The Impact of Entropic Drag on Command and Control

12th ICCRTS
"Adapting C2 to the 21st Century

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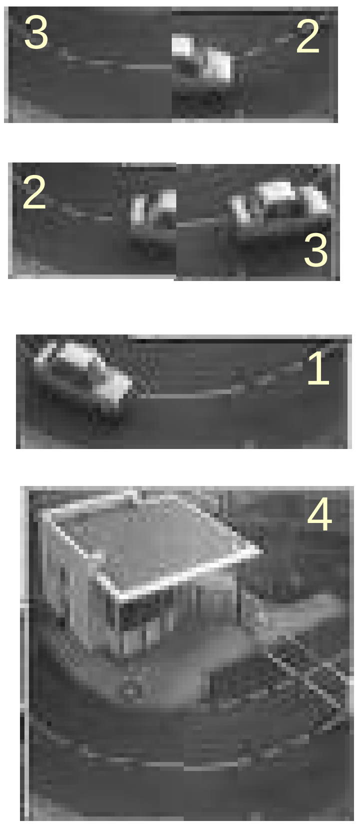
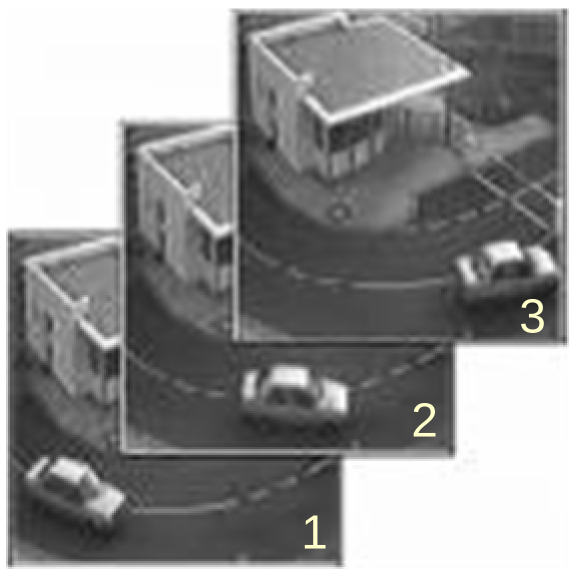
Information and Command and Control Time and Fidelity

Which Information is Most Useful?

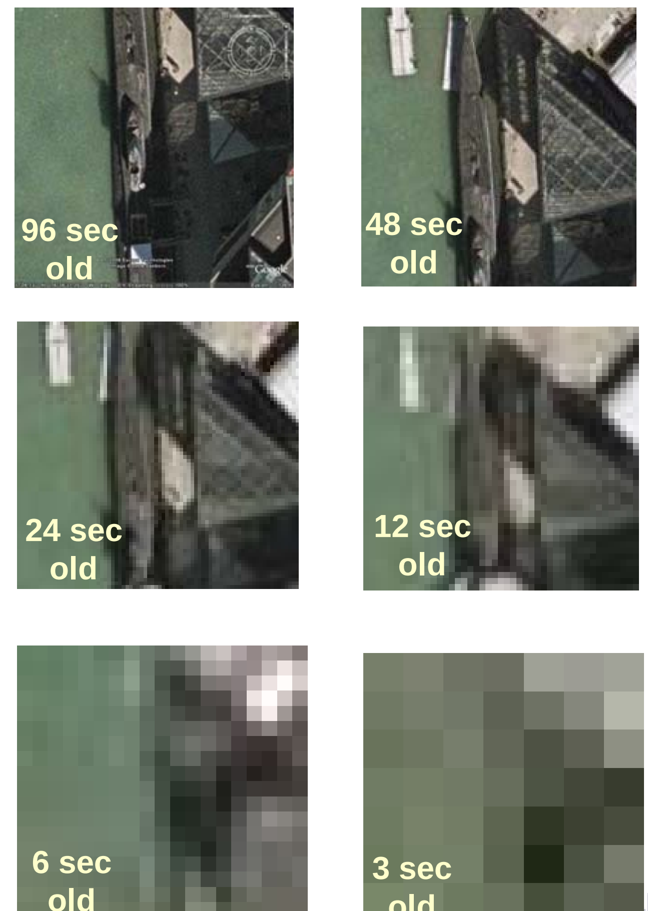


=size of data packet



Current Reality

Which Information is Most Useful?



