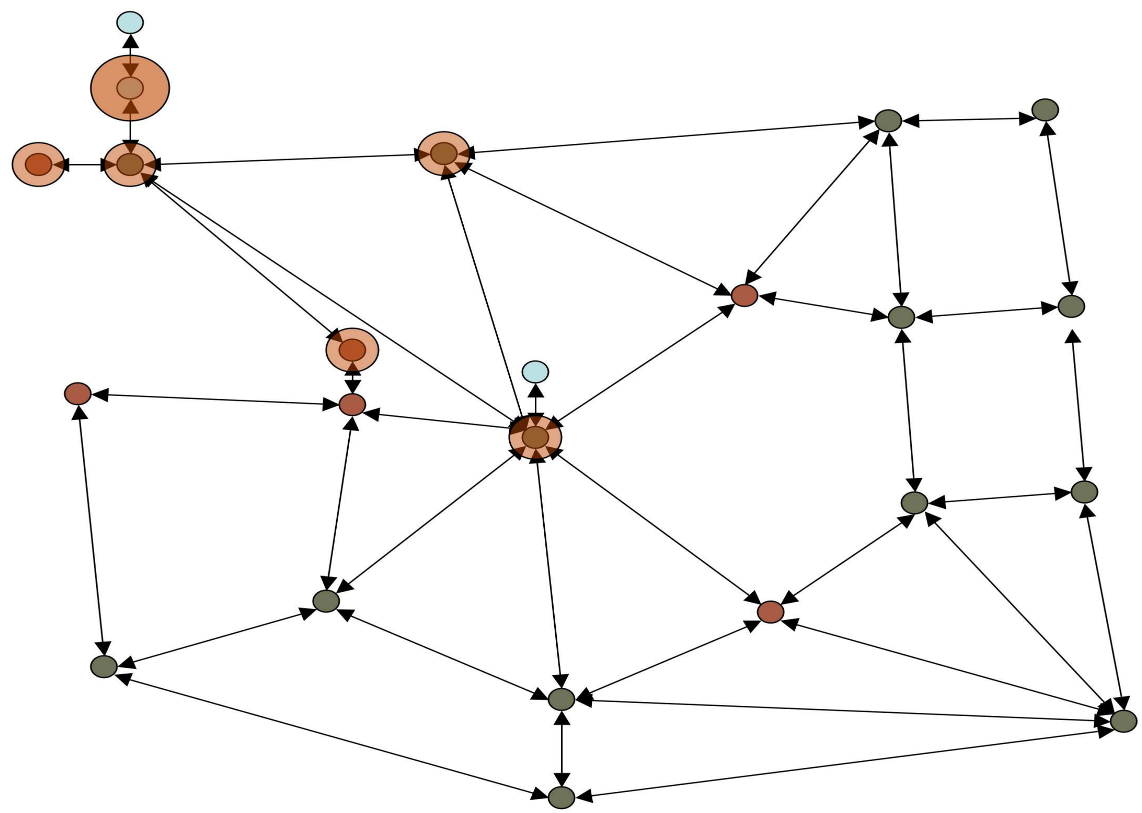


An Initial Strength Mass



After dynamics flow/diffusion



Further dynamics flow/diffusion

Observation Processing

- Without observations, the strength distribution would typically tend toward a rather flat distribution over many, many nodes (not knowing virtually anything about the opponent position).
- Observations must conserve strength mass (i.e., keep $\sum_{l \in \mathcal{L}} [S_t]_l = N$).
- Let the strength distribution before observation at time t be S_t, and after the observation(s), $\widehat{S}_t$.

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Observation Processing

- We choose an estimator form, using Bayes rule as a guide.
- Suppose we had a proabilitiy distribution over $\mathcal{L}$, with p_l representing the probability that the one (and only one) object is at l.
- After observing the object at l, the a posteriori distribution would be

$$\hat{p}_l = \frac{\beta/\alpha}{1 + (\beta/\alpha - 1)p_l} p_l, \qquad \hat{p}_\lambda = \frac{\beta/\alpha}{1 + (\beta/\alpha - 1)p_l} p_\lambda \qquad \forall\, \lambda \neq l$$

where α is probability of a false positive, and $1 - \beta$ is probability of a false negative.

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Observation Processing

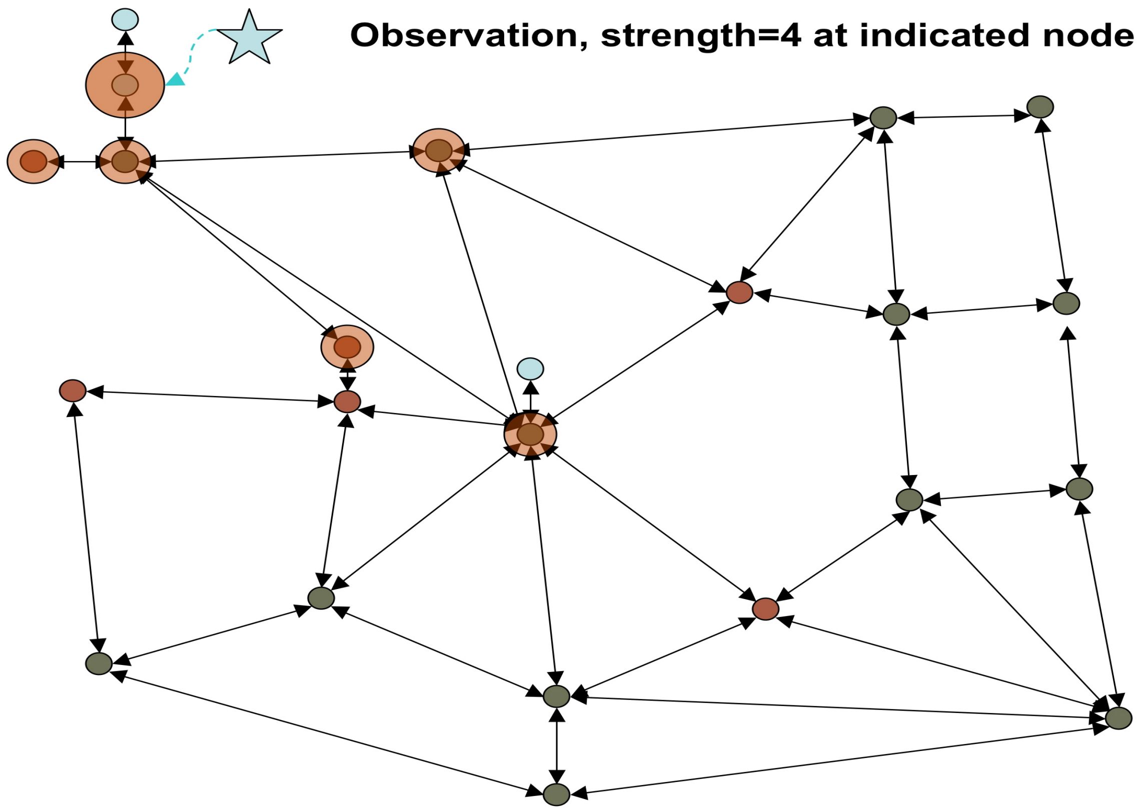
- Suppose we observe strength $y \in \{0, 1, 2 \ldots N\}$ at node l are time t.
- Suppose the a priori strength at node l at time t is $[S_t]_l = S_l$.
- The a posteriori strength distribution is defined to be

$$[\widehat{S}_t]_l = \frac{1 + k\frac{y - S_l}{S_l}}{1 + k\frac{y - S_l}{N}}[S_t]_l \doteq G(y, S_l)[S_t]_l$$