Information Basics

If all states are equally probable then the amount of information provided by defining a state is:

$$\text{Information} = \text{I} = \log_2|\text{P}|$$

$\text{I} = \log_2 52 = 5.7$ bits

Six bits are required to encode I

	a	b	c	d	e	f
Heart	0	0	*	*	*	*
Diamond	0	1	*	*	*	*
Club	1	0	*	*	*	*
Spade	1	1	*	*	*	*
Ace	*	*	0	0	0	0
Two	*	*	0	0	0	1
Three	*	*	0	0	1	0
Four	*	*	0	0	1	1
Five	*	*	0	1	0	0
Six	*	*	0	1	0	1
Seven	*	*	0	1	1	0
Eight	*	*	0	1	1	1
Nine	*	*	1	0	0	0
Ten	*	*	1	0	0	1
Jack	*	*	1	0	1	0
Queen	*	*	1	0	1	1
King	*	*	1	1	0	0

$$\text{P}_k = \text{Val}_k$$

$$\text{P}_\text{w} = \text{P}_{k_1} \times \text{P}_{k_2} \times \ldots \times \text{P}_{k_n}$$

$$\text{P}_f \cap \text{P}_i = \varnothing$$

$$\text{P}_w = \text{P}_f \cup \text{P}_i$$

$$\text{I} = \log_2|P|$$

Information Basics – Cat World

In what room is the cat?

A three bit question; I= log8 = 3 bits

Entropy = 3 bits

A two bit answer! “In room ?01”

Information = 2 bits

Entropy = 1 bit

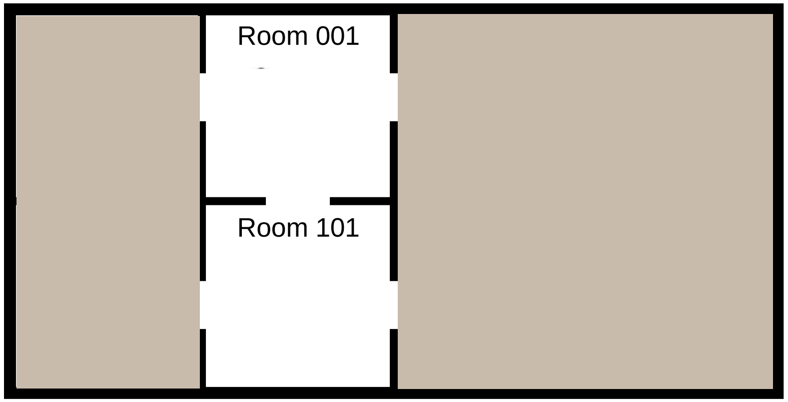
Here’s the cat! “In room 001”

Information = 3 bits

Entropy = 0 bits

But where is the cat?

- 9 bits to the nearest foot
- 11 bits to know the direction the cat is facing
-
-
- 10^{31} bits to know the location of each atom to 10^{-8}m



The limit of knowledge generated by the coding scheme is the *Representational Uncertainty*

$$\left|P_{f_{\mathrm{I}_m}}\right| = 2^{(\log_2 |P_w| - 2^{\mathrm{I}_m})}$$

$$\left|P_{f_{\mathrm{I}_m}}\right| = \frac{|P_w|}{2^{\mathrm{I}_m}}$$

$$h(a) = \log_2 \frac{1}{\Pr(a)}$$

$$H = \log_2 |P_f| = \log_2 |P_w| - I_m$$

$$\Delta H = \begin{cases} -I_m; |P_w| \geq 2^{I_m} \\ \log_2 P_w; |P_w| < 2^{I_m} \end{cases}$$

$$\Delta H = \log_2 | P_f \cap p(I_m) | - I_m$$

$$\varepsilon_r = \frac{\sqrt{n}}{2}\hat{r}; r \in Z^n$$

$$|P_k| = \prod_{x \in X} \left| \frac{x_i - x_0}{\hat{r}_x} \right|$$

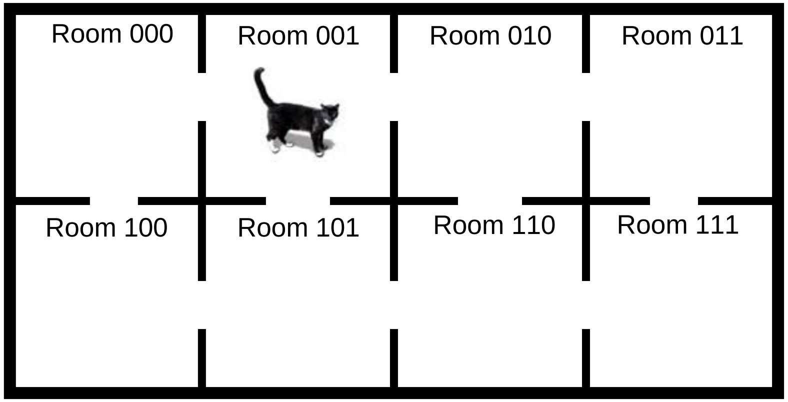
$$|P_k| = \left| \frac{x_i - x_0}{\hat{r}} \right|^n$$

$$|P_w| = \left| \frac{x_i - x_0}{\hat{r}} \right|^{|K| \cdot n}$$

Cat World
Cat Wakes Up, Information is Lost

- **Cat can move at a rate of one room per minute***

- **Time = 0**
  - Information = 3 bits
  - Entropy = 0 bits
- **Time = 1 minute**
  - Information = 1 bit
  - Entropy = 2 bits
- **Time = 2 minutes**
  - Information = 0.2 bits
  - Entropy = 2.8 bits
- **Time = 3 minutes**
  - Information = 0 bits
  - Entropy = 3 bits



Information is lost through Entropic Drag, which is the log of the rate at which the system changes state in an unpredictable manner.

$$\Gamma_w(t) = H_w(I)\frac{dI}{dt} = \log_2 |P_f|(x)\frac{dx}{dt}$$

$$I_w = (I_0 + I_m)\cdot\left(1 - \int_0^{t_m}\Gamma(t)dt\right)$$

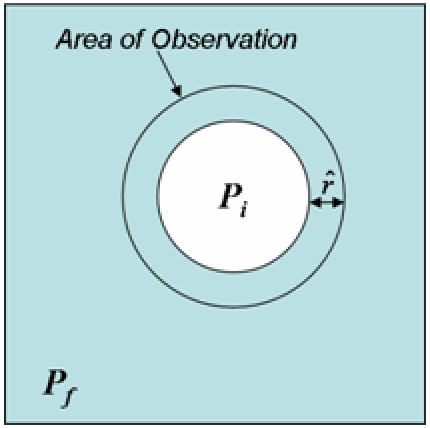
*Assumes random cat distribution throughout all possible rooms

Information Space
Positive and Negative Information

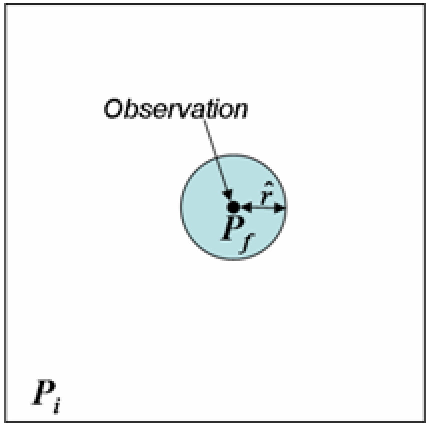
- The State Space of a System may be viewed as an N-dimensional topological field (iff transitions are Markovian)
- This Space may be divided into contiguous sets of feasible space (P_f) and infeasible space (P_i)
- Observations increase the infeasible space
- Entropy increases the feasible space
- Information May be Positive or Negative

$$P^{+} = \{x_1, x_2, .., x_n\}$$

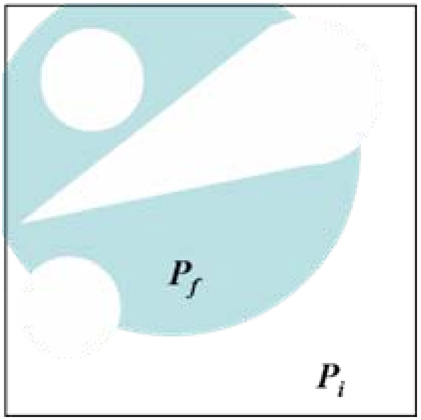
$$P^{-} = \{\neg x_1, \neg x_2, .., \neg x_n\}$$