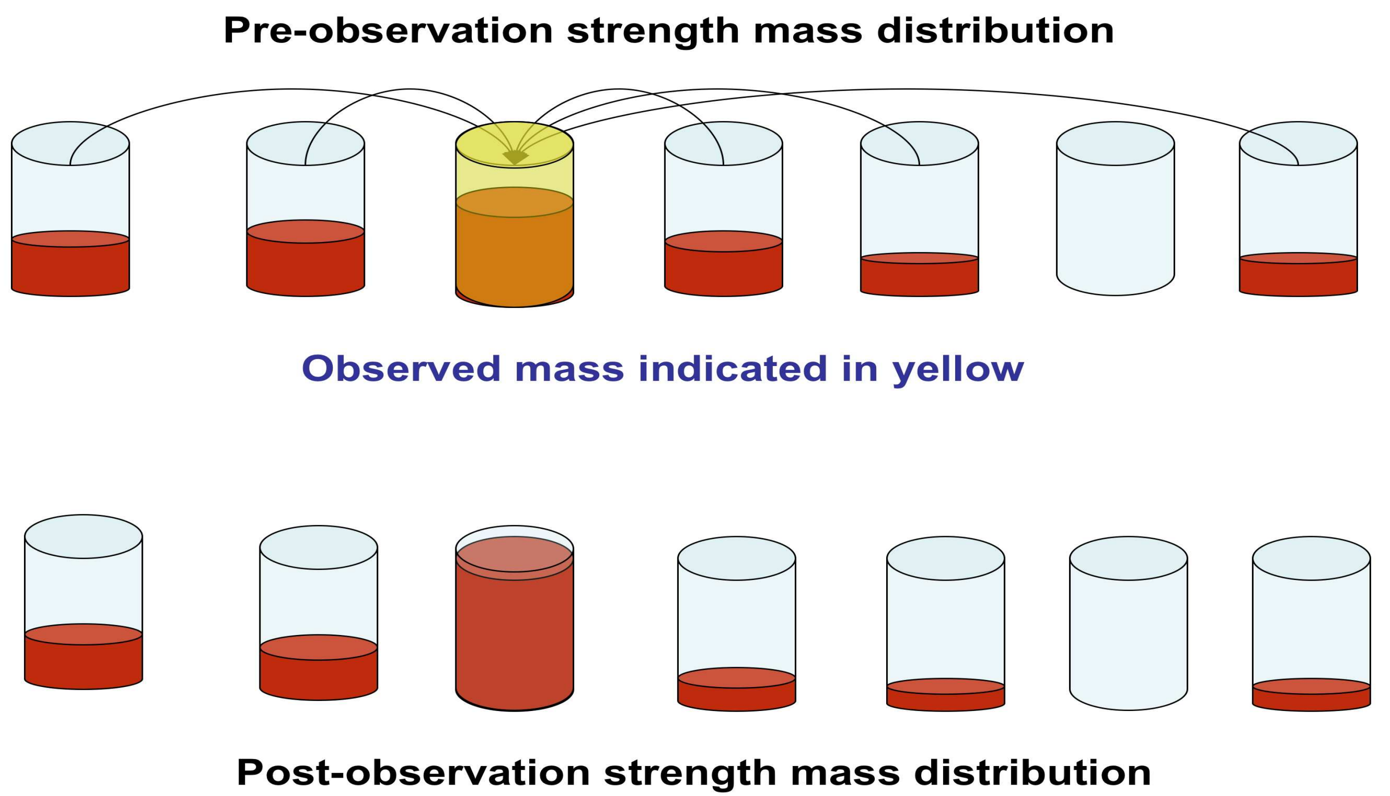
$$[\widehat{S}_t]_\lambda = \frac{1}{1 + k\frac{y - S_l}{N}}[S_t]_\lambda \doteq F(y, S_l)[S_t]_\lambda \qquad \forall\, \lambda \neq l.$$