Negative Information



Positive Information



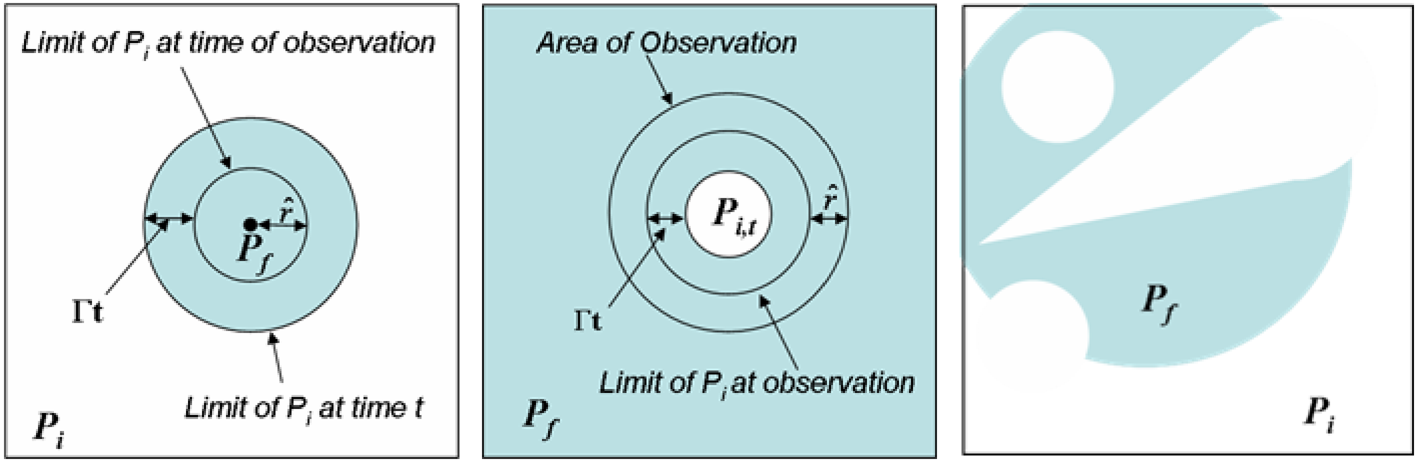
Complex Information

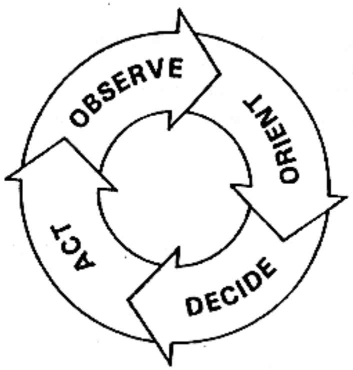
The OODA Loop and Information

- Each step of the OODA loop takes some finite amount of time.
- During that time information is lost through entropic drag.
- Entropic Drag accumulated during an observation impacts all information, not just the information gained from the observation
- Entropic Drag decay information at a constant rate along the information frontier

$$I_m^- = \log_2 \left| P^- \right|$$

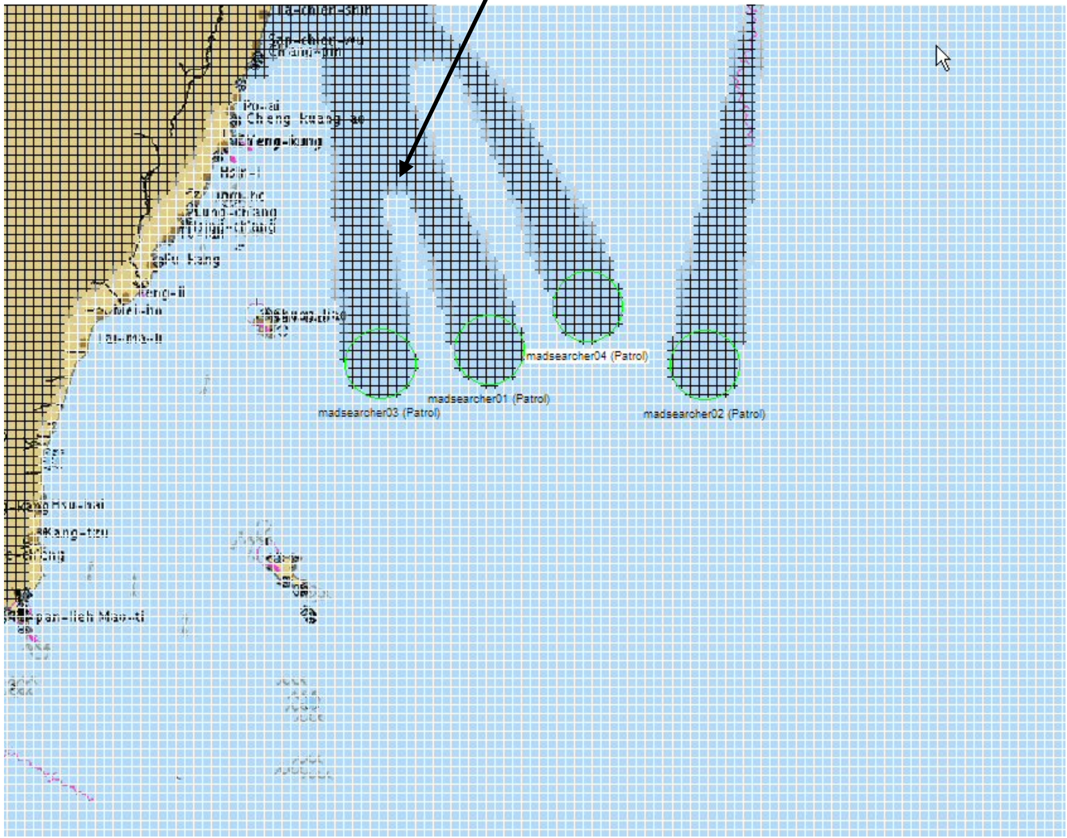
$$I_m^+ = \log_2 \left| P_w \right| - \log_2 \left| P^+ \right|$$





An example

Note: Inference can be used to increase knowledge beyond the original observations