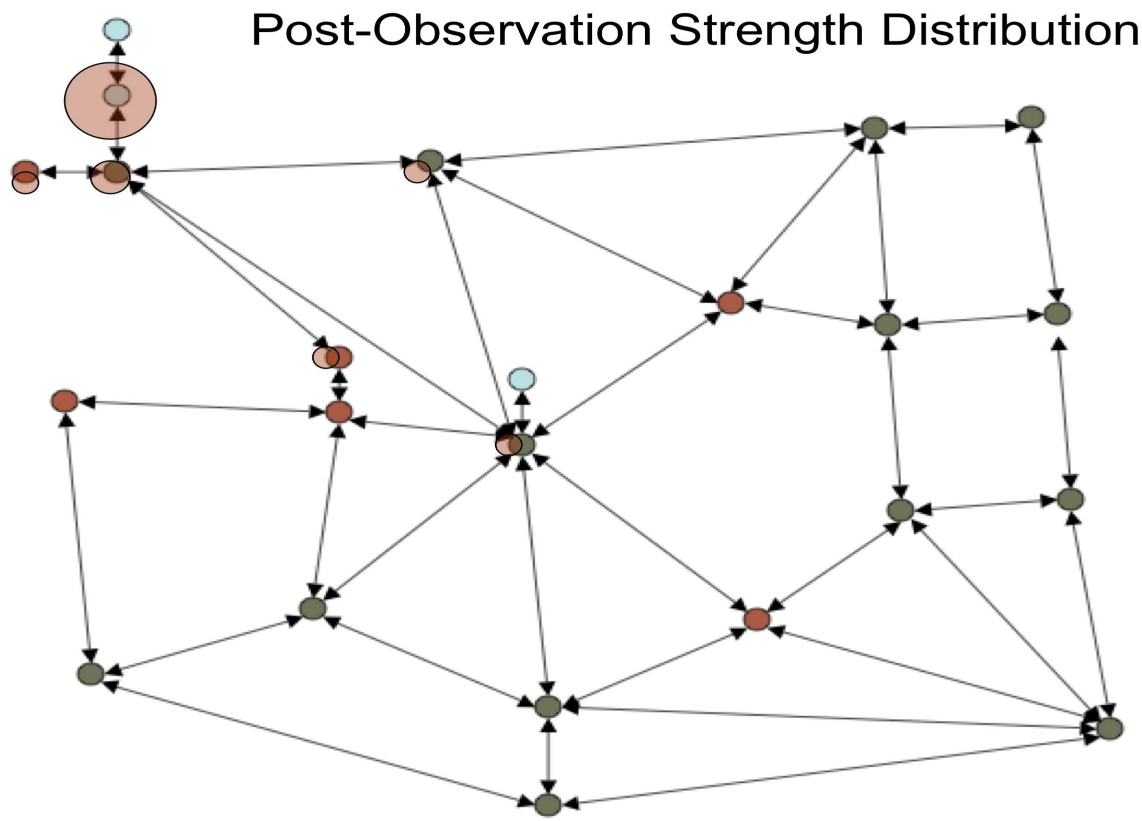
Observation Processing

- Note that $\sum_{\lambda\in\mathcal{L}}[\widehat{S}_t]_\lambda = \sum_{\lambda\in\mathcal{L}}[S_t]_\lambda = N$.
- Further, it can be shown that, given repeated observation of y at node l, with no intervening dynamics, one has $[\widehat{S}_t]_l \to y$; this observation-processing form leads the estimator to converge to the observed strength.



Observation Occurs


Pre-observation strength mass distribution
Observed mass indicated in yellow
Post-observation strength mass distribution


Post-Observation Strength Distribution

Robustness

- The above implies, roughly speaking, that this strength estimator functions as an “observer”.
- We would also like the strength estimator to have some predictable behavior limits as a function of noise in the observations and dynamics.
- Define the norm $\|S_t\| \doteq \sum_{\lambda \in \mathcal{L}} |[S_t]_\lambda|$.
- For the dynamics component, we prove that

$$\|S_t - \mathrm{E}\{H_t\}\| \leq \|S_0 - \mathrm{E}\{H_0\}\| + N \sum_{r=0}^{t-1} \left\| U_r^T \right\|$$

where the $\|\cdot\|$ in the last term is the induced norm on the matrix.