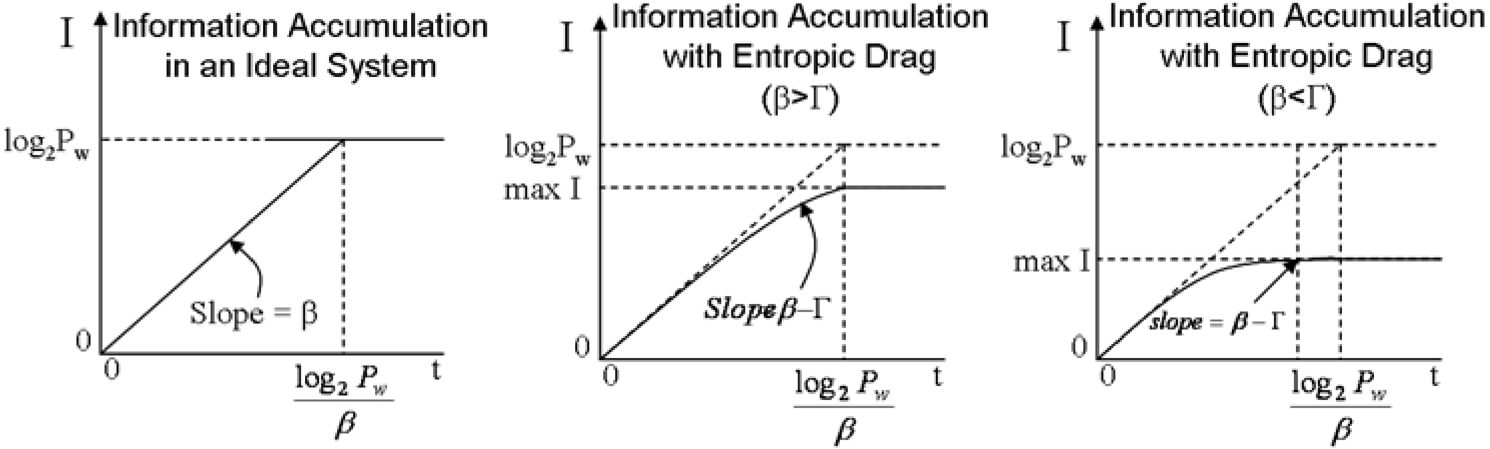
Information Channels

- Sensors can provide continuous streams of information, called Information channels.
- Information channels are measured in bits/second
- Entropic Drag decays the aggregate information and limits the total amount of possible knowledge

$$I = I_0 + \int_0^{\frac{I_m}{\beta}} \left(C(t) - \Gamma(t)\right)dt$$

$$t_{\max} \geq \frac{\log_2(P_w)}{\beta}$$

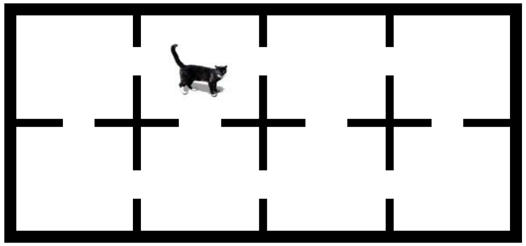
$$\max I = \log_2 P_w - \Gamma\left(\frac{\log_2(P_w)}{\beta}\right)$$



Minimizing Uncertainty
Back to Cat World

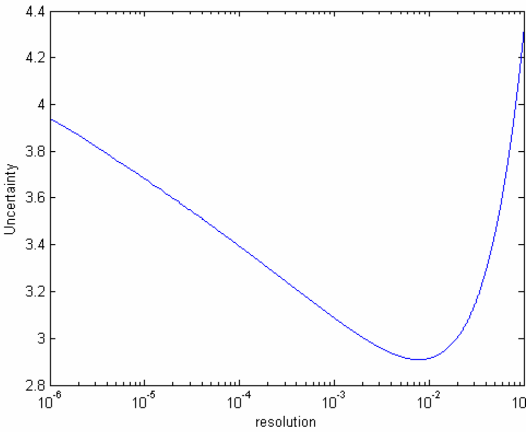
How many bits should we use to represent the cat?

- Room only? (3 bits)
- ft^2 (8 bits)
- ft^2 + orientation (11 bits)



A fundamental trade-off exists between Representational Uncertainty and Uncertainty derived from Entropic Drag

An optimal level of fidelity exists. Any attempt to measure the system more precisely will result in more information being lost from Entropic Drag than is than gained from the observation



$$U = U_r + \hat{r} \cdot |P_f|$$

$$U_r = |S_t| \cdot \varepsilon_r$$

$$U = |S_t| \cdot \varepsilon_r + \hat{r} \cdot |P_f|$$

$$\delta = \frac{\log_2 |P_w|}{\beta} = \frac{|K|}{\beta} \cdot \log_2\left(\prod_{x \in X} \left| \frac{x_i - x_0}{\hat{r}_x} \right| \right)$$

$$\delta = \frac{|K| \cdot n}{\beta} \cdot (\log_2 a - \log_2 r)$$

$$P_f = |K| \int_0^{\delta} \vec{v}(t)\,dt$$

$$S_t = |K| \oint_{\delta} \vec{v}(t)\,dt$$

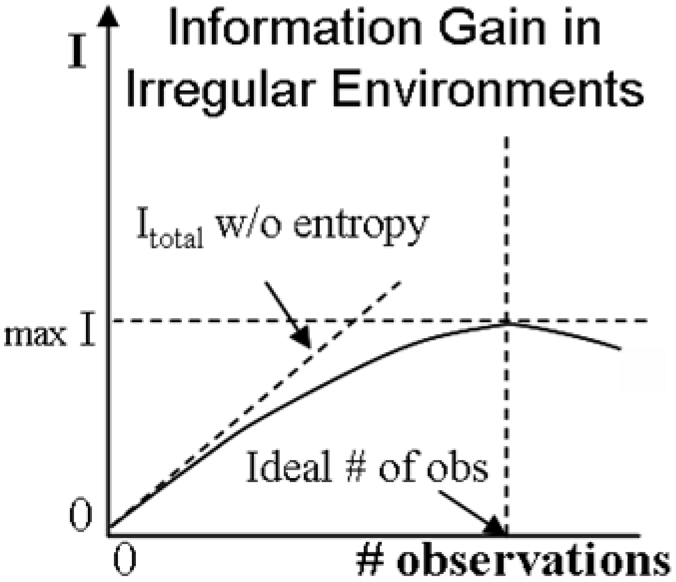
$$U_r = \frac{|K|\hat{r}\sqrt{n}}{2} \left| \oint_{\delta} \vec{v}(t)\,dt \right|$$

$$U = \frac{|K|\hat{r}\sqrt{n}}{2} \oint_{\delta} \vec{v}(t)\,dt + \hat{r} \cdot |K| \left| \int_0^{\delta} \vec{v}(t)\,dt \right|$$

Information Gain in Irregular Environments

“Yes, I looked in the ...” **“Did you look in the ...?”**

• Not all observations and actions are created equal
• The information bandwidth from well coordinated surveillance strategies diminishes over time
• In dynamic environments entropic drag dictates an optimal amount of observations. Additional observations will cause a net loss of information



Why do we care?

- Sensor Uncertainty >> Entropic Uncertainty
- 2.4 GHz data rates are common
- Terahertz communications are emerging
- I don’t need to track my cat to the nearest nanometer

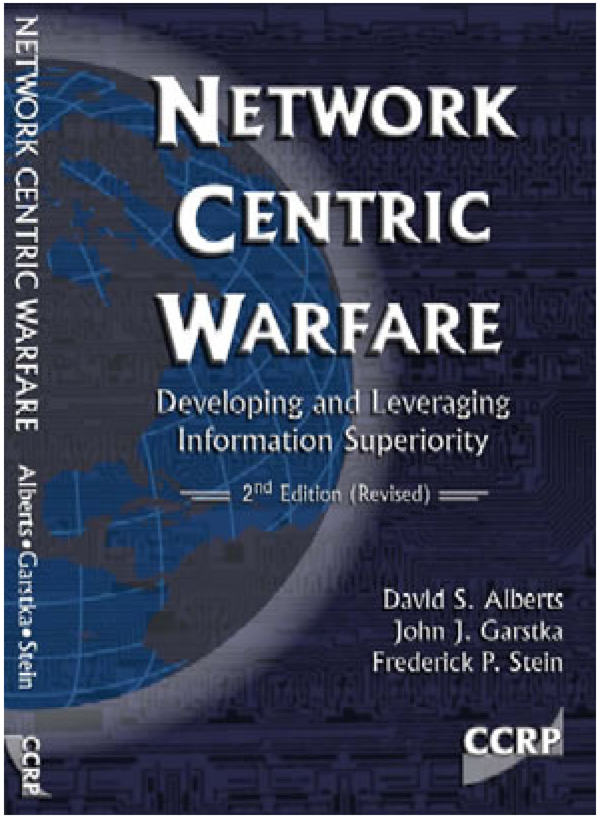
In most real-world applications the impact of entropic drag is minimal. It only becomes significant when the combination of the rate of unpredictable change and complexity of the environment become large.

Why we care

- In most real-world applications the impact of entropic drag is minimal. It only becomes significant when the combination of the rate of unpredictable change and state space of the environment become large compared to the available bandwidth

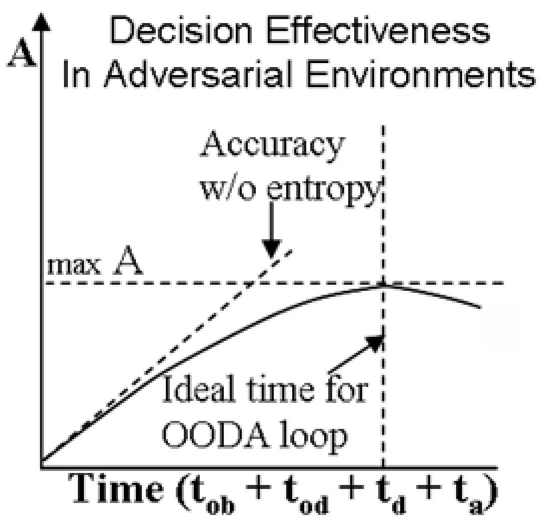
- The state space increases exponentially with the number of unpredictable dimensions