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Robustness

- The effects of the dynamics noise are cumulative, and need to be offset by the observations in order to have good performance.
- Let $y_l^c = y_l^c(s_l, H_l)$ be the observation value that would cause $[\widehat{S}_t]_l = [H_t]_l$, i.e., that would cause the filter to end up with exactly the correct strength estimate at node l after the observation (of y_l^c).
- After much work, one obtains (in the case $[H_t]_l \geq [S_t]_l$)

$$\left\| H - \widehat{S} \right\| \leq \begin{cases} \|H - S\| - \frac{H_l - S_l}{N - S_l} |H_l - S_l| & \\ \qquad\qquad +2\left[G(y, S_l) - H_l\right] & \text{if } y > y^c, \\ \|H - S\| - (1 - F(y, S_l))|H_l - S_l| & \text{if } y^c \geq y \geq S_l, \\ \|H - S\| + 2k|y - H_l| & \text{if } y < S_l. \end{cases}$$

Robustness

- In particular, when $y = H_l$,
  $|H - \widehat{S}| \leq \|H - S\| - (1 - F(y, S_l))|H_l - S_l|$.

- Repeating,

$$
\left\| H - \widehat{S} \right\| \leq \begin{cases} \|H - S\| - \frac{H_l - S_l}{N - S_l}|H_l - S_l| \\ \qquad\qquad +2\left[G(y, S_l) - H_l\right] & \text{if } y > y^c, \\ \|H - S\| - (1 - F(y, S_l))|H_l - S_l| & \text{if } y^c \geq y \geq S_l, \\ \|H - S\| + 2k|y - H_l| & \text{if } y < S_l. \end{cases}
$$

- Note that the first two cases, the previously built-up errors are attenuated, and in the last case, the current observation errors are attenuated.

- These are error bounds, one typically obtains attenuation of both.

Main Convergence and Robustness Results

- If one repeatedly observes y at node l without intervening dynamics, the estimator converges to $[S_t]_l = y$ (sanity check).
- Given noise in the dynamics and in the observation, there is a bound on the expected estimator error in terms of the size of the non-stochastic noise input norm.

$$\|S_t - \mathrm{E}\{H_t\}\| \leq \|S_0 - \mathrm{E}\{H_0\}\| + K_1 \sum_{r=0}^{t-1} \left[\|U_r^T\| + \sum_{\lambda \in \mathcal{O}_r} |[y_r]_\lambda - [S_r]_\lambda| \right]$$

where $\mathcal{O}_r$ is the set of locations that were observed at time r.

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Computational Issues

- One does not use an entire $\mathcal{F}$ matrix, as it is almost all 0's.
- Even with this tremendous complexity reduction, the computations may still burden a real-time system running tasking controllers on top of the estimator.
- For further reduction, developed an approximator where the strength distribution was ϵ and 0 for most of $\mathcal{L}$, and only those locations where the value is above ϵ are explicitly propagated.
- For other points with non-zero strength, one only needs to migrate in and out of the ϵ-value set.
- Strength estimator is embedded into a large-scale commander-support system, which is currently being studied.