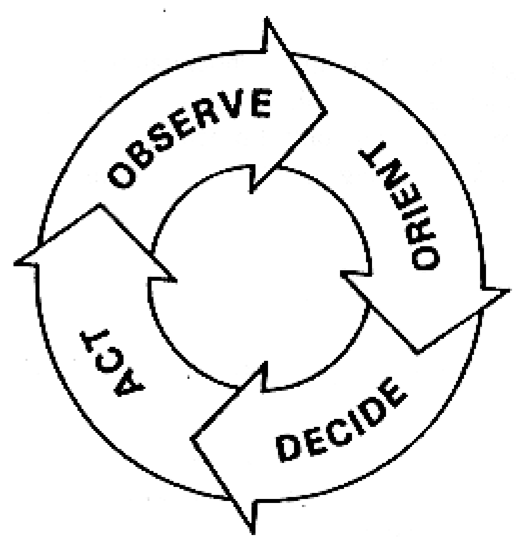
- Entropic Drag impacts those applications that:
  - Change rapidly
  - Change unpredictably
  - Require the tracking of many elements



What about Decide and Act?

1. The more detailed we attempt to make a command decision, the longer it takes to make the decision.
2. The longer it takes to make a decision, the more the premises upon which the decision is made are likely to change.
3. The more the premises upon which a decision are made change, the less effective the decision.





Decision Functions

- For Signal-following Control:

1. Time to decide is fixed
2. Output quality is dependent upon input quality (which is impacted by entropic drag)
3. Correctness is dependent upon complexity

- For Predictive Control:

1. Time to decide is dependent upon complexity
2. Output quality is dependent upon input quality AND time dedicated to decision processing
3. Correctness is dependent upon time dedicated to decision processing

Networks

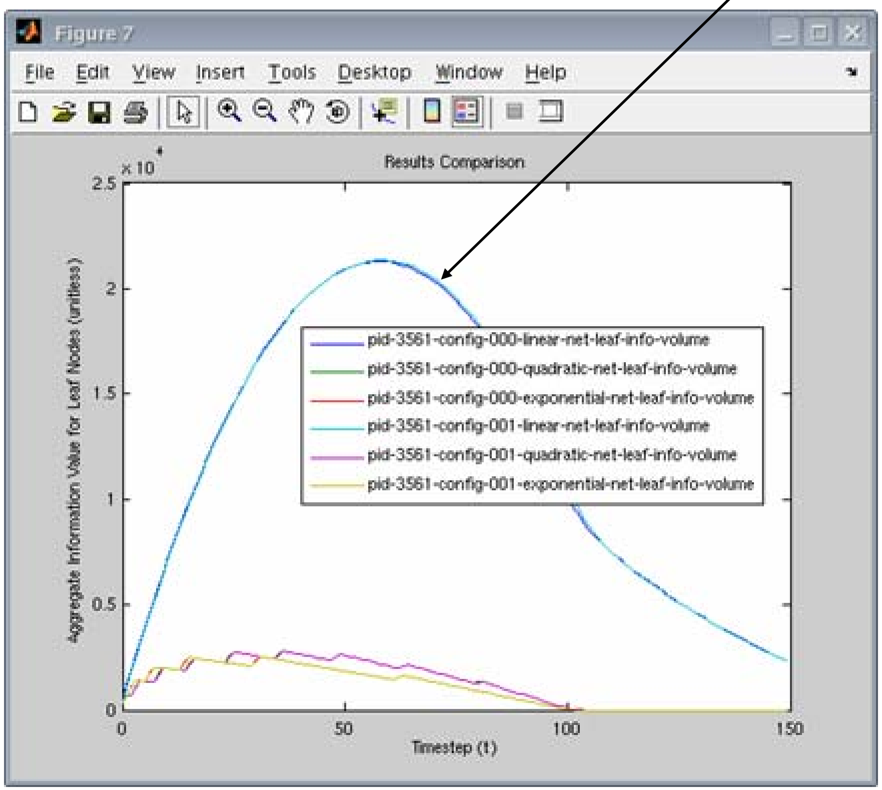
When communities are collaborating, the amount of information that is known both individually and collectively is impacted by entropic drag along the following principles:

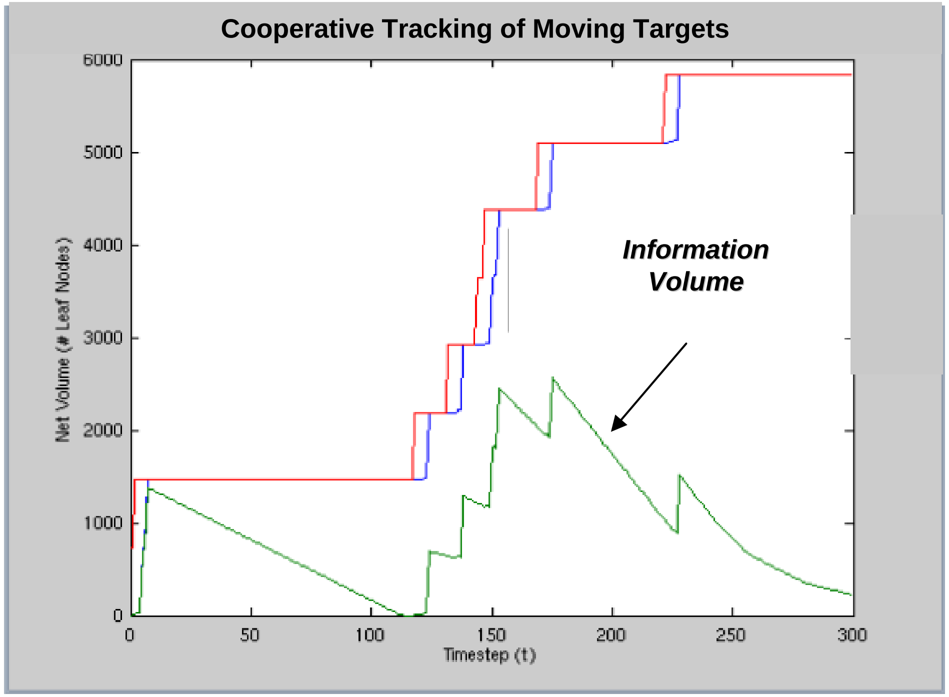
- The greater the collaboration, the greater the entropic drag
- The more centralized the network, the greater the entropic drag
- An optimal scoping of information distribution and collaboration exists for any given environment (e.g. comprehensive information sharing does not provide the most information across the community)

Networks that are optimized for pair-wise distribution of information (e.g. scale-free power-law networks such as the internet) are not optimal networks for collaboration

Networks

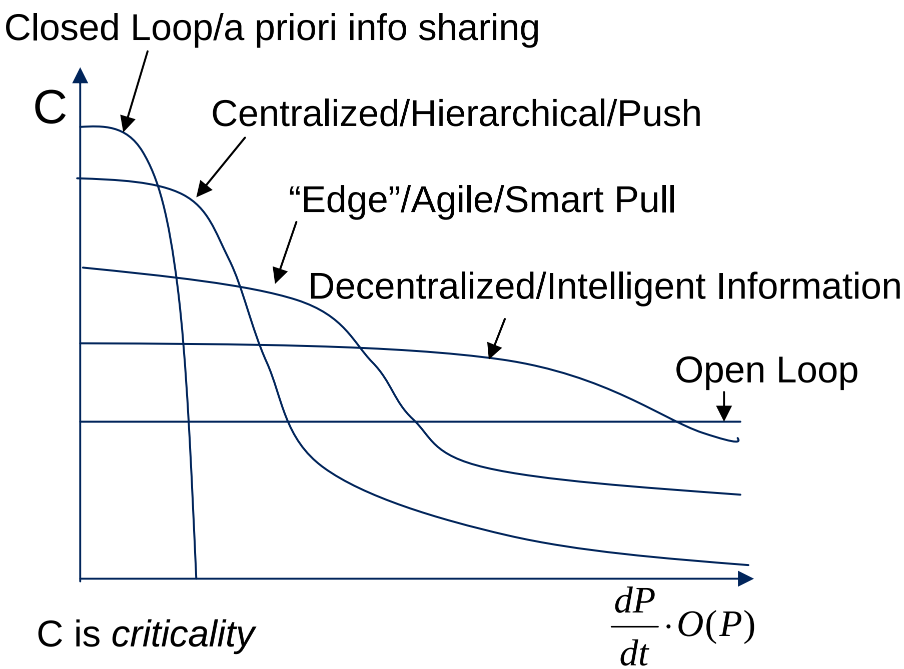
“traffic jam”





Networked Decision Communities

- When communities are collaborating, the amount of information that is known both individually and collectively is impacted by entropic drag along the following principles:
  - The greater the collaboration, the greater the entropic drag
  - The more centralized the network, the greater the entropic drag
  - Optimal scoping of information distribution and collaboration exists for any given environment (e.g. comprehensive information sharing may not provide the most information across the enterprise)



How do we devise an experiment to prove this hypothesis?

Future (Ongoing) Work

- Entropic Reduction Through Inference (Orientation)
- Mathematical Study Control Theoretic Dual
  - Taxonomy of Decision-Making Techniques
  - Game Theoretic vs. Passive Environments
- Topology Impact on Entropy
- Adaptive C2
- Real-World Examples