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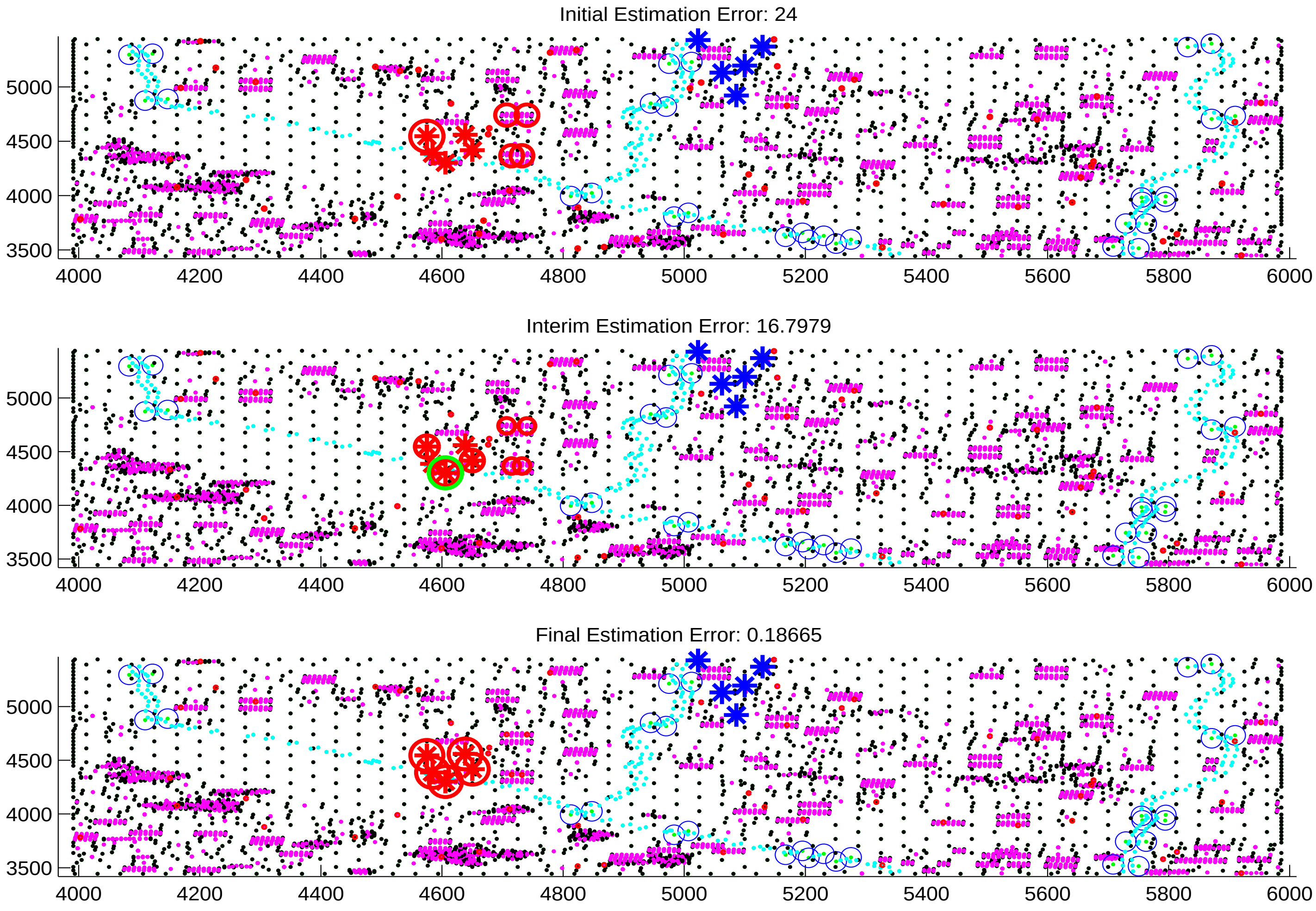
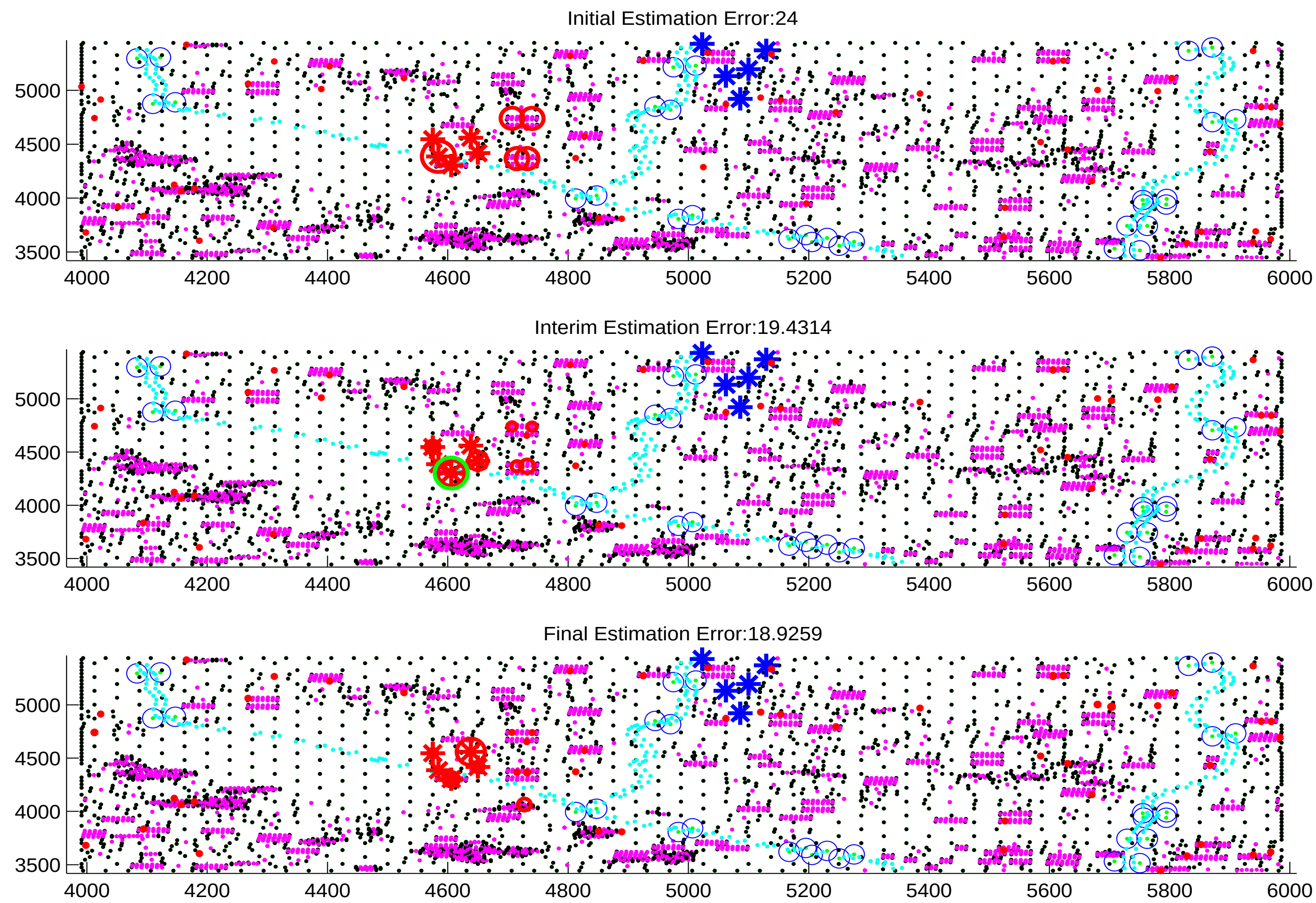


Initial Estimation Error: 24
5000
4500
4000
3500
4000
4200
4400
4600
4800
5000
5200
5400
5600
5800
6000
Interim Estimation Error: 16.7979
5000
4500
4000
3500
4000
4200
4400
4600
4800
5000
5200
5400
5600
5800
6000
Final Estimation Error: 0.18665
5000
4500
4000
3500
4000
4200
4400
4600
4800
5000
5200
5400
5600
5800
6000



